

Constraining massive binary formation and evolution from compact object mergers

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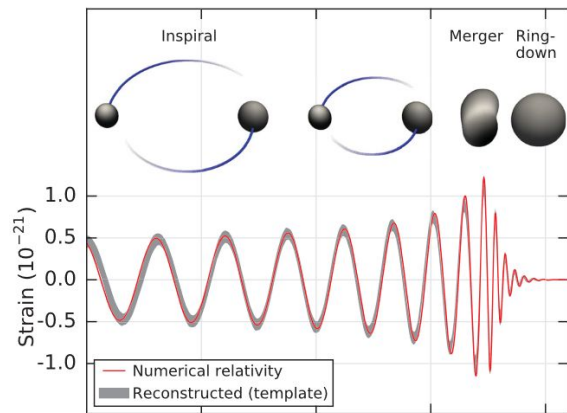
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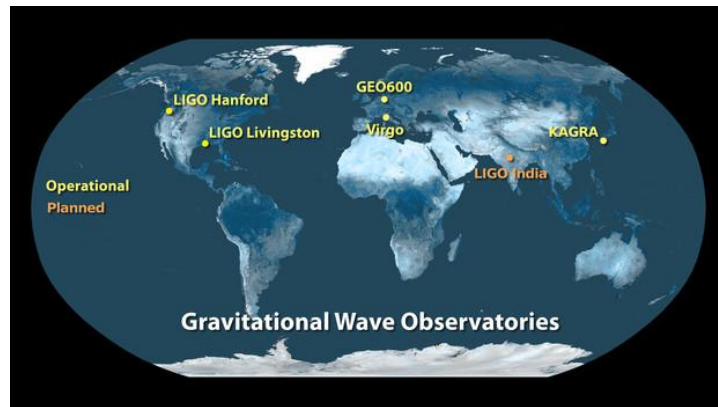
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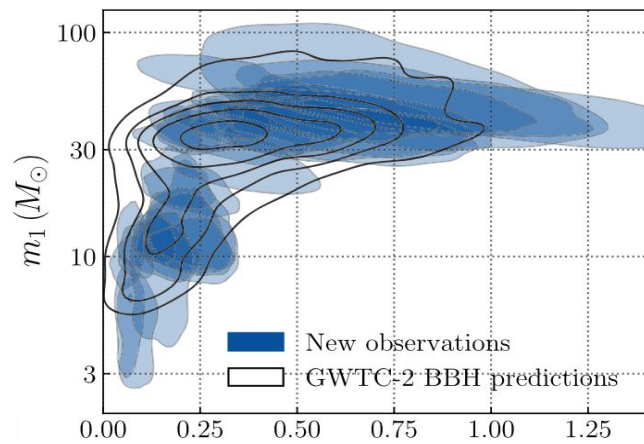
The challenge of forming CBMs



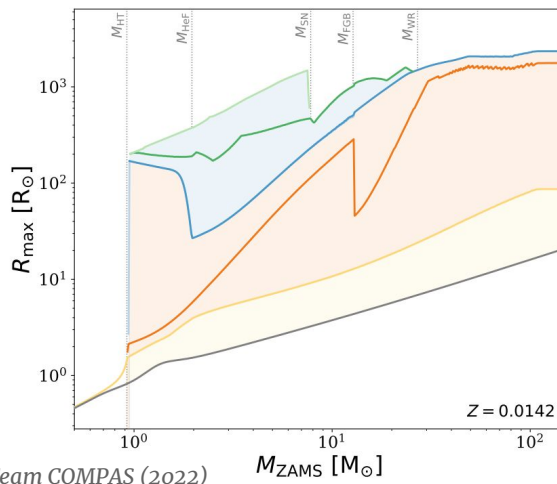
LV Collaboration (2016)



Current GW interferometers are capable of detecting the coalescence of BCOs, descended from massive binaries, and are particularly sensitive to BBHs.



LVK Collaboration (2023) z

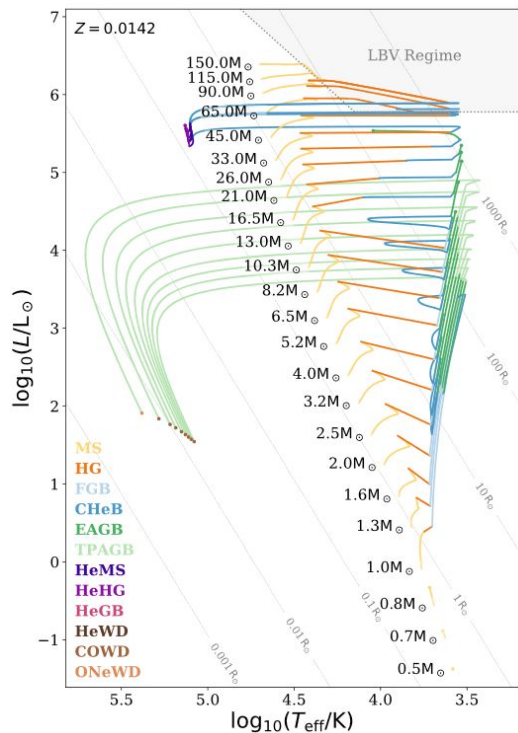


Team COMPAS (2022)

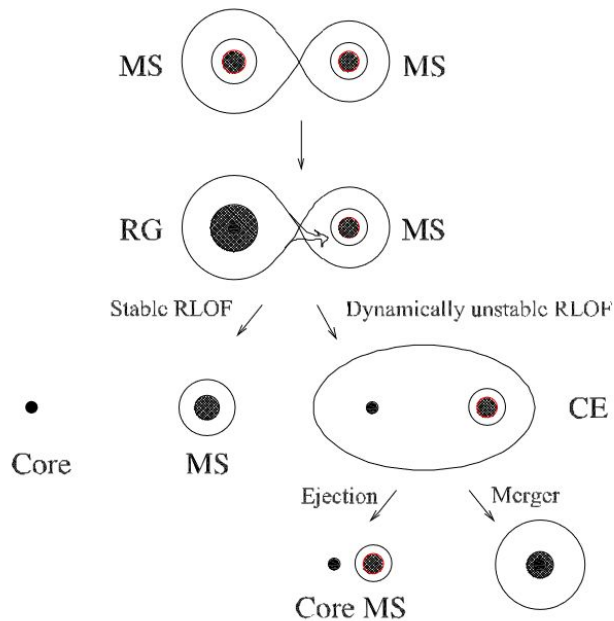
Massive binaries present a fundamental challenge for binary evolution: how to fit massive stars within an orbit that will lead to merger within a Hubble time.

Isolated binary evolution: population synthesis (BPS)

Simple fits and prescriptions to stellar/binary evolution.



Team COMPAS: Riley+2022



Han+2020

BPS trades individual precision for **statistical results**.

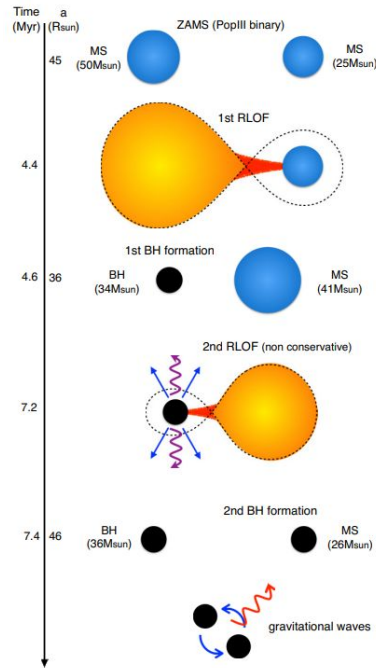
- Detailed evolution: ~ 1 h per binary
- BPS: ~ 0.1 s per binary

Simplified models are very imprecise but allow **mapping out uncertainties** in other domains.

Isolated binary evolution: avoiding expansion

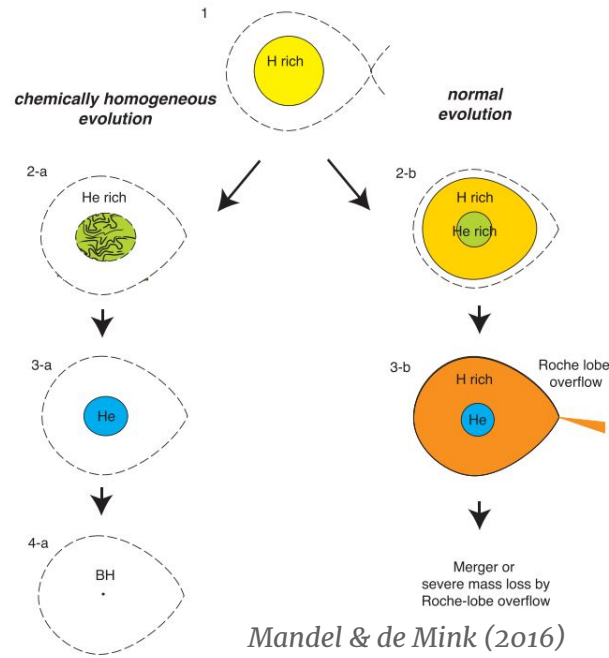
Alternatively, it could be possible to avoid expansion altogether.

Population III stars



Inayoshi et al. (2018)

Chemically homogeneous evolution (CHE)

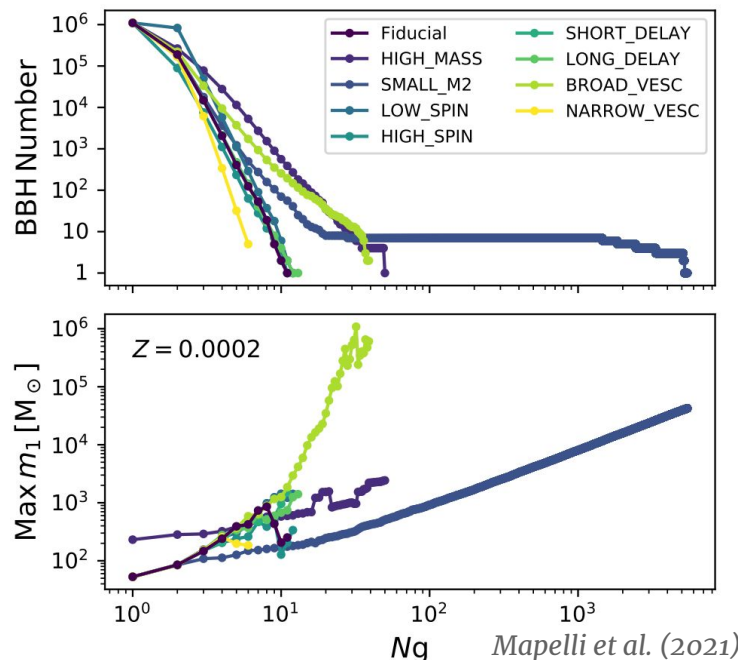
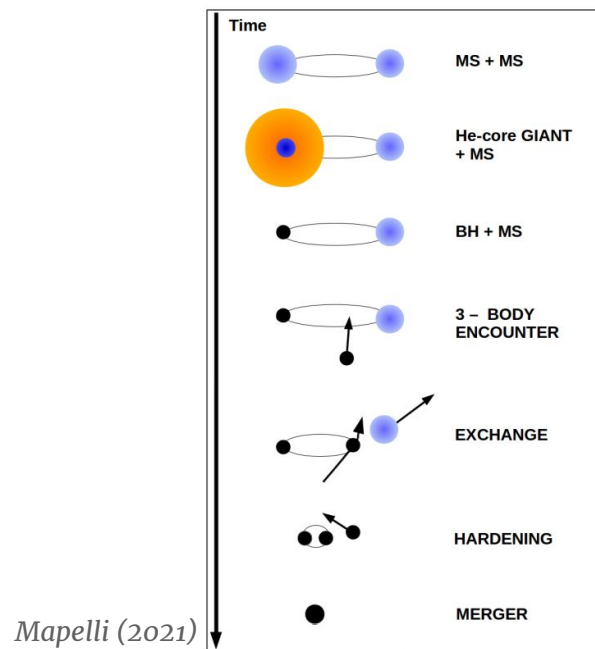


Mandel & de Mink (2016)

These channels **disfavor mass loss**, and are good candidates for producing massive BHs*.

An alternative: dynamical formation

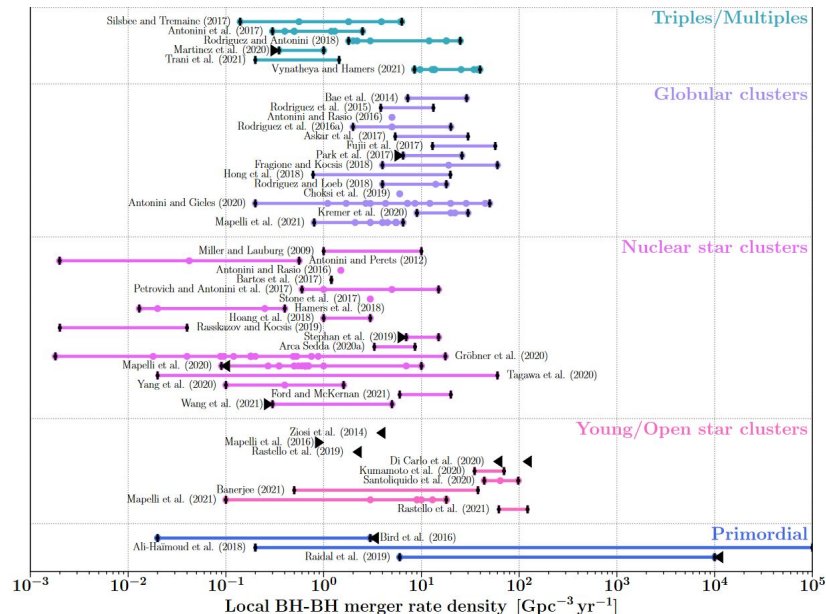
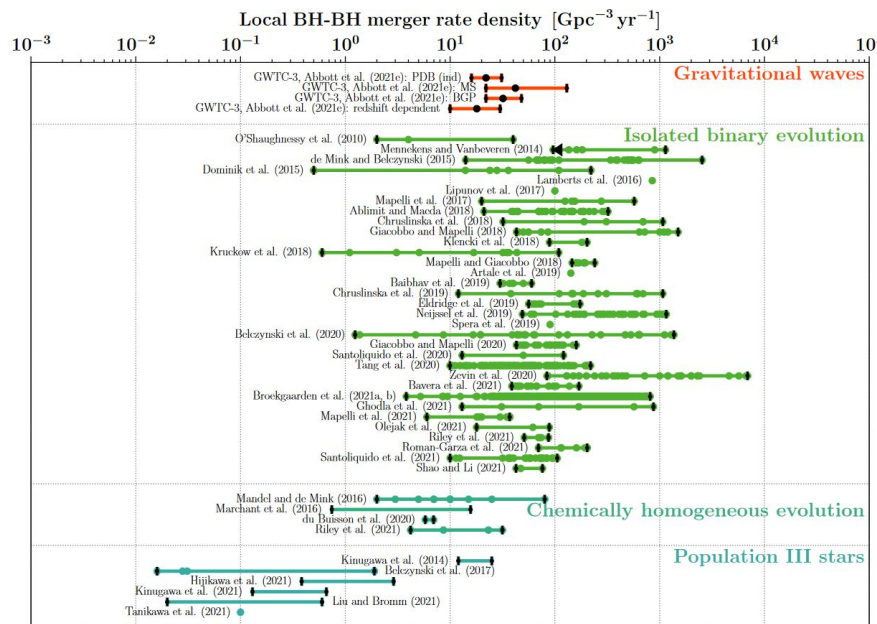
Encounters in dense environments (e.g., GCs, YSCs) can eject lighter components, pair massive components and harden orbits.



Similar effects can be present in **higher-order multiples**. Hierarchical mergers are candidates for producing massive BHs, and possibly IMBHs.

The big picture

Current uncertainties still yield widely varying predictions from different channels. As such it is difficult to establish firm constraints on models for particular stages.

