

Hydrodynamical view of massive stars' winds



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History

Massive Stars Winds

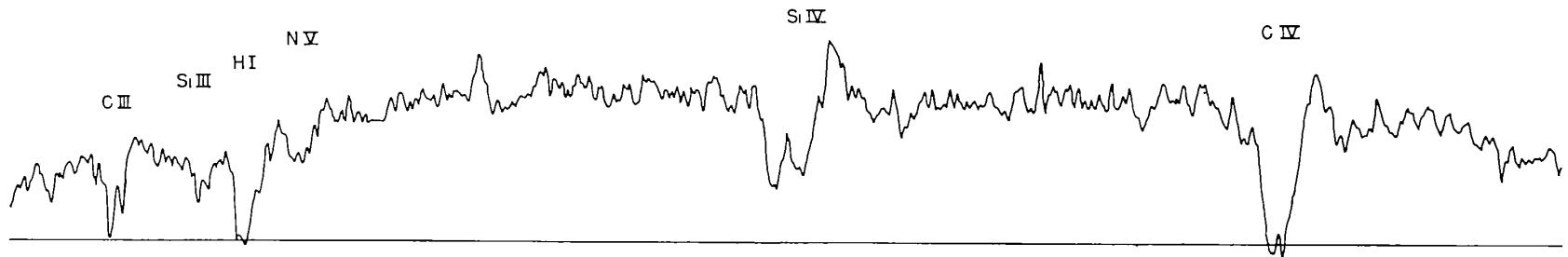


Fig. 1. Densitometer tracing, on an intensity scale, of the far-ultraviolet spectrum of ζ Orionis, photographed by Princeton on September 10, 1966. The distribution of intensity with wavelength includes the unknown response of the spectrograph. Wavelengths increase towards the right from 1140 to 1630 Å. The HI line is interstellar, but all the other identified absorption features are circumstellar with large Doppler shifts to shorter wavelengths.

History

Massive Stars Winds

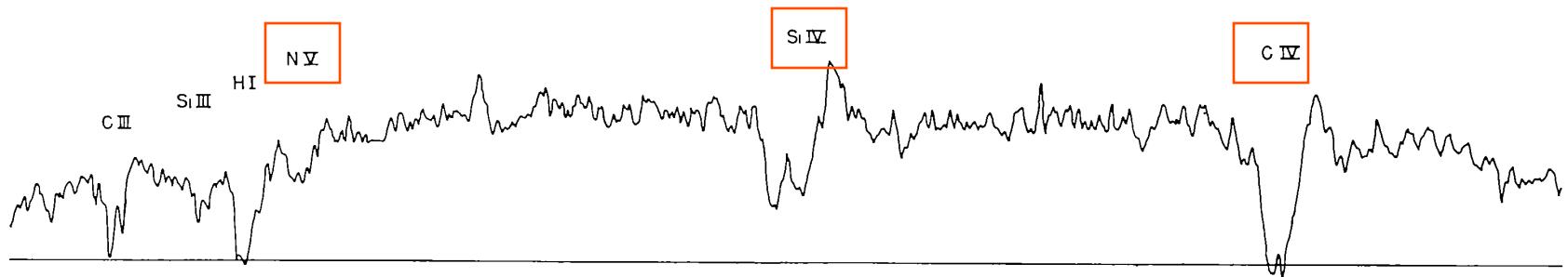
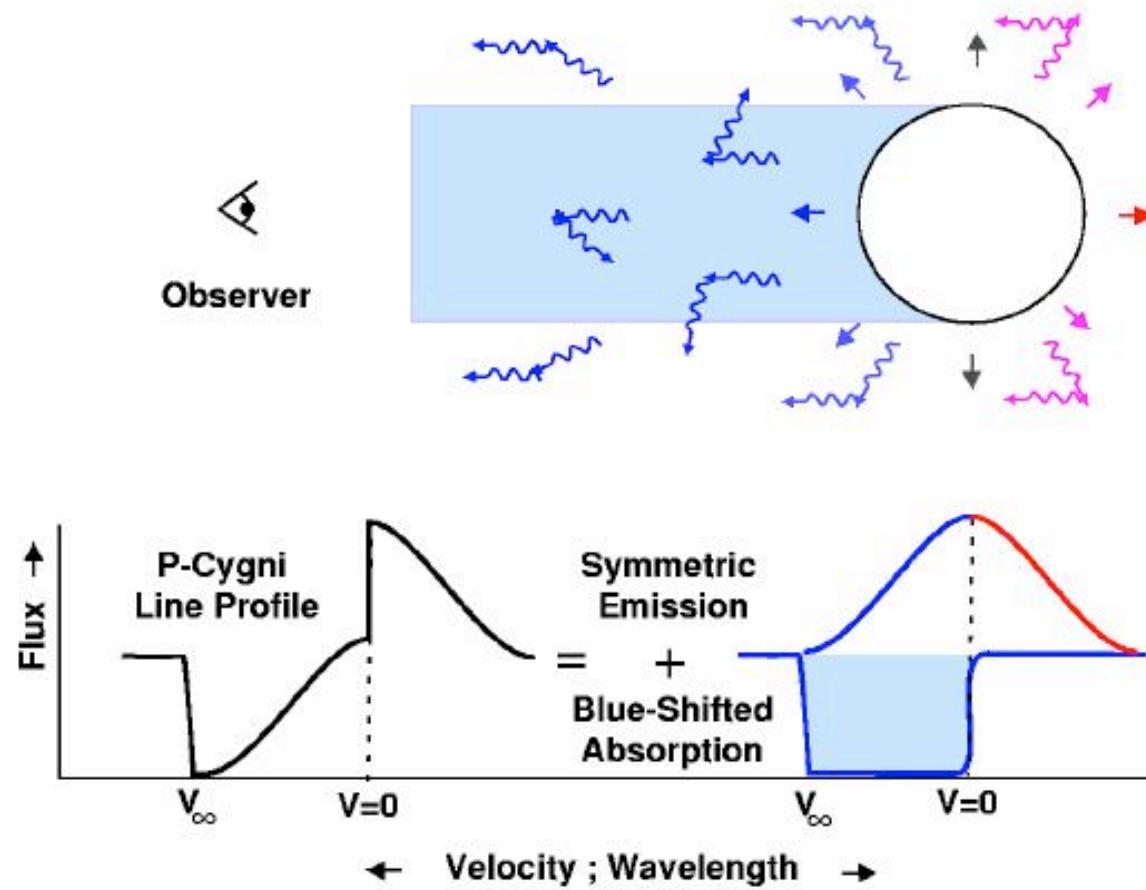