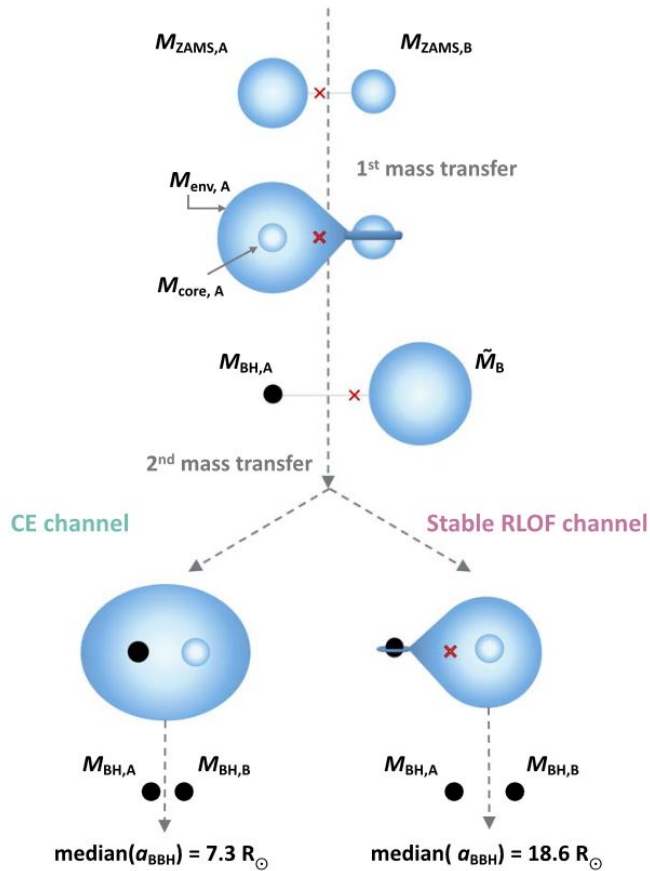


Mandel & Broekgaard (2022)

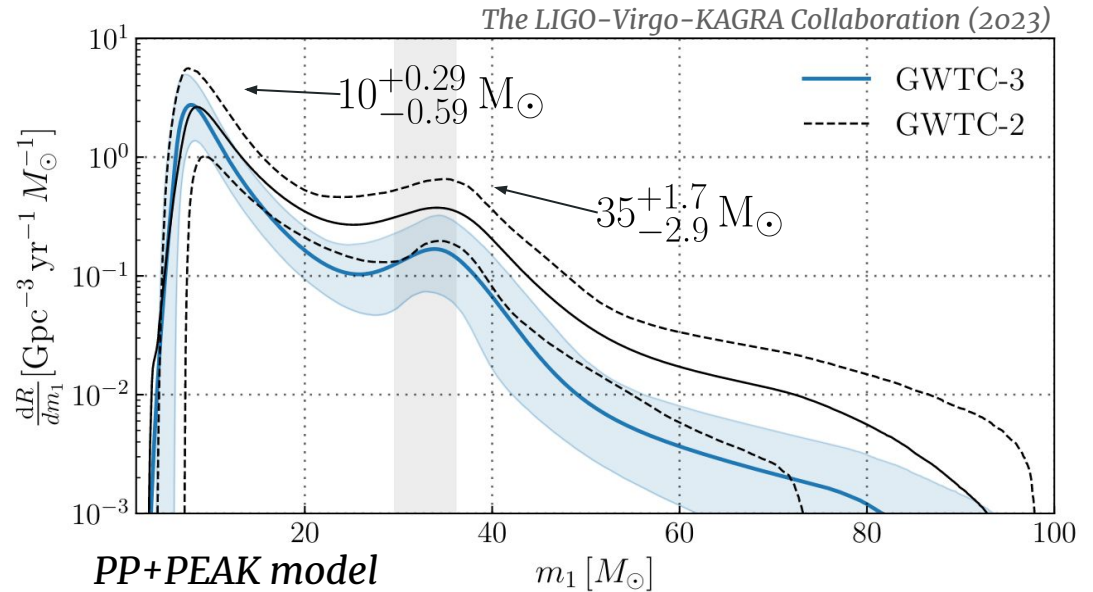
Difficult, but not impossible. The case of mass transfer provides an ongoing example.

The case of mass transfer

Mass transfer

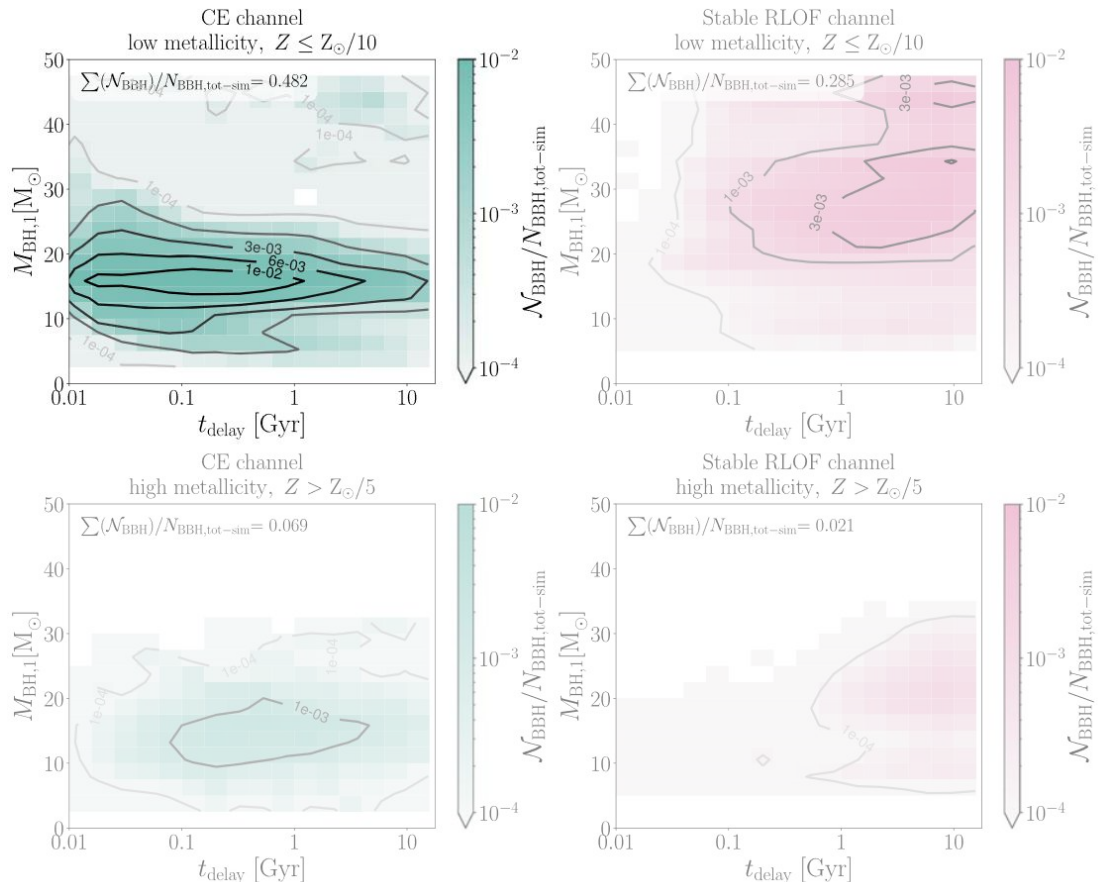


Both stable RLOF and CE can shrink the orbit. However, they are expected to robustly produce **different** systems which might leave an **imprint** in their properties.



Our main guide is the **primary mass distribution** of BBH mergers, as characterized in GWTC-3.

Mass transfer



Beyond mass, each channel yields characteristic delay times

Broadly, for BBHs from the CE channel,

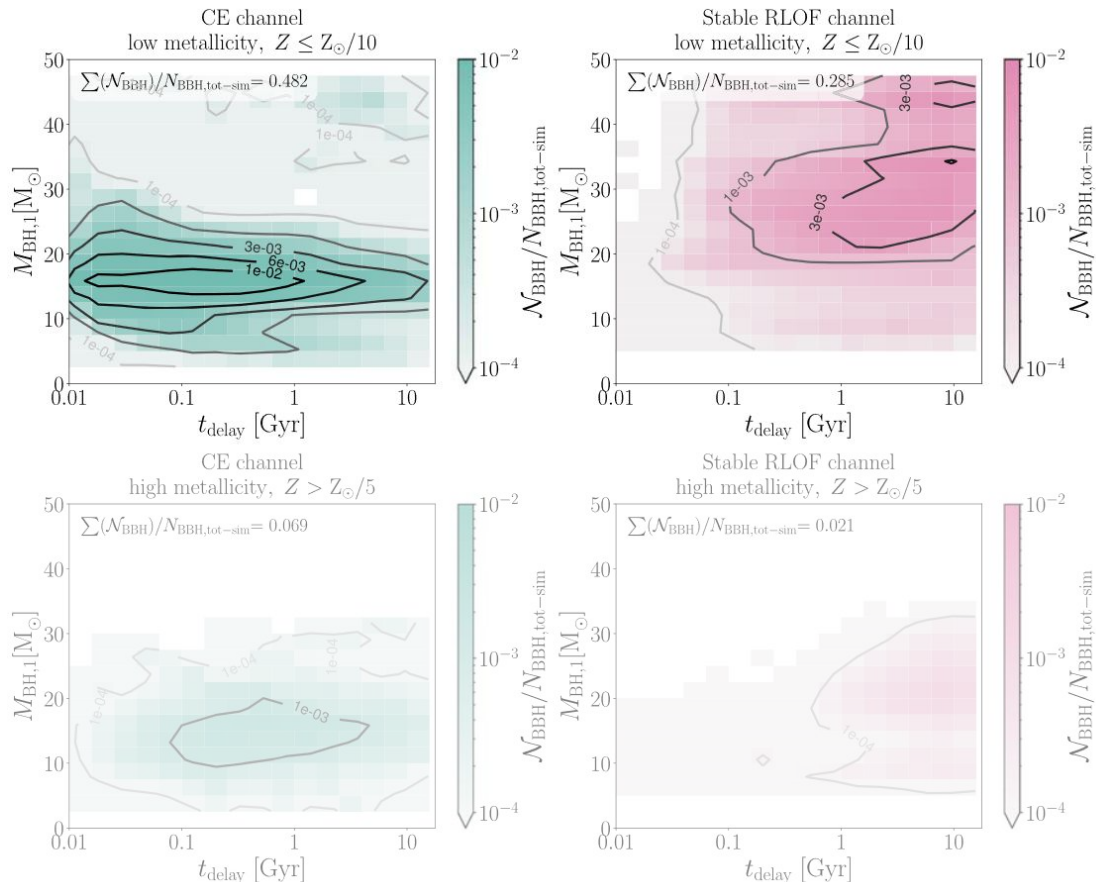
- $m_1 \sim 10-20 M_{\odot}$,
- $t_d \sim 0.01-1$ Gyr,

while for those from the stable RLOF channel,

- $m_1 \sim 20-50 M_{\odot}$,
- $t_d \sim 1-10$ Gyr.

Stellar winds suppress massive BH formation at high metallicity (Vink et al., 2001; Vink & de Koter, 2005).

Mass transfer



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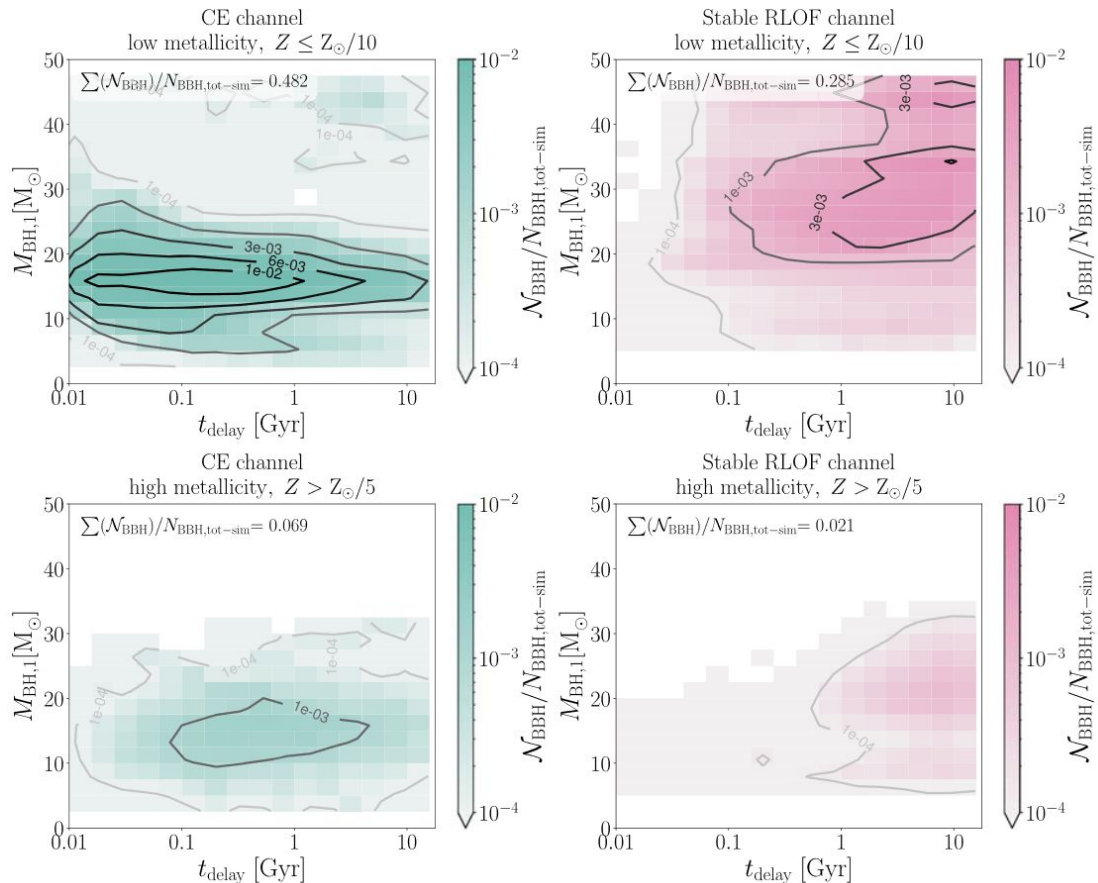
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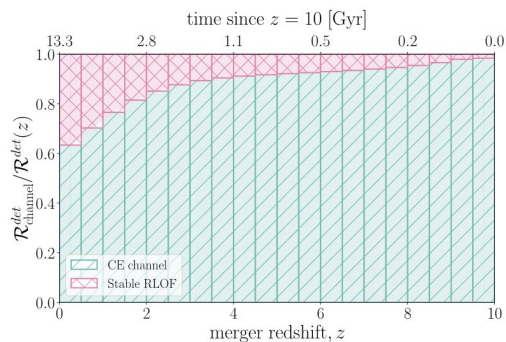
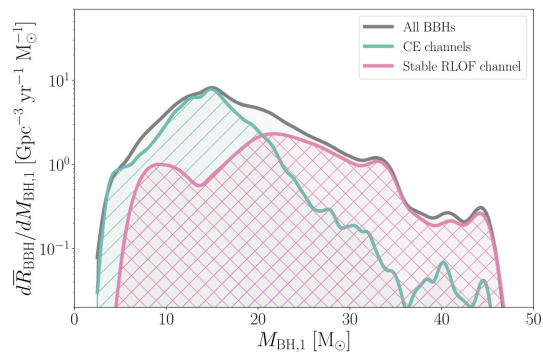
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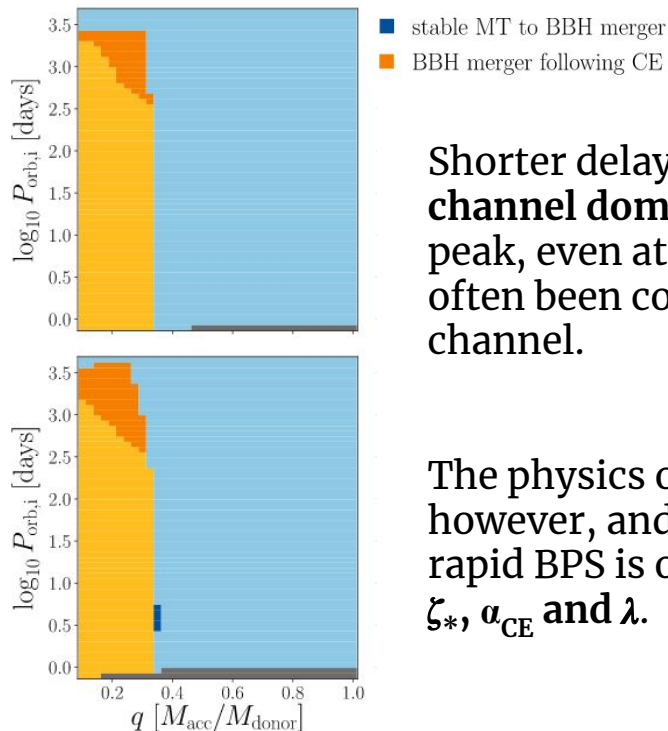
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Mass transfer



van Son et al. (2022)



- stable MT to BBH merger
- BBH merger following CE

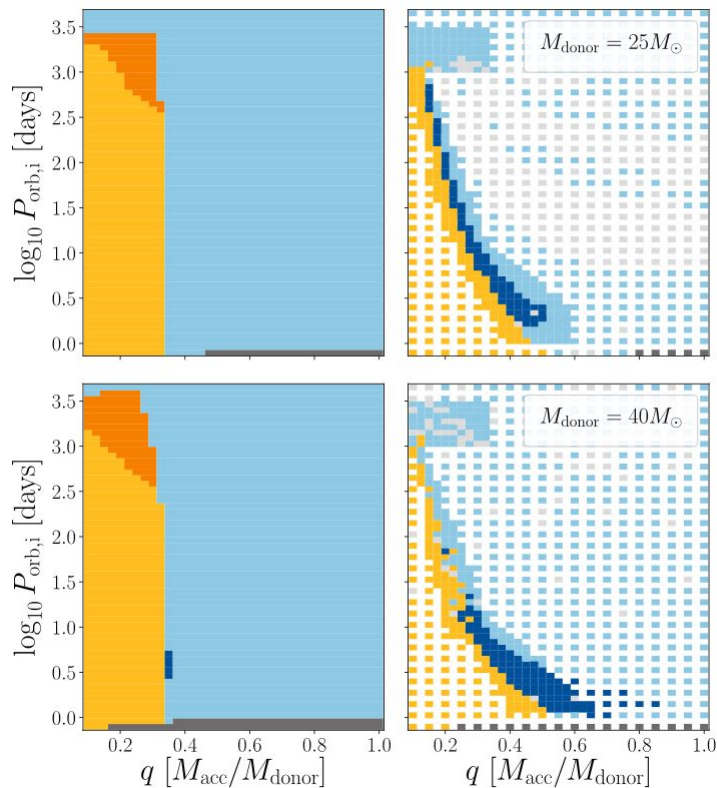
Shorter delay times tend to make the **CE channel dominant**, yielding no $10 M_{\odot}$ peak, even at lower redshift. This has often been considered the “classical” channel.

The physics of RLOF are highly uncertain, however, and their implementation in rapid BPS is often simplistic, with fixed ζ_* , α_{CE} and λ .

Gallegos-Garcia et al. (2021)

Mass transfer

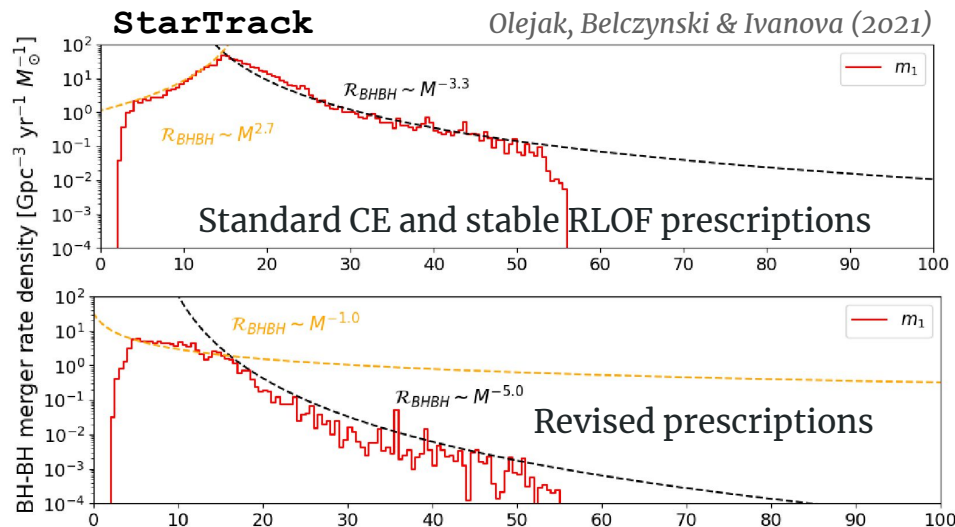
- merger during CE
- stable MT to BBH merger
- error
- wide binary
- RLOF during ZAMS
- BBH merger following CE



Gallegos-García et al. (2021)

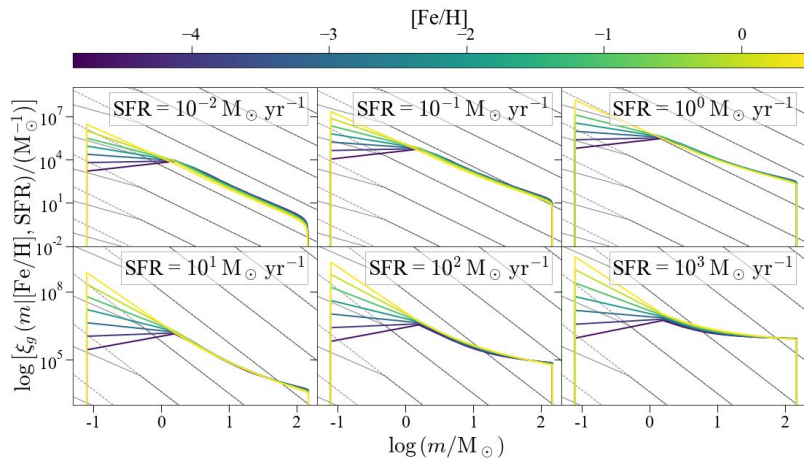
Improved prescriptions and detailed simulations increasingly suggest that the CE contribution has been **overestimated**, and that **stable RLOF** might in fact be the dominant formation channel for BBH mergers.

However, that is not the only factor in setting the balance between channels.



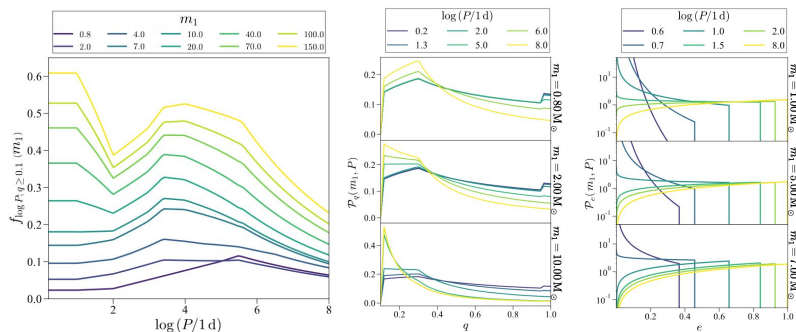
CBMs over cosmic time

Working backward: stellar formation



Different channels access different regions of the initial parameter space. These are affected by

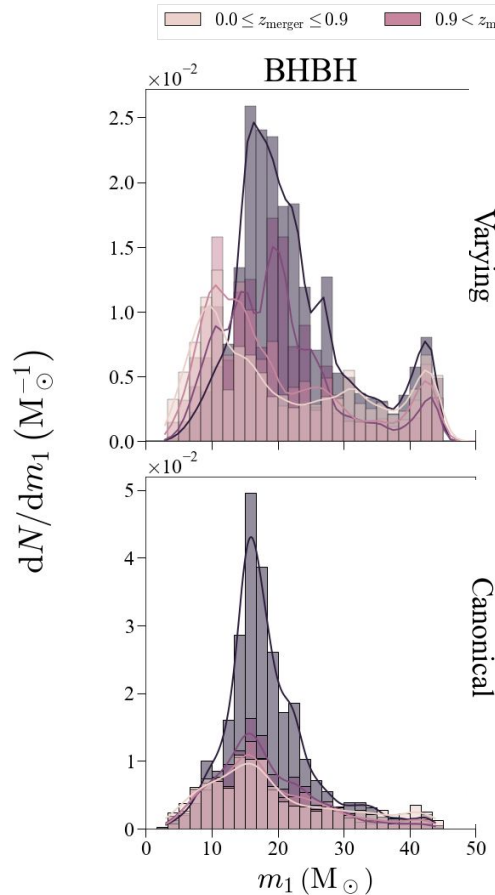
- Initial mass function (IMF),
 - Universal (Kroupa, 2001) x SFR- and -metallicity dependent (Jerabkova et al. 2018).
- Orbital parameters: P , q and e .
 - Uncorrelated (Öpik, 1924; Sana et al., 2012) x Correlated (Moe & Di Stefano, 2017).
- The metallicity-specific cosmic star formation history (Chruślińska et al, 2019, 2020).



de Sá et al. (submitted)

With COMPAS, we compared BBH merger yields from the **Canonical** and **Varying** distributions. 16

On the masses



In the **Canonical** model, the distribution is always dominated by $\sim 15 M_\odot$.

In the **Varying** model, it is

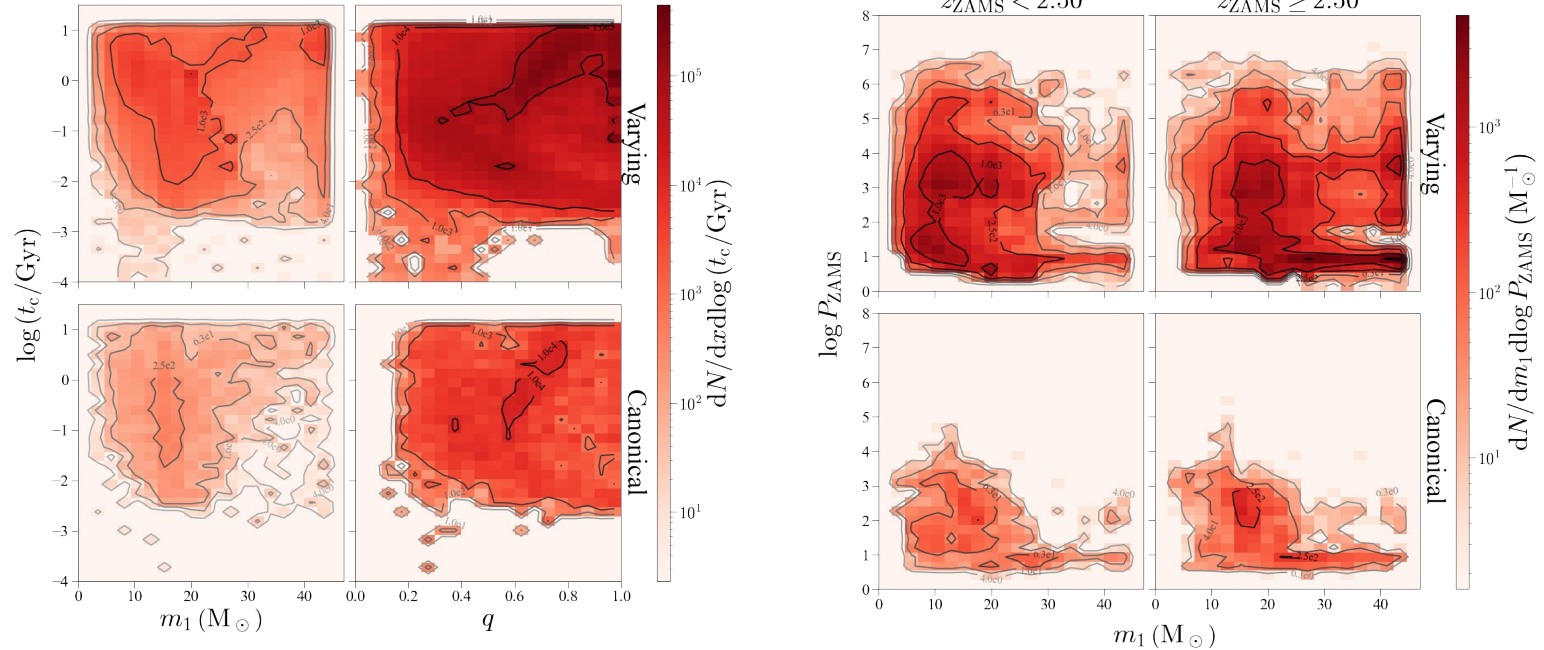
- Dominated by $\sim 15 M_\odot$ for $z_{\text{merger}} > 1.6$,
- Dominated by $\sim 10 M_\odot$ for $z_{\text{merger}} < 1.6$,
- Characterized by a growing high-mass tail at lower z_{merger} ,
- Characterized by the $\sim 45 M_\odot$ PPSiNE at all z_{merger} .

The high-mass tail binaries being generated at high z_{ZAMS} but merging at low z_{merger} suggests long coalescence times.

But how do the orbital parameters set this behavior?

On the formation channels

Each formation channel seems to map particular regions of the initial parameter space to the final parameter space. The stable RLOF channel seems to favor shorter initial periods.



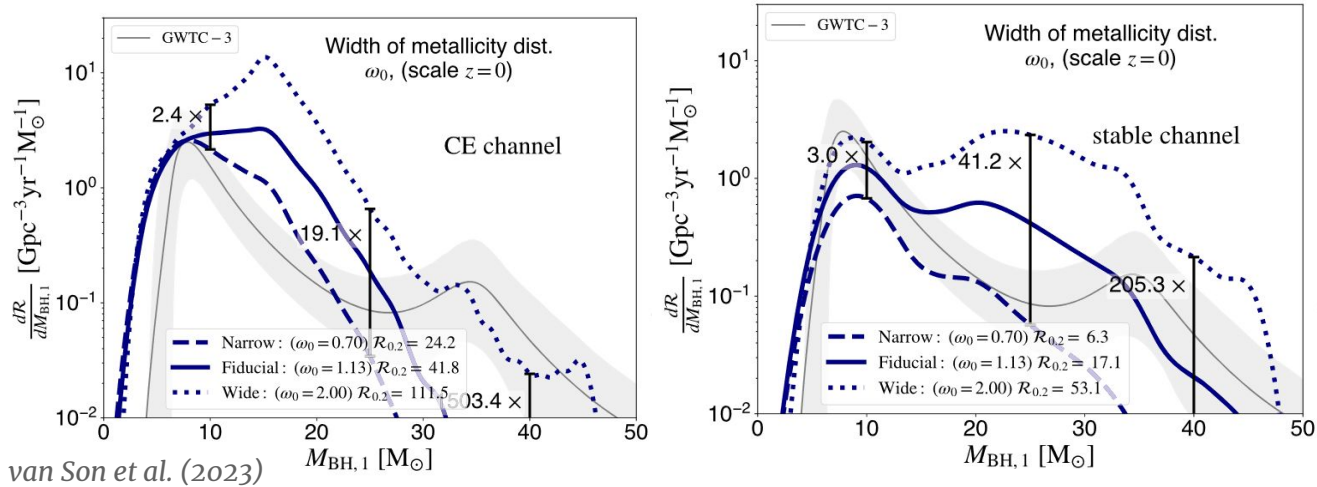
For BBHs, our results indicate that the formation channels are **robust against variations of the initial conditions**. The Varying model favors the **stable RLOF channel**.

Initial conditions X Evolution models

We propose a framework for studying model variations in full, where, in terms of the final parameter space,

The set of evolution models sets the **location of key features**, and the set of initial condition models their **relative weights**.

This has already been verified for BBHs and SFRD variations.



- Initial condition uncertainties can have a similar impact to evolution uncertainties,
- A clear delineation of roles would help with simultaneous constraining.

Conclusions

To take home...

While binary population synthesis is a highly **degenerate** problem, the fine tuning necessary to produce CBMs means that we can track **well-defined formation channels**.

For BBHs, in terms of evolution models, in classical isolated binary evolution,

1. **Mass transfer** is the key phase,
2. The **stable RLOF and CE channels** have distinctive yields,
3. The BBH detections by LVK are helping direct updated models for **MT stability and CE**,

In terms of initial conditions,

1. **Initial conditions** matter because formation channels map particular regions of the initial parameter space,
2. We suggest initial condition permutations set the **relative weights** of key features of final parameter distributions,

There are many other important elements (SNe, winds, pulsations, multiplicity...), but there is **a framework that makes constraining them possible**.

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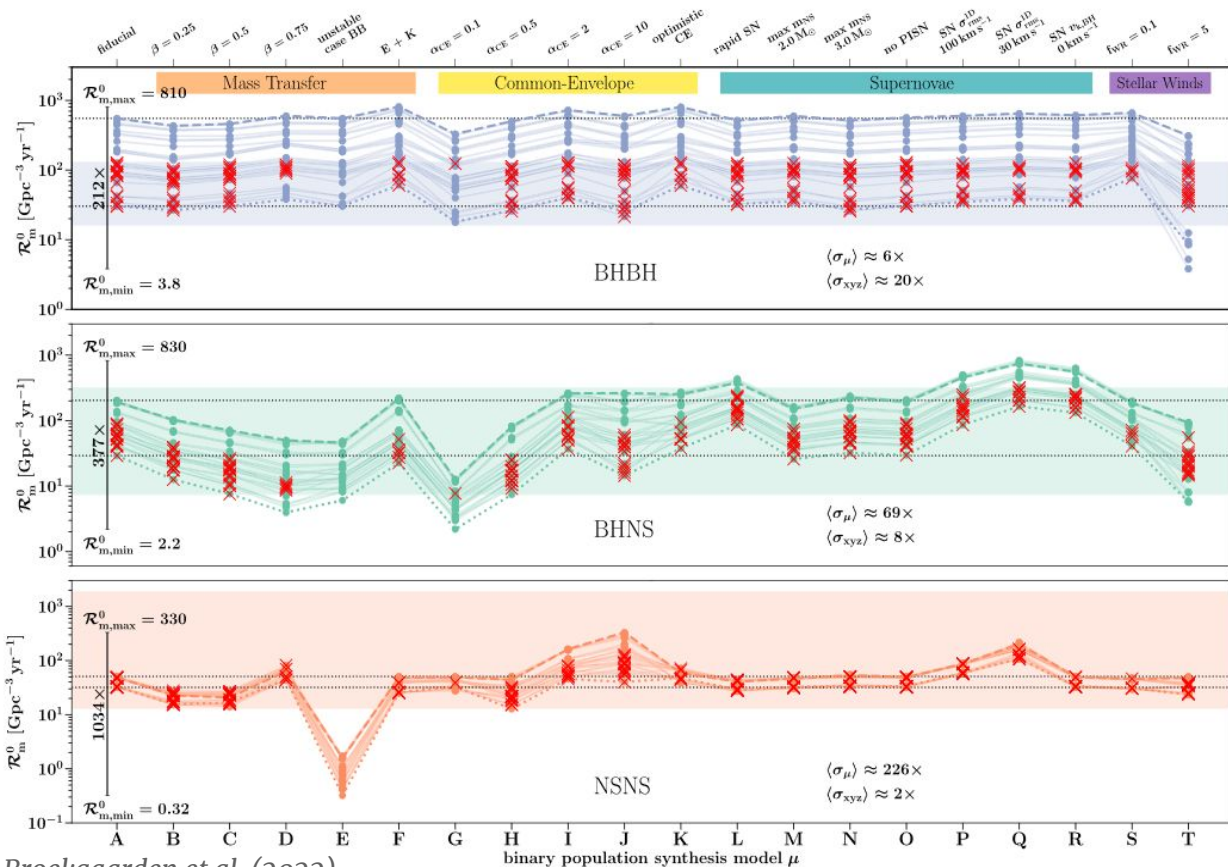
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Thank you!

Appendix

Other examples of constraining massive binary evolution

The limitations of population synthesis



Broekgaarden et al. (2022)

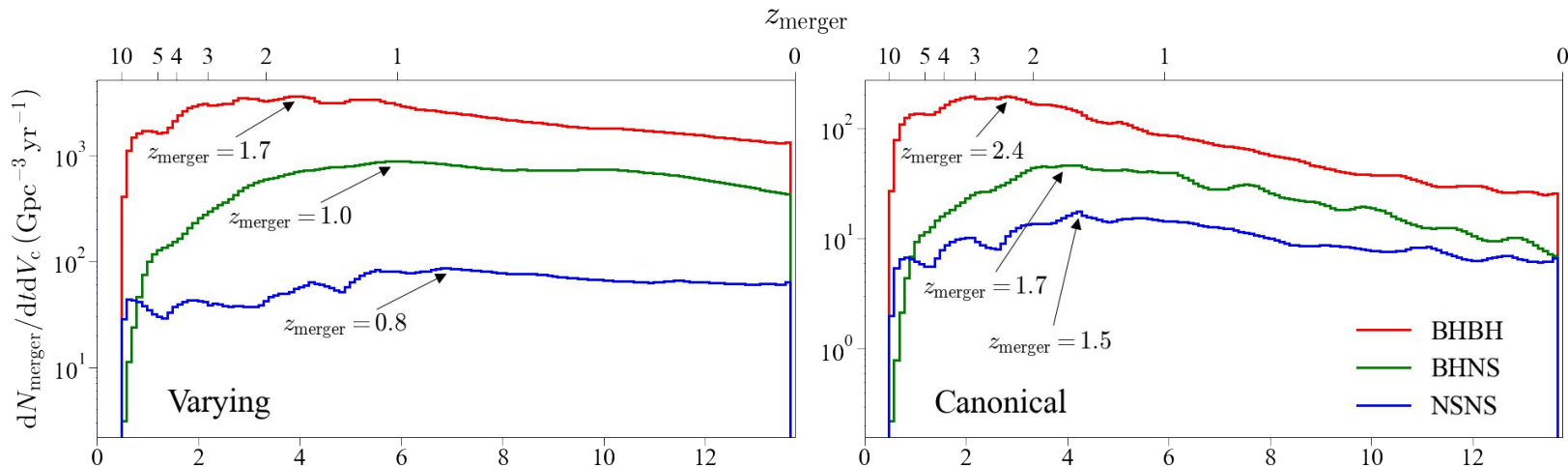
Population synthesis takes us from ZAMS to merger for a set of assumed models, but constraining evolution models from populations is a highly degenerate problem.

Large model grids are useful in finding common patterns, but less so in constraining individual models.

Because the final population is a convolution of many processes, it is important to identify specific formation channels.

On the merger rates

Comparing local merger rates suggests that uncertainties in the initial conditions have an impact comparable to evolution uncertainties.



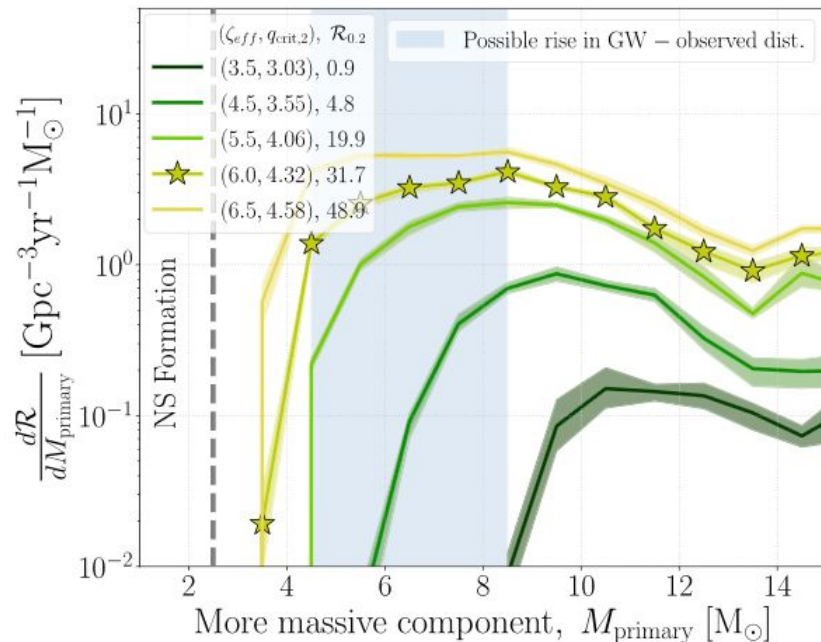
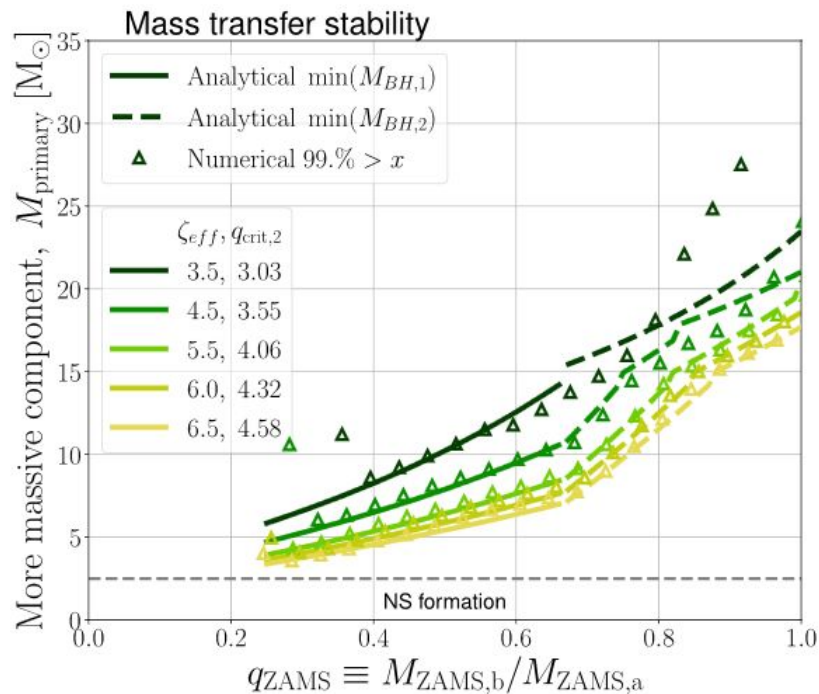
Source	Local rate density ($\text{Gpc}^{-3} \text{yr}^{-1}$)		
	BHBH	BHNS	NSNS
Varying	1314.4	432.5	64.0
Canonical	25.5	7.0	6.6
GWTC-3 (90%)	16–61	7.8–140	10–1700

Age_{Universe}@Merger (Gyr)

- Merger rates follow the shape of the SFR, which is flatter in the Varying model.
- The Varying model *vastly* overestimates BH production — BHBH and BHNS rates are **100 times greater**.

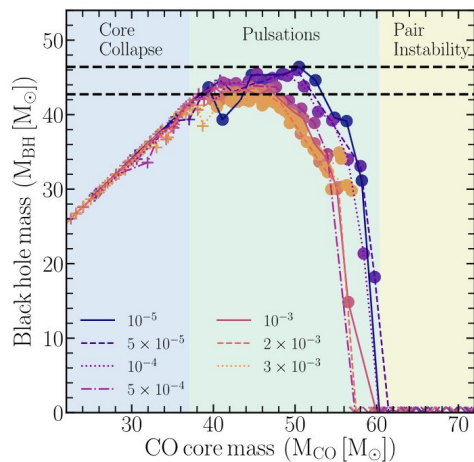
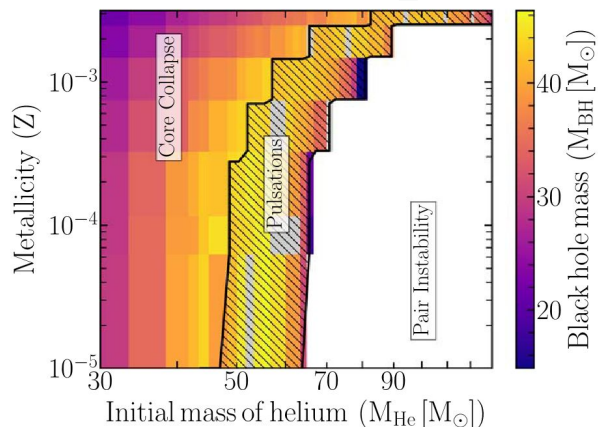
Mass transfer: the lower mass gap

One particular region of the mass spectrum might further constraint the balance between the stable and unstable MT channels: the **lower mass gap**.



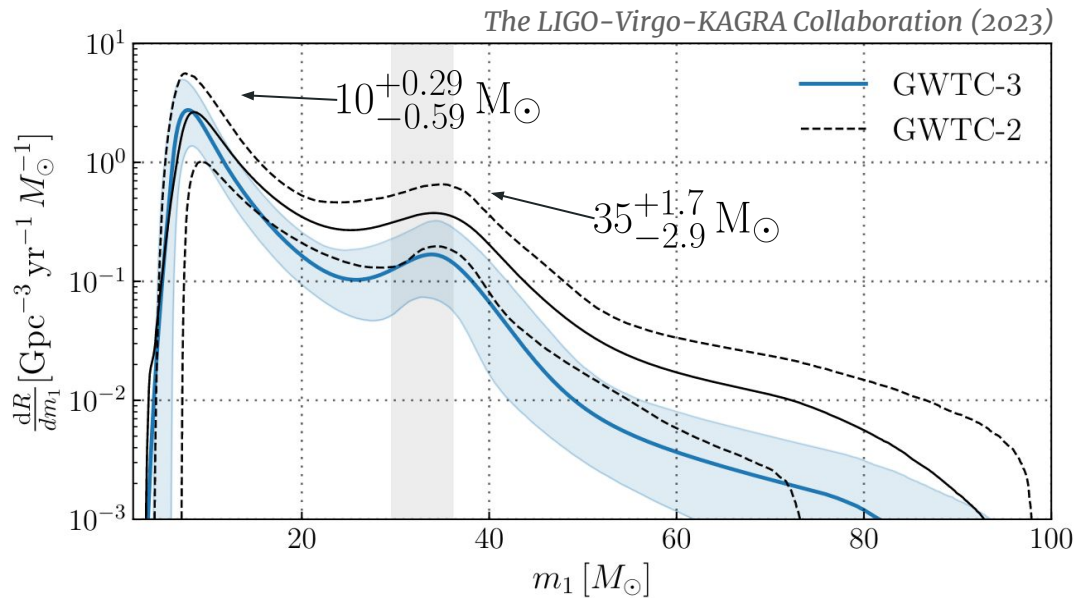
van Son et al. (2022)

Pulsational pair-instability SNe

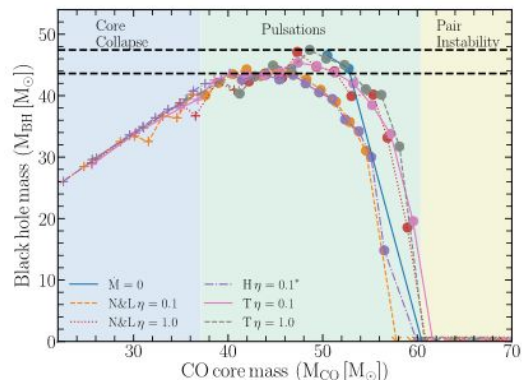


CBMs can naturally probe the **upper mass gap**, which is thought to start at the **PPISNe pile-up**.

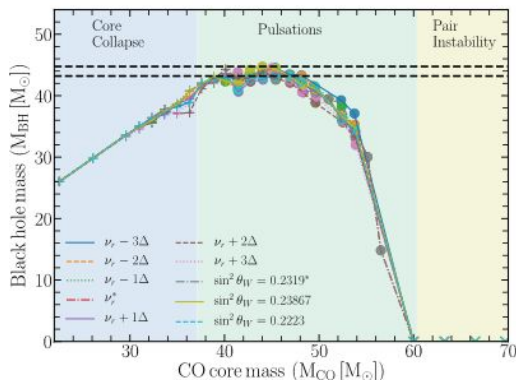
There is, however, no evidence of a $\sim 45 M_{\odot}$ pile-up in the empirical distribution.



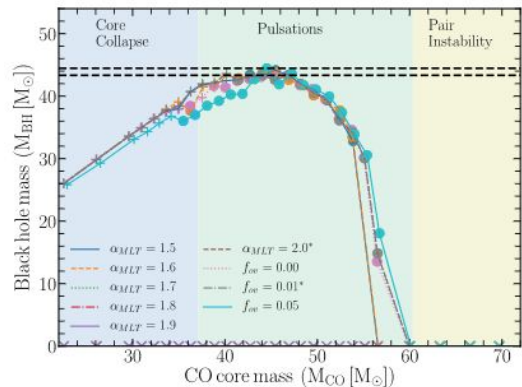
Pulsational pair-instability SNe: the upper mass gap



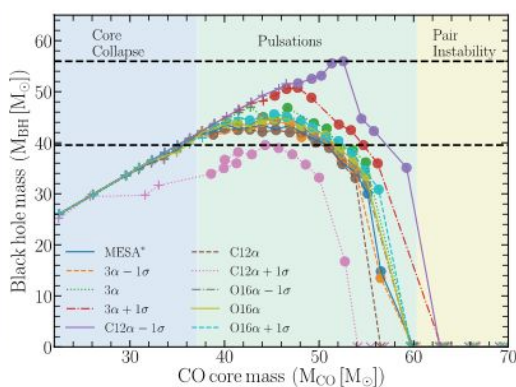
(a) Winds



(b) Neutrinos



(c) Mixing



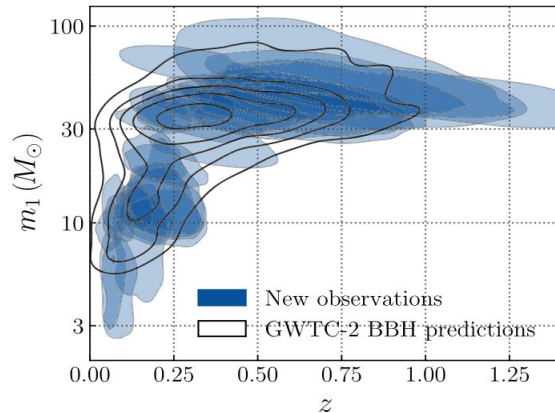
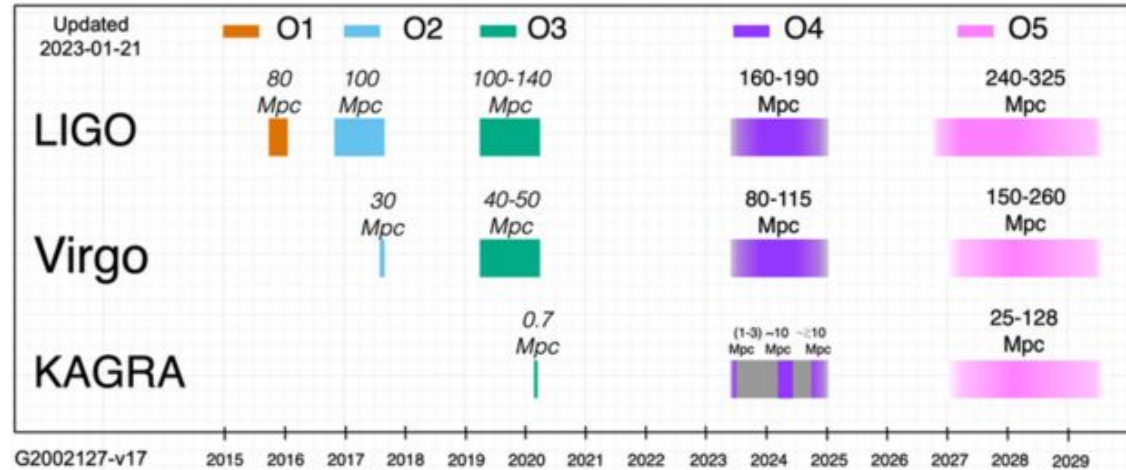
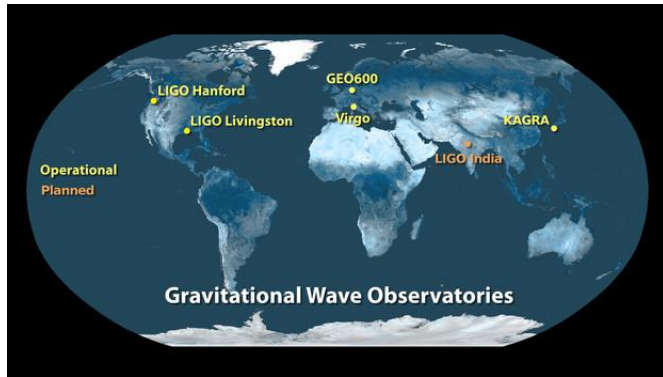
(d) Nuclear reaction rates

Mounting evidence against PPSiNe as the source of the 35 M_{\odot} bump has stimulated the investigation of other means of forming massive BBHs, such as **hierarchical mergers and CHE**.

This has left us in the situation where the peak was motivated by PPSiNe, yet the **only peak is unexpected and cannot be explained by PPSiNe**.

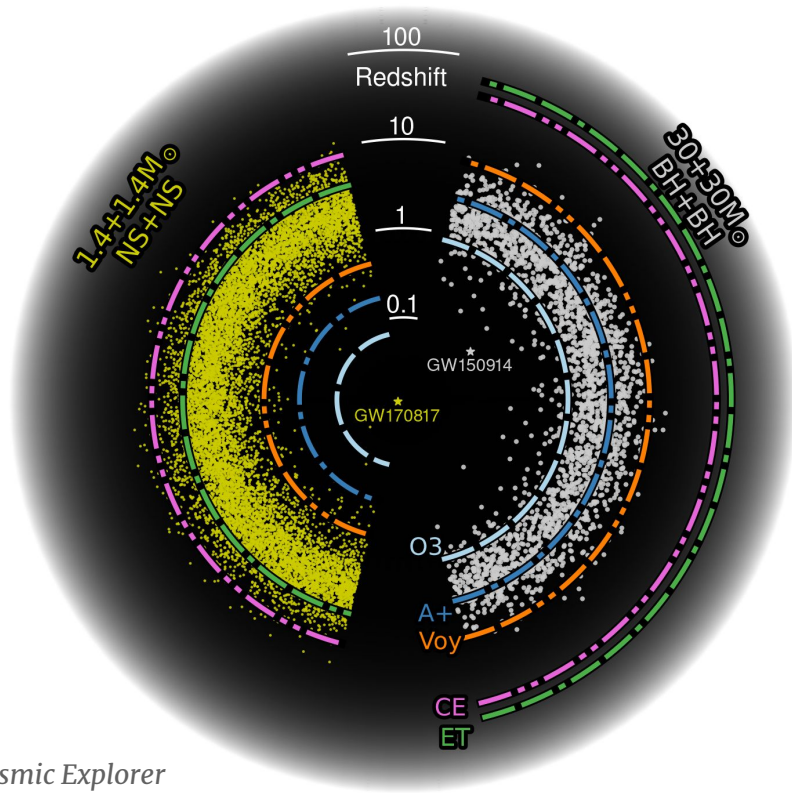
Extra introduction slides

Gravitational-wave astronomy: where we are and where we are going

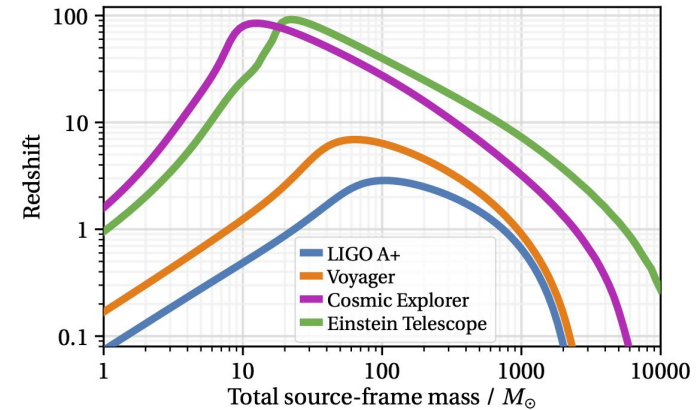


In GWTC-3 there are 90 BHBH detections, but only 4 BHNSs and 2 NSNSs. We expect more from all cases in O4, but **more sensitive detectors** are still needed.

Gravitational-wave astronomy: where we are and where we are going



Cosmic Explorer



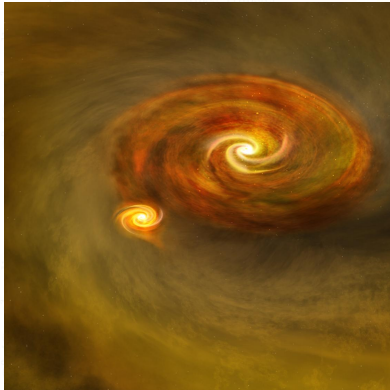
Third generation detectors are expected to observe all BHBH mergers and most NSNSs up to and beyond $z = 10$.

Due to long coalescence times, many merging binaries may be as old as star formation.

Compact mergers tell us about how binaries evolve and how they formed in the past.

How we do it: constraining uncertainties

Most work how so far has evaluated **binary evolution** uncertainties. We are now starting to extend the same treatment to **initial conditions**.



B. Saxton, NRAO/AUI/NSF



Team COMPAS: Riley+ (2022)

For fixed evolution models, test impact of different initial conditions on the compact merger population up to $z = 10$.

- IMF,
 - Orbital parameters,
 - Multiplicity,
 - Environment.
- **Question #1:** How much do stellar formation uncertainties affect compact mergers?
 - **Question #2:** Which model permutations can we rule out?
 - **Question #3:** How does this play out with evolution uncertainties?

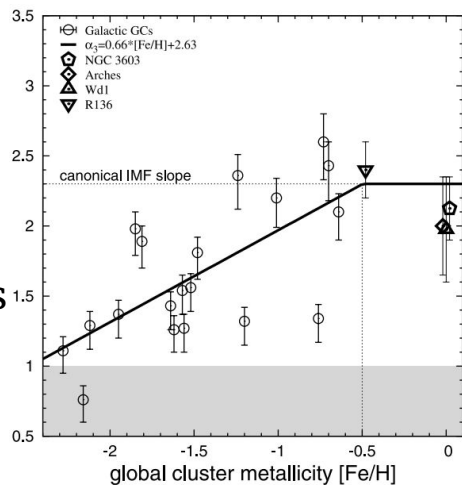
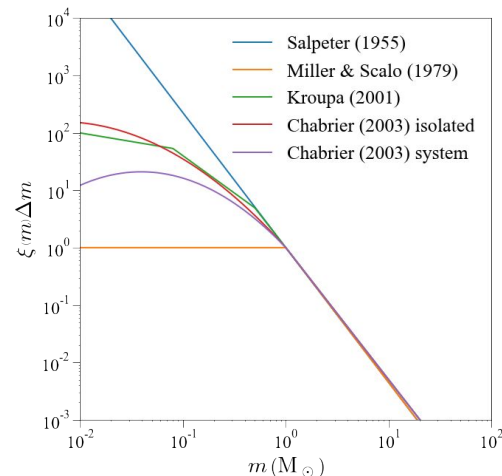
Stellar formation uncertainties

The Initial Mass Function (IMF)

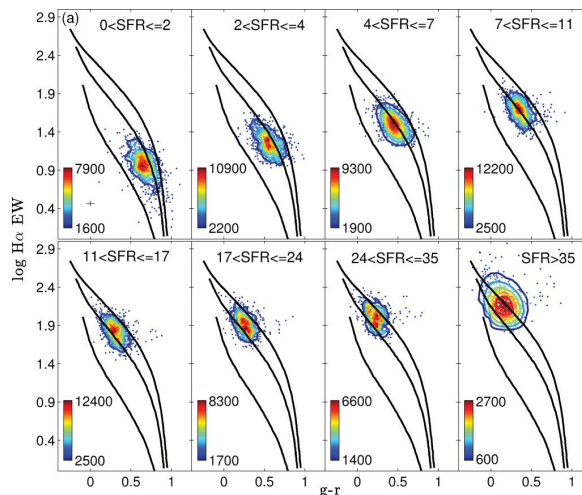
- Usual choices: the Salpeter (1955) IMF and its descendants.
- Theoretical challenges mainly from the **Jeans mass**,

$$M_J \propto \rho^{-1/2} T^{3/2}$$

- Lower metallicity \rightarrow less efficient cooling \rightarrow higher M_J
 - Higher SFR \rightarrow heating from massive stars \rightarrow higher M_J
- Observational support is recent, but we now have **testable models**.



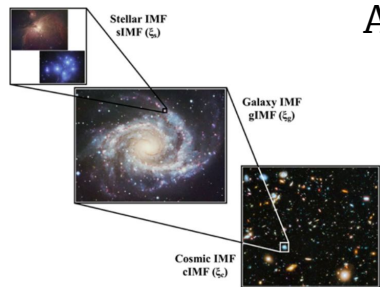
Marks+2012
top-heavy at
low metallicities



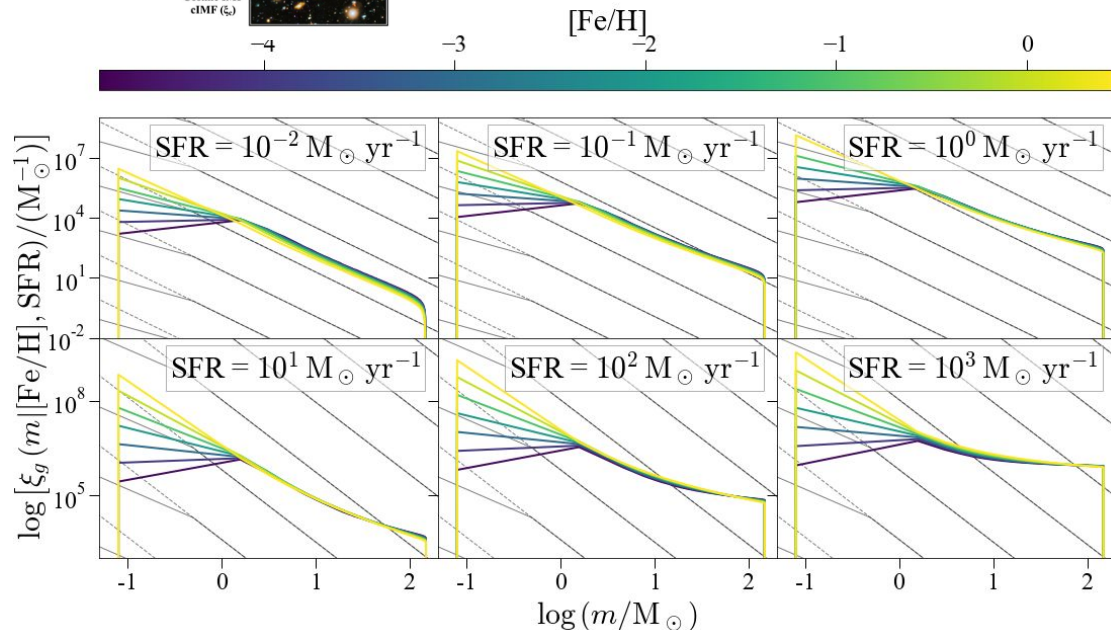
Gunawardhana+2011
top-heavy at high SFRs

The Initial Mass Function (IMF)

Alt.: Integrated galaxy-wide IMF theory (IGIMF, *Kroupa&Weidner2003*)



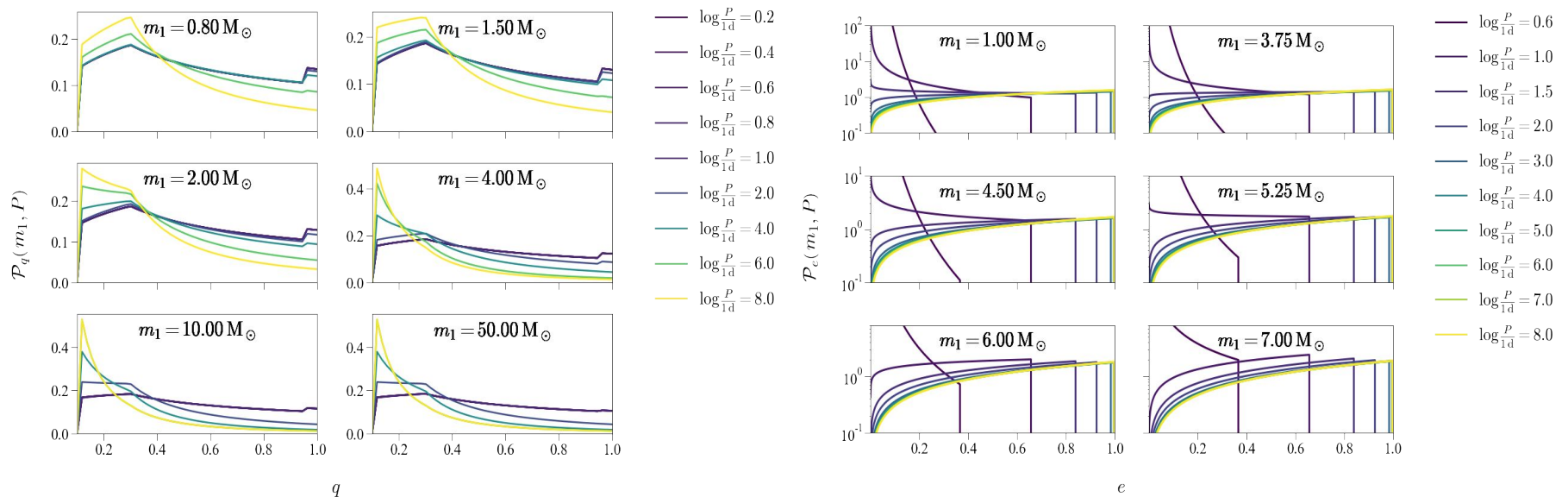
$$\underbrace{\xi_g(m_* | Z, \text{SFR})}_{\text{galaxy IMF}} = \int_{\text{gal}} \underbrace{\xi_s(m_* | Z, M_{\text{ecl}})}_{\text{stellar IMF}} \underbrace{\xi_e(M_{\text{ecl}} | \text{SFR})}_{\text{cluster IMF}} dM_{\text{ecl}}$$



- In *Jerabkova+2018*, with fits from *Marks+2012* and *Gunawardhana+2011*.
- + Simplifying assumptions:
 - Chemically homogenous galaxy,
 - Single star formation epoch,
 - Constant SFR for 10 Myr.

Mass ratio and eccentricity at ZAMS

- Usual choices:
 - Uniform $0.01 \lesssim q \leq 1$ (Sana+2012),
 - Log-uniform $0.4 \leq \log(P/d) \leq 3$ (Öpik1924),
 - Circular orbits.
- From *Moe&Di Stefano2017*: pre-ZAMS evolution leads to piecewise correlated functions.

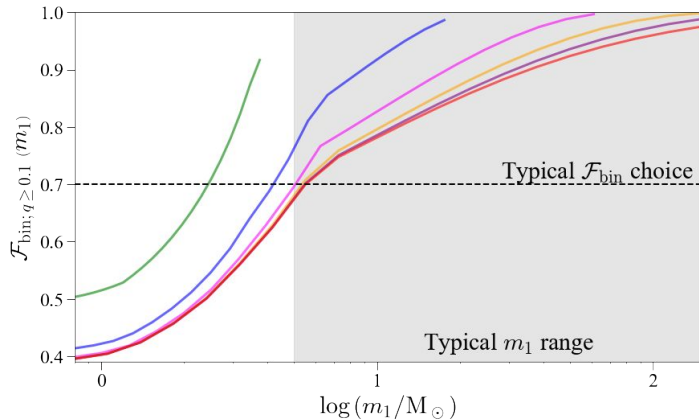


Orbital period at ZAMS and multiplicity

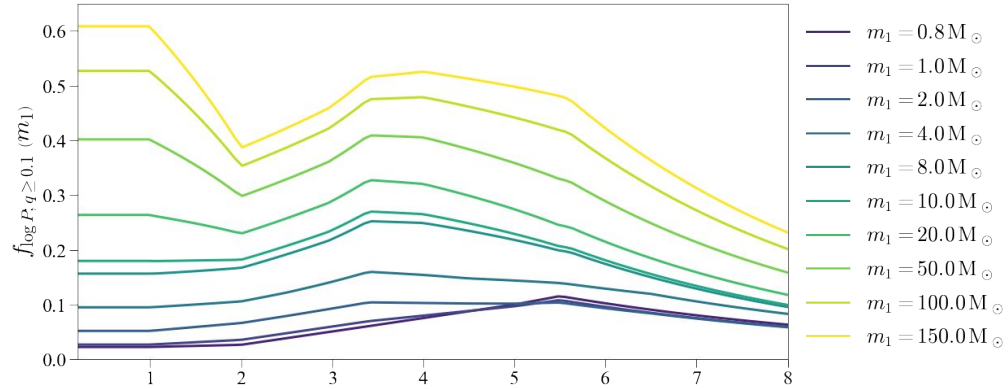
Moe & DiStefano2017: dominance of **higher-order multiples** for massive primaries.

Companion frequency,
not a straightforward probability distribution

$$f_{\log P; q \geq q_{\min}}(m_1) = \frac{dN_{\text{cp}}}{dN_1 d \log P}$$



- $n_{\text{cp}}^{\text{max}} = 1$
- $n_{\text{cp}}^{\text{max}} = 2$
- $n_{\text{cp}}^{\text{max}} = 3$
- $n_{\text{cp}}^{\text{max}} = 4, \langle \mathcal{F}_{\text{bin}} \rangle_{m_1 \geq 5} = 0.75$
- $n_{\text{cp}}^{\text{max}} = 5, \langle \mathcal{F}_{\text{bin}} \rangle_{m_1 \geq 5} = 0.74$
- $n_{\text{cp}}^{\text{max}} = 6, \langle \mathcal{F}_{\text{bin}} \rangle_{m_1 \geq 5} = 0.74$



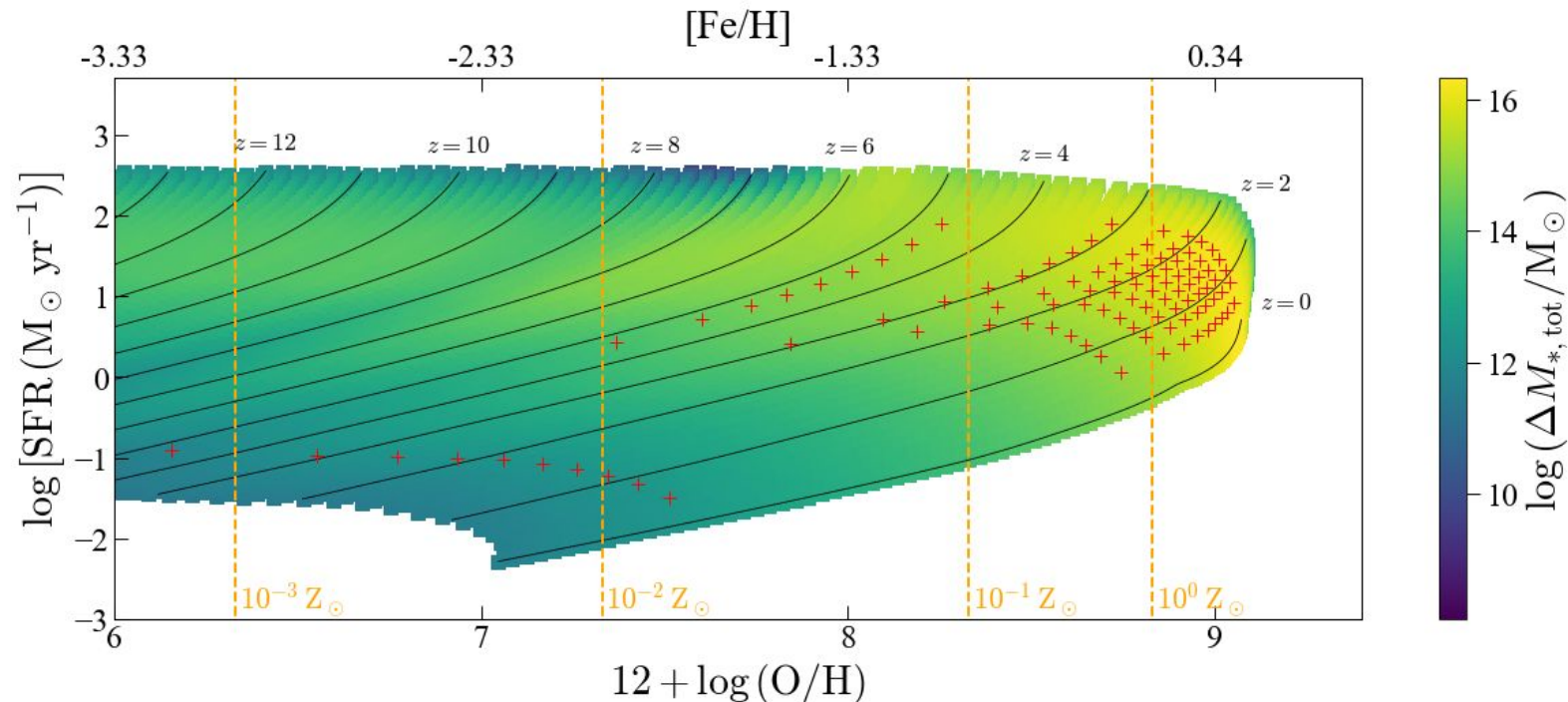
Multiplicity frequency and fractions

$$f_{\text{mult}; q \geq 0.1}(m_1) = \int_{0.2}^8 d \log P f_{\log P; q \geq 0.1}(m_1) = \mathcal{F}_{n_{\text{cp}}=1; q \geq 0.1}(m_1) + 2\mathcal{F}_{n_{\text{cp}}=2; q \geq 0.1}(m_1) + \dots$$

We can compute a m_1 -dependent binary fraction, but the full picture will require higher-order multiples.

The metallicity-specific star formation history

From *Chruslinska+2020*: GSMF + MZR + SFMR \rightarrow metallicity-specific cSFH

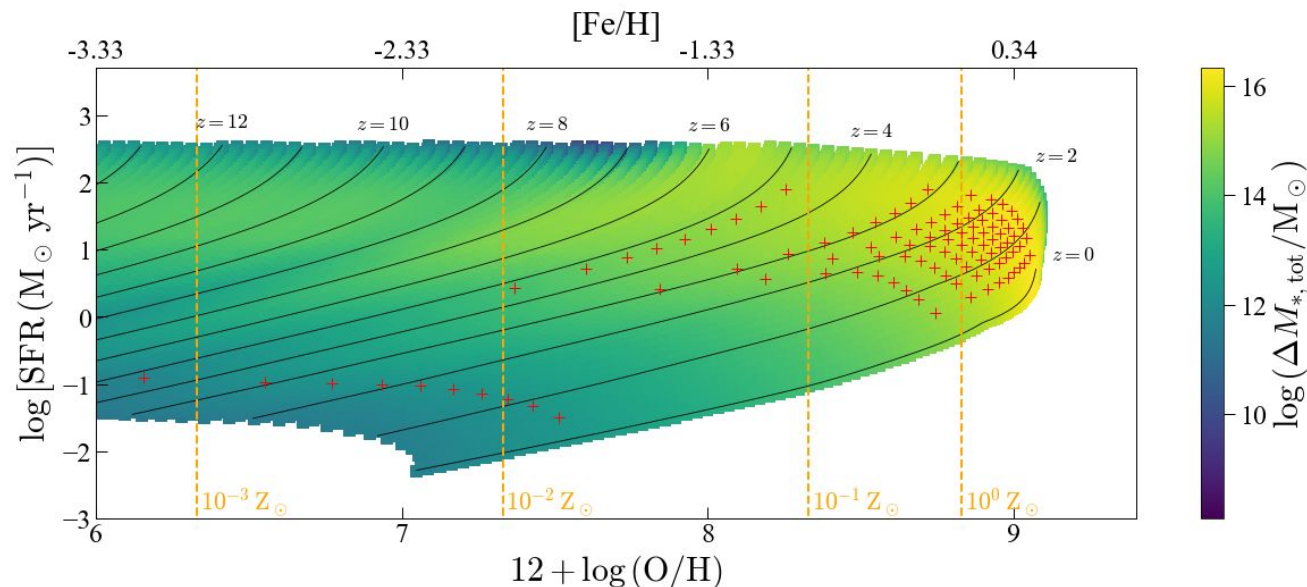


SFR measurements must be **corrected** for the IMF.

What we have tested (so far)

Base grid:

- 10 z_{ZAMS} from the cSFH, plus 0.01 and 10.0 (boundaries).
- 10 metallicities per z_{ZAMS} from the cSFH.
- $\sim 10^6$ binaries per (z_{ZAMS}, Z) .



Base models:

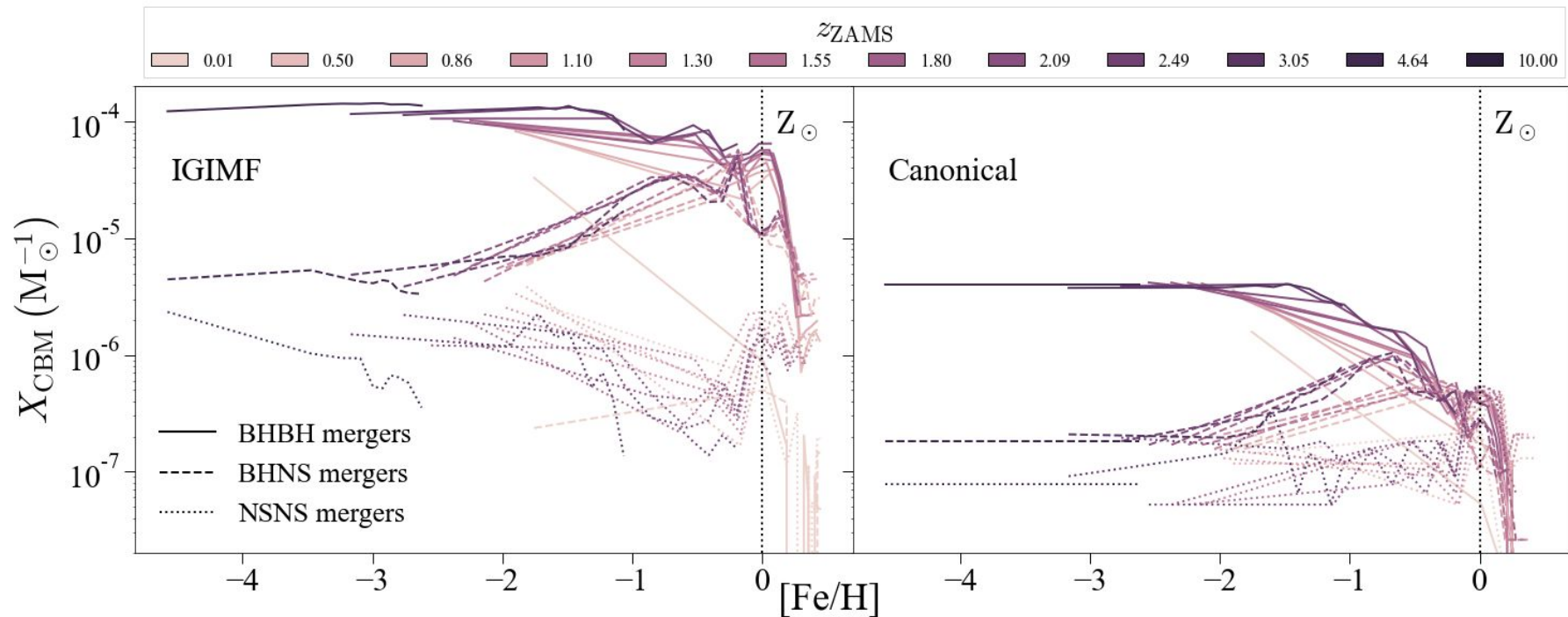
- **Canonical** IMF and orbital parameters,
- **Varying** IMF and orbital parameters,

both with,

- Standard COMPAS settings,
- Single and binaries only,
- Intermediate metallicity cSFH from *Chruslinska+2020*.

Implications for compact mergers

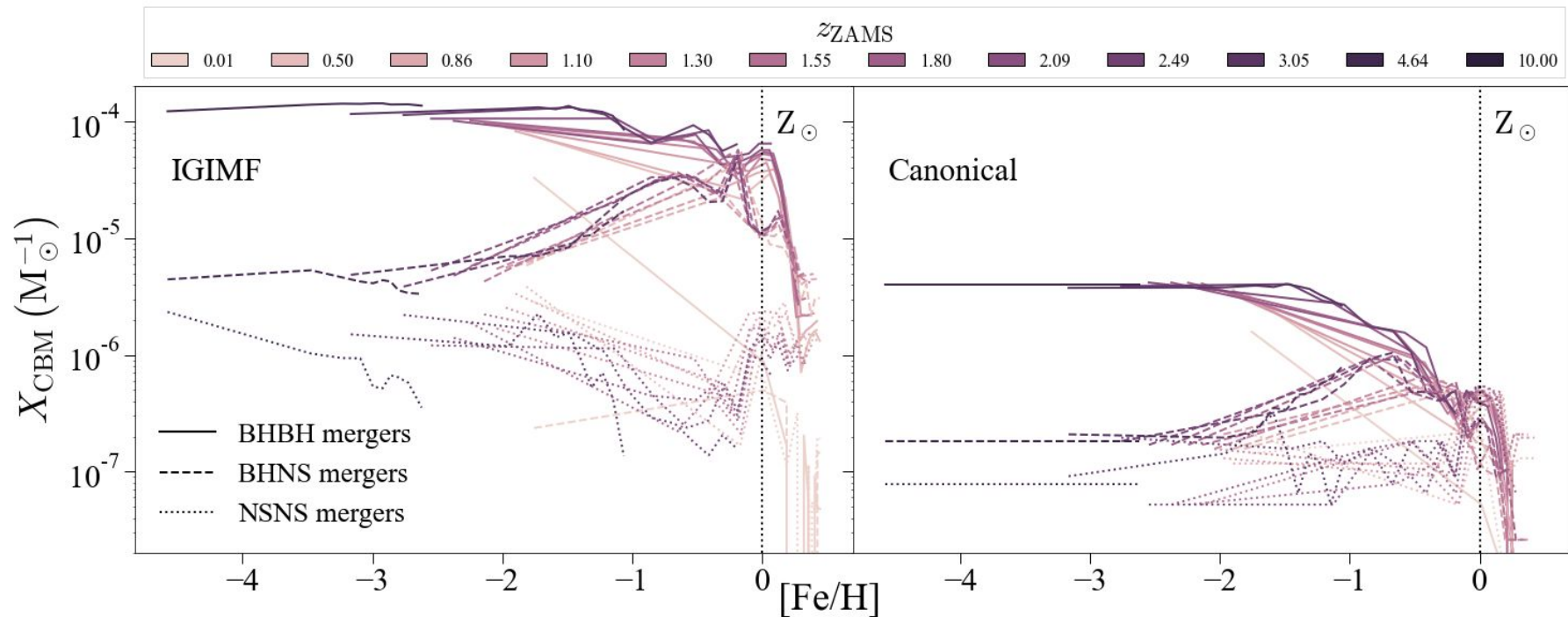
On the formation efficiency



In common:

- BHBHs favor lower metallicities, BHNSs intermediary. Both disfavored around solar.
- NSNS are the least sensitive to metallicity, but slightly favor solar.

On the formation efficiency

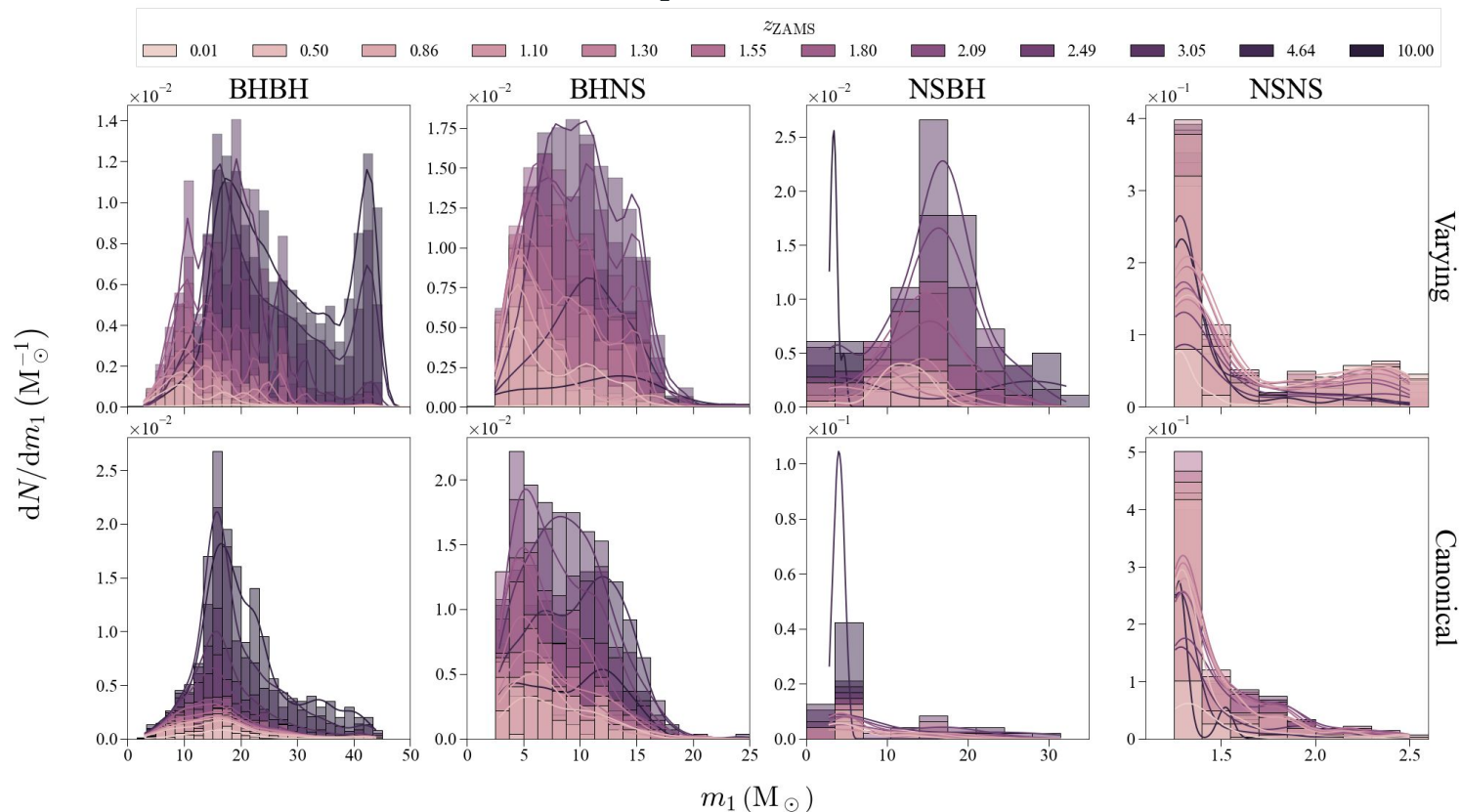


However:

- The Varying model amplifies CBM formation, strongly favoring BHs .
- This happens **even at solar and supersolar metallicity**. It is **not** the sole effect of the IMF.

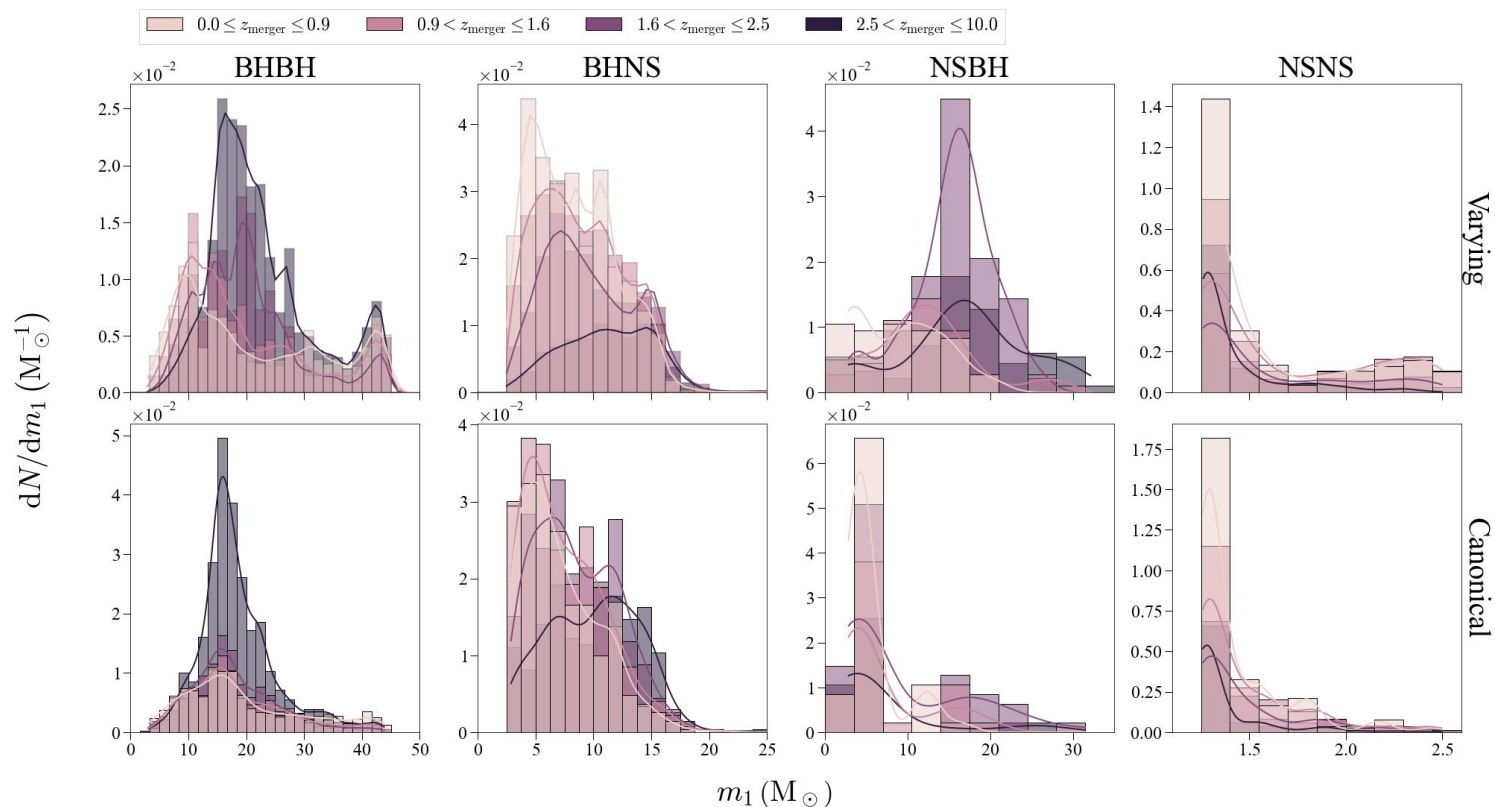
On the masses

We can see this in the shift in the m_1 distributions.



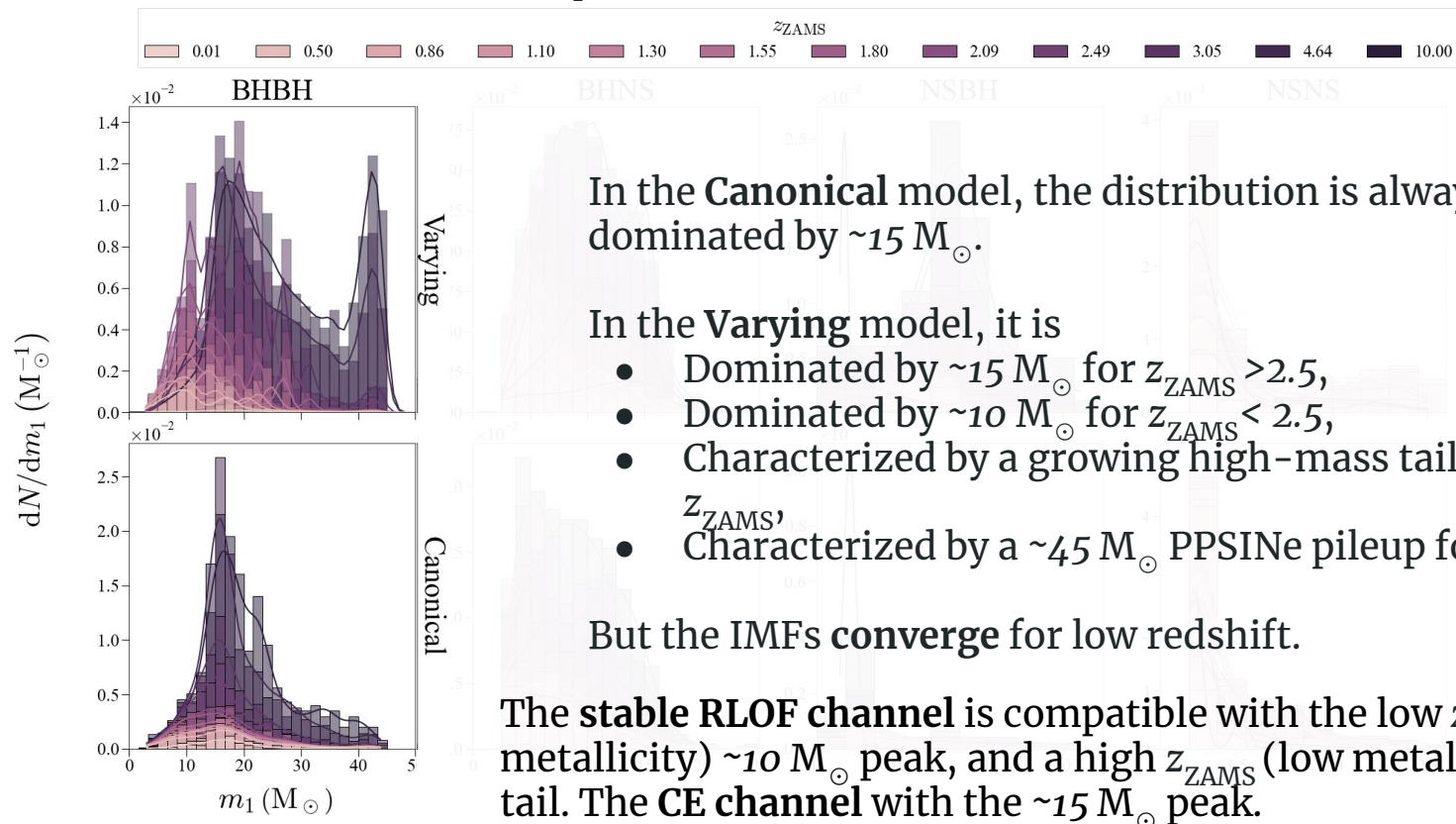
On the masses

We find a similar situation in terms of z_{merger} .



On the masses

We can see a the shift in the m_1 distributions.



In the **Canonical** model, the distribution is always dominated by $\sim 15 M_{\odot}$.

In the **Varying** model, it is

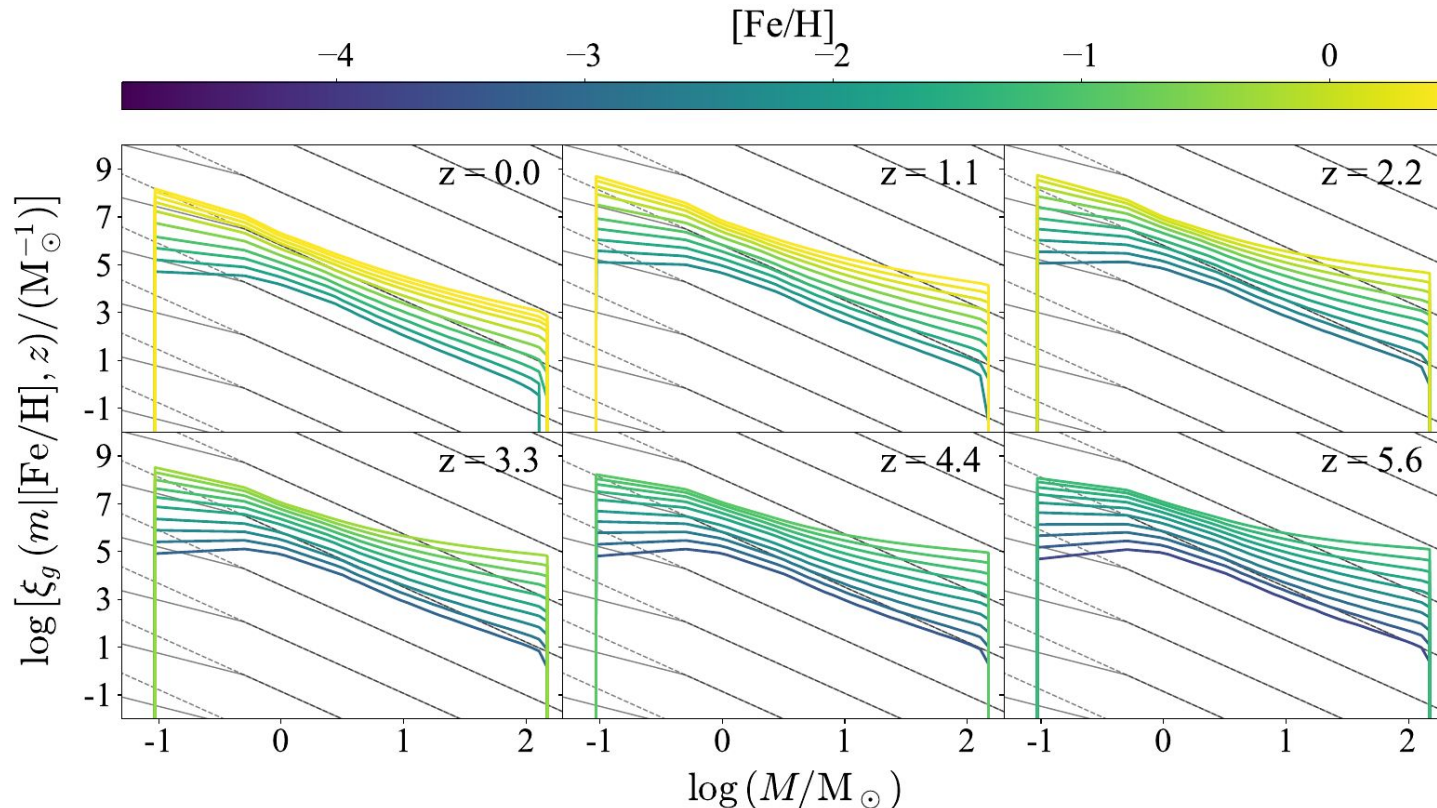
- Dominated by $\sim 15 M_{\odot}$ for $z_{\text{ZAMS}} > 2.5$,
- Dominated by $\sim 10 M_{\odot}$ for $z_{\text{ZAMS}} < 2.5$,
- Characterized by a growing high-mass tail at higher z_{ZAMS} ,
- Characterized by a $\sim 45 M_{\odot}$ PPSINE pileup for $z_{\text{ZAMS}} > 4.6$.

But the IMFs **converge** for low redshift.

The **stable RLOF channel** is compatible with the low z_{ZAMS} (high metallicity) $\sim 10 M_{\odot}$ peak, and a high z_{ZAMS} (low metallicity) high-mass tail. The **CE channel** with the $\sim 15 M_{\odot}$ peak.

Metallicity-specific cSFRD

The redshift-dependent gIMF



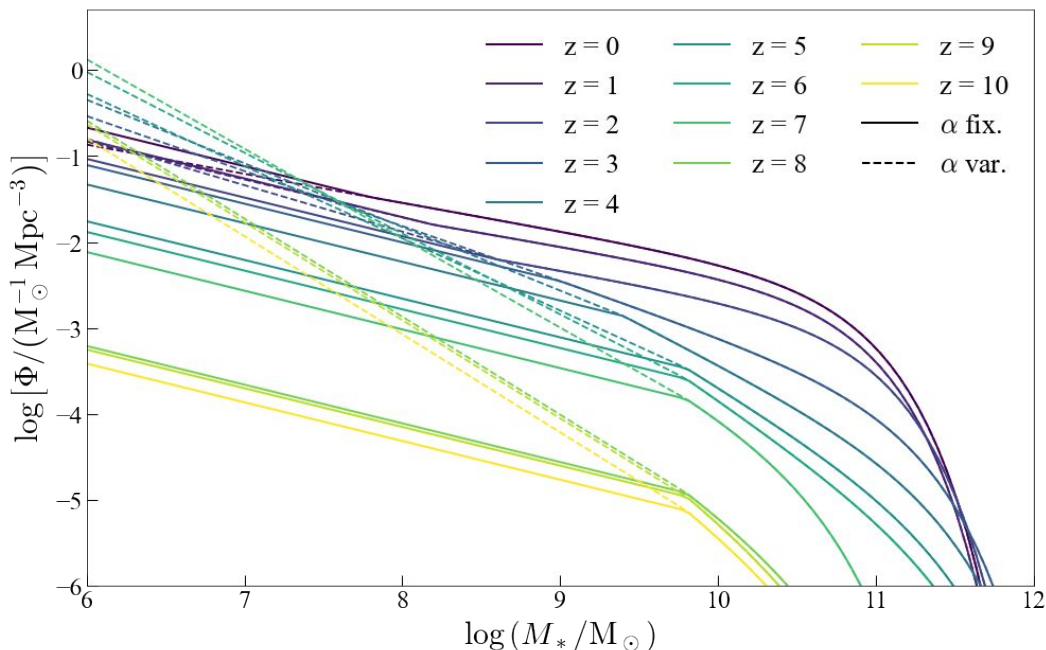
- The SFR-dependence dominates at high masses.

First ingredient: environmental conditions

Chruślińska & Nelemans (2019), Chruślińska+ (2020)

Simple analytical empirical distributions for average star-forming galaxy properties up to $z=10$ (~ beginning of star formation)

Galaxy stellar mass function



$$\frac{dn_{\text{gal}}}{dM_*} = \Phi(M_*) = \Phi_* e^{-M/M_c} \left(\frac{M_*}{M_c}\right)^{-\alpha_{\text{GSMF}}}$$

Avoiding “overcommitment”,

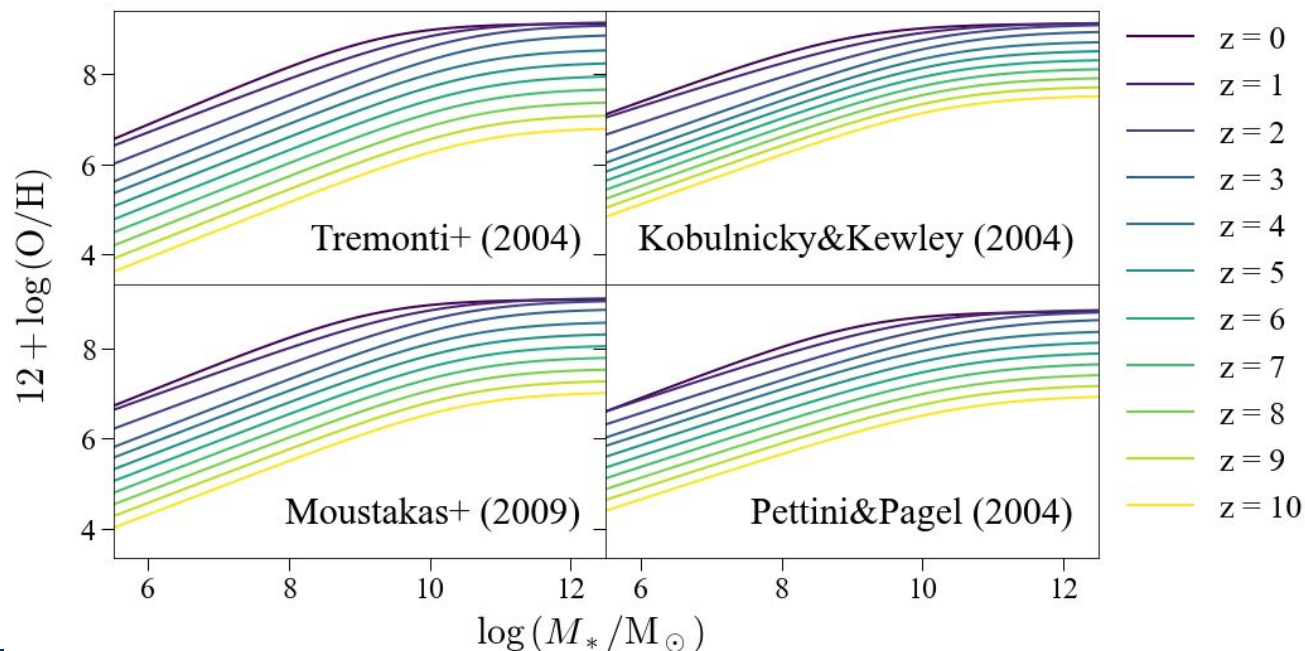
- Average over many fits at different redshift, for Φ_* , M_c , α_{GSMF}
- Two models for the low-mass power-law, α

First ingredient: environmental conditions

Mass-metallicity relation

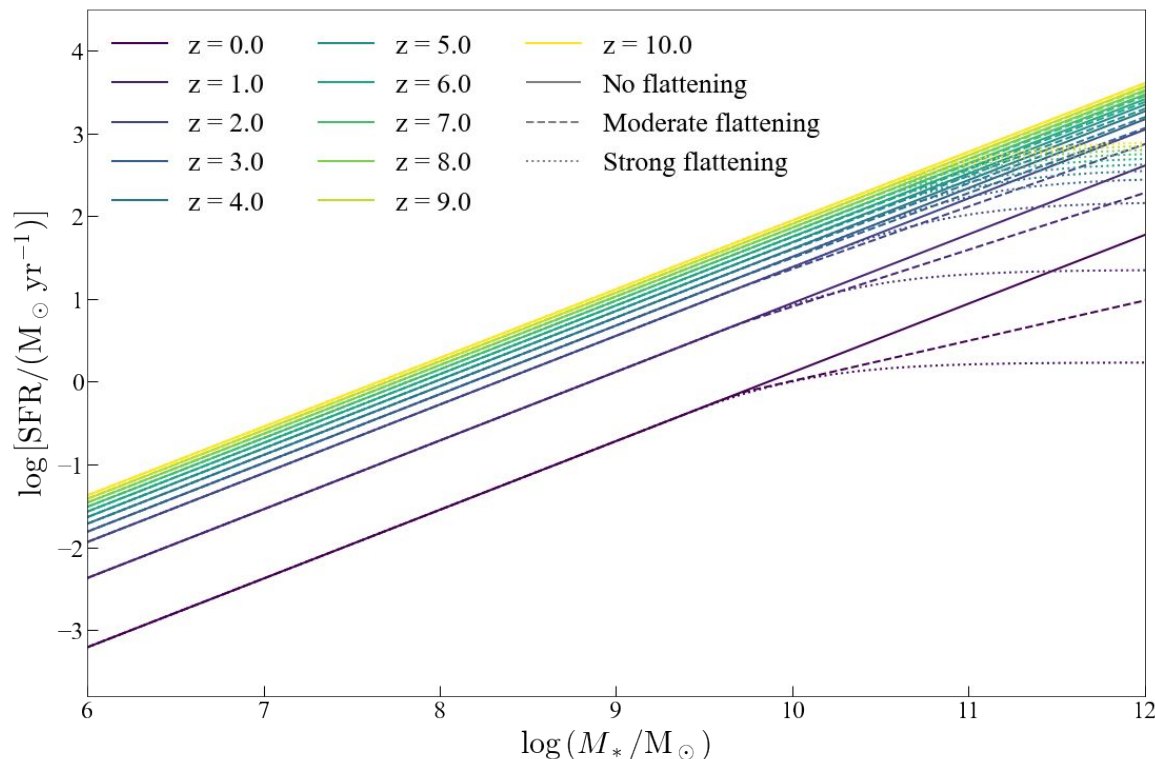
- Different calibrations for converting from HII region observations to metallicity

$$12 + \log(O/H) = Z_{O/H,a} - \log \left[1 + \left(\frac{M_*}{M_{TO}} \right)^{-\gamma} \right]$$



First ingredient: environmental conditions

Star formation-mass relation



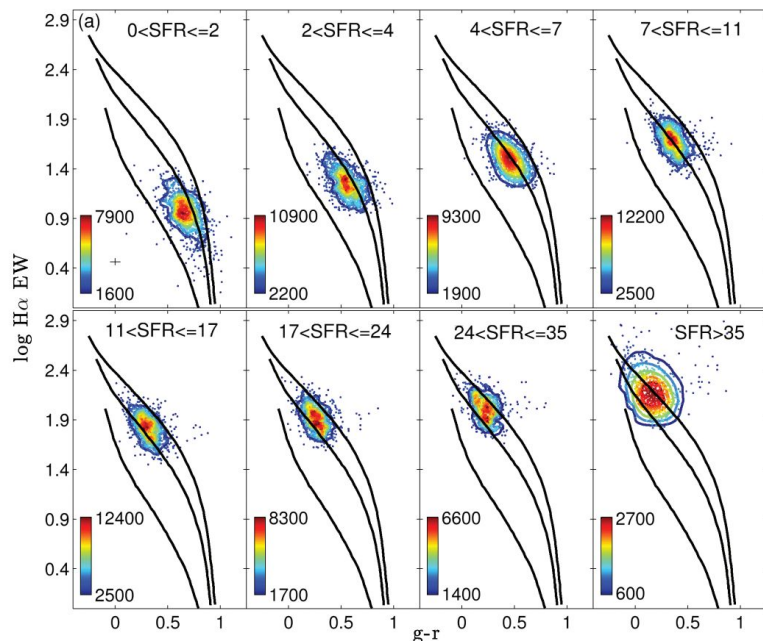
$$\log\left(\frac{\text{SFR}}{M_\odot \text{ yr}^{-1}}\right) = a \times \log\left(\frac{M_*}{M_\odot}\right) + b$$

- Accounting for disagreement on the flattening at high masses,
- $\text{H}\alpha$ -SFR relation depends on the IMF choice. Chruślińska+ (2020) correct the SFMR for the **IGIMF**.

Second ingredient: the initial mass function (IMF)

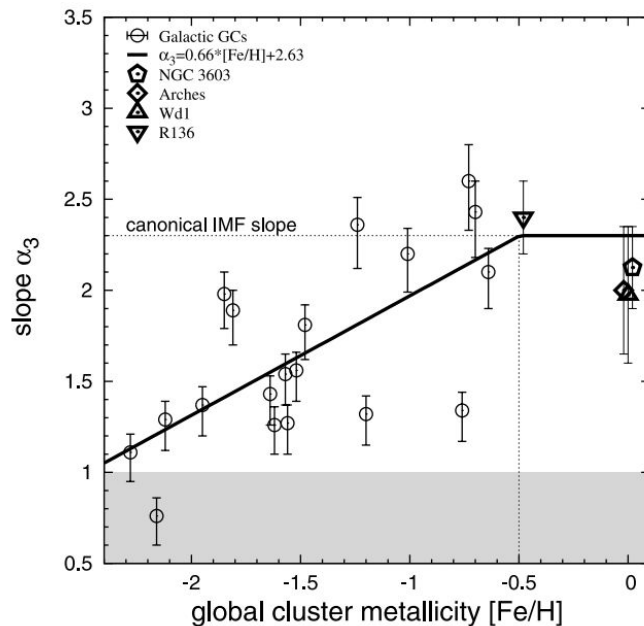
Observational support is recent: top-heavy/bottom-light IMFs from

High star formation rate (SFR)



Gunawardhana+ (2011)

Low metallicity



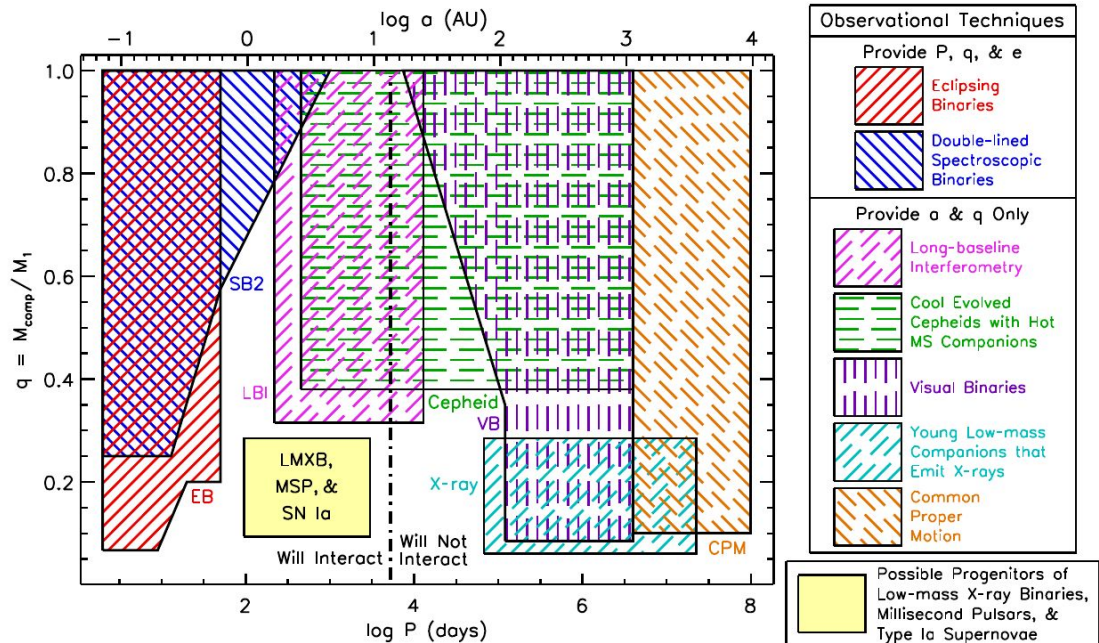
Marks+ (2012)

Third ingredient: Orbital parameters

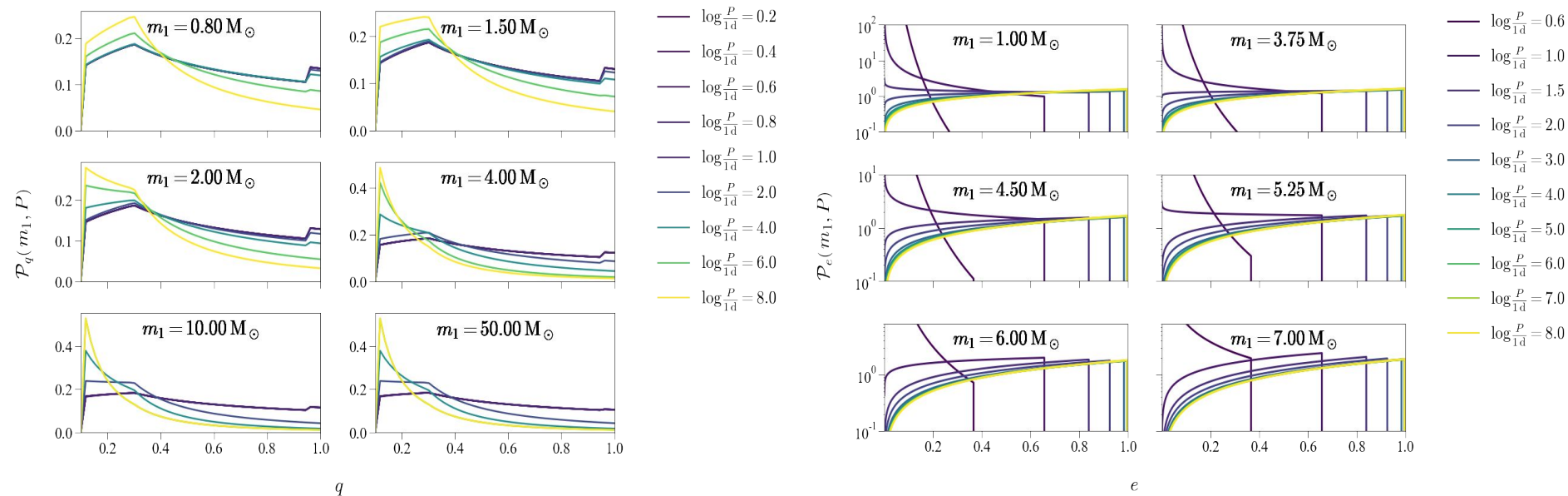
Usual choices:

- Uniform $0 \leq q \leq 1$,
- Log-uniform $10^{-2} \text{ AU} \leq A \leq 10^3 \text{ AU}$, or log-uniform $0.4 \leq \log(P/d) \leq 3$,
- Circular orbits ($e = 0$).

However, Moe & Di Stefano (2017) found that they are significantly correlated.



Third ingredient: Orbital parameters

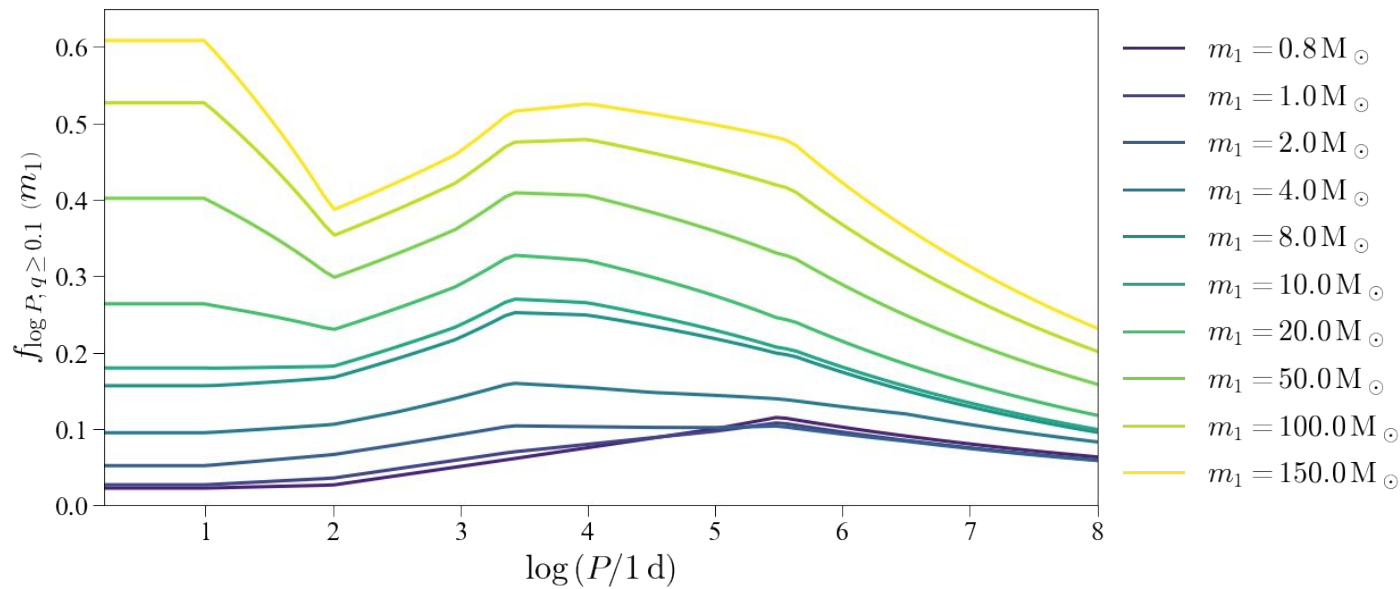


- Correlations probably related to pre-ZAMS evolution and/or dynamical interactions

Third ingredient: Orbital parameters

Main quantity: companion frequency,

$$f_{\log P; q \geq q_{\min}}(m_1) = \frac{dN_{\text{cp}}}{dN_1 d \log P}$$



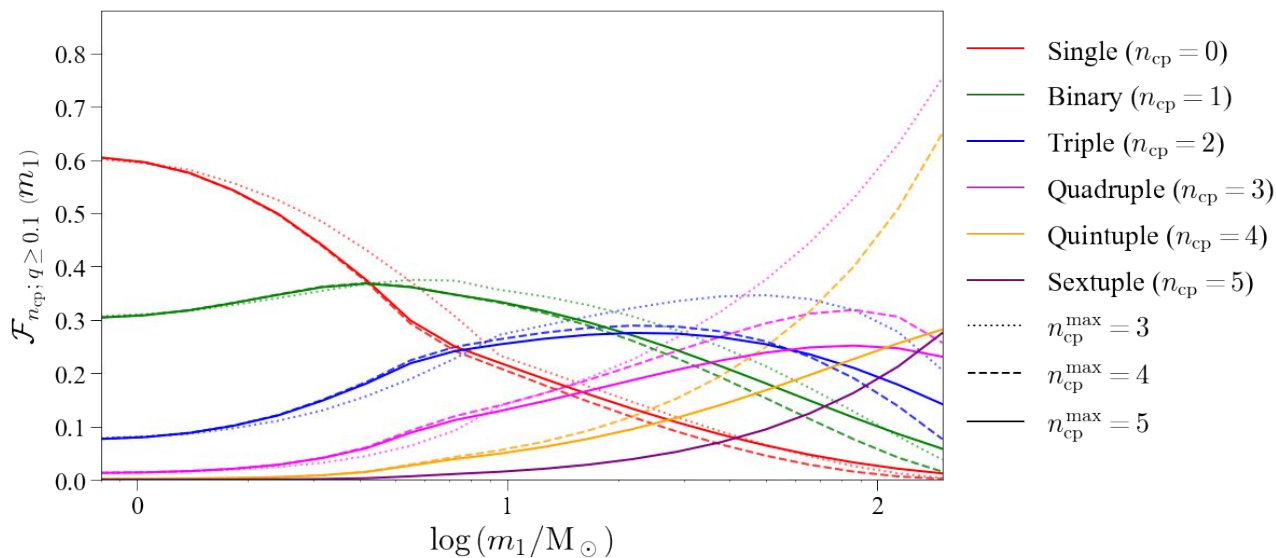
- Does not distinguish between binaries and higher-order multiples

The **multiple fractions** are constrained by

$$f_{\text{mult}; q \geq 0.1}(m_1) = \int_{0.2}^8 d \log P f_{\log P; q \geq 0.1}(m_1) = \mathcal{F}_{n_{\text{cp}}=1; q \geq 0.1}(m_1) + 2\mathcal{F}_{n_{\text{cp}}=2; q \geq 0.1}(m_1) + 3\mathcal{F}_{n_{\text{cp}}=3; q \geq 0.1}(m_1) + \dots$$

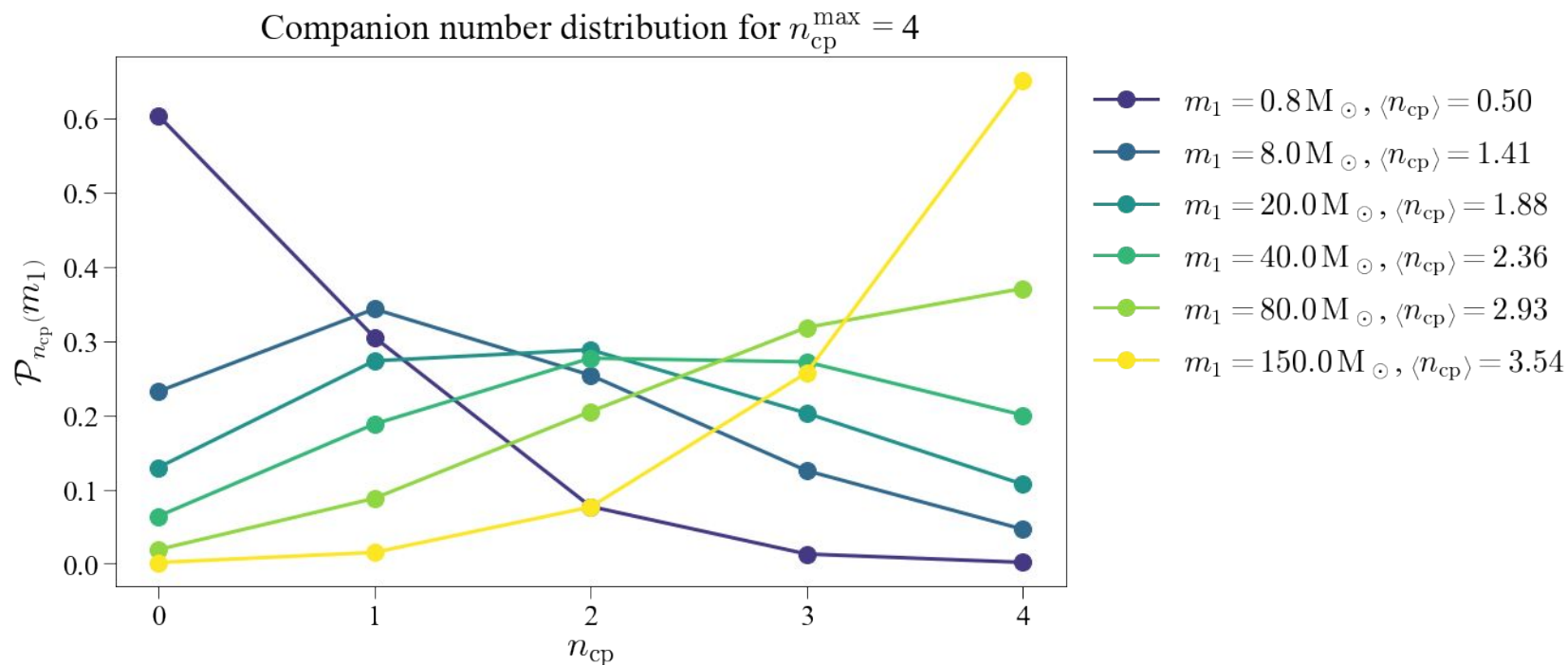
extra information required if we allow $n_{\text{cp}} > 1$

→ Extrapolate from the Poissonian behavior of $\mathcal{P}_{n_{\text{cp}}}(m_1)$ observed for solar-type stars.



The higher m_1 , the higher $n_{\text{cp}}^{\text{max}}$ is required.

For a maximum $m_1 = 150 M_\odot$,



- We can keep track of all companion masses, but only evolve inner binaries.
- The total **star-forming mass** affects volumetric rates.