Fig. 1. Densitometer tracing, on an intensity scale, of the far-ultraviolet spectrum of ζ Orionis, photographed by Princeton on September 10, 1966. The distribution of intensity with wavelength includes the unknown response of the spectrograph. Wavelengths increase towards the right from 1140 to 1630 Å. The HI line is interstellar, but all the other identified absorption features are circumstellar with large Doppler shifts to shorter wavelengths.

Massive Stars Winds

Formation of a P-Cygni Line- Profile



Theory

Driven Mechanism

Only known Theory in the '60:

Parker's Model for the Solar Wind

(Parker, E.N.: 1960, ApJ 132, 821)

For O stars => $T_{\text{eff}} = 10^7 \text{ K}$

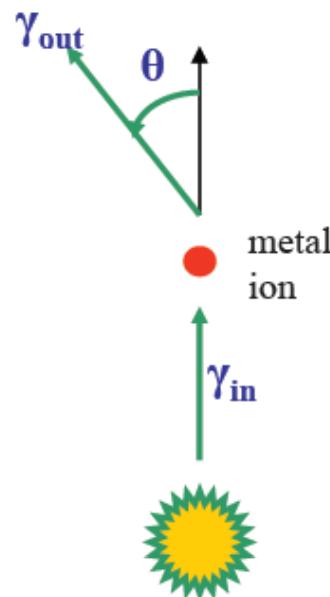
But at this Temperature C IV - N V - Si IV Don't Exist

Destroyed by collisional ionisation

Massive Stars Winds

Driven Mechanism

Lucy & Solomon (1970, ApJ, 159, 870):
Wind driven by resonance lines
Obtained only mass loss rates of about 1/100th of the observed values



$$\begin{aligned}\cos \theta_{\text{in}} &\approx 1 \\ \text{isotropic reemission} \\ \langle \cos \theta_{\text{out}} \rangle &= 0\end{aligned}$$

Massive Stars Winds

Driven Mechanism

Castor, Abbott & Klein (1975, ApJ, 195, 157)

Wind driven by an **ensemble of lines** (scattering)

They obtained a qualitative agreement with observational values



Massive Stars Winds

Driven Mechanism

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The Standard Model (m-CAK)



Line Force

Contribution from an **ensemble** of lines

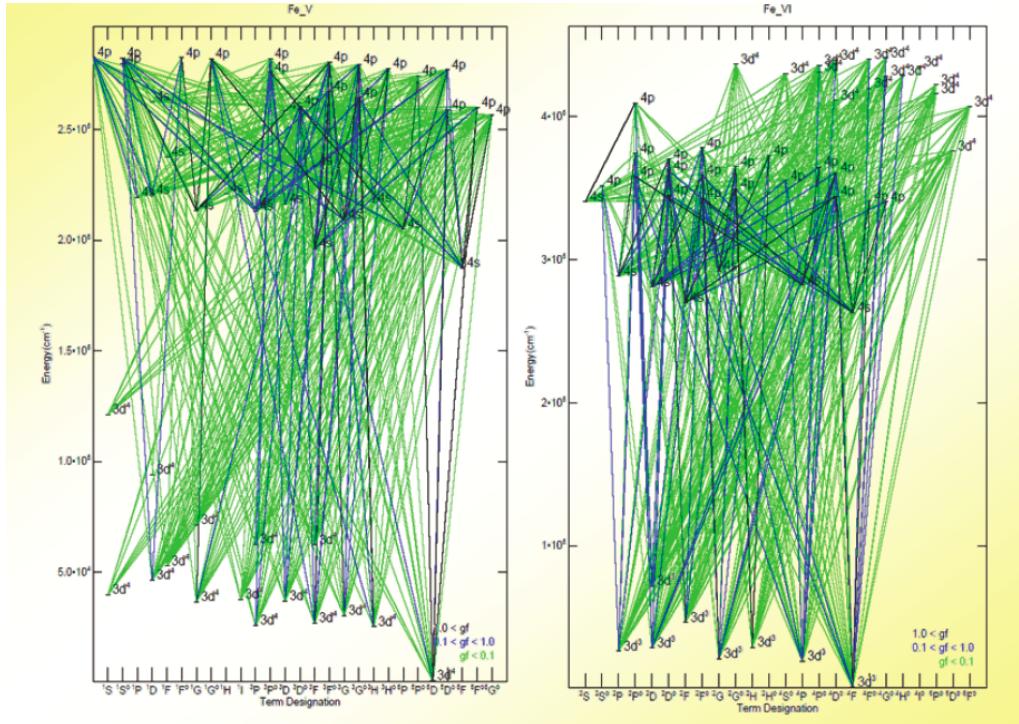
Currently: ~Mega lines, ~150 ionisation stages (H –Zn),



Line Force

Contribution from an **ensemble** of lines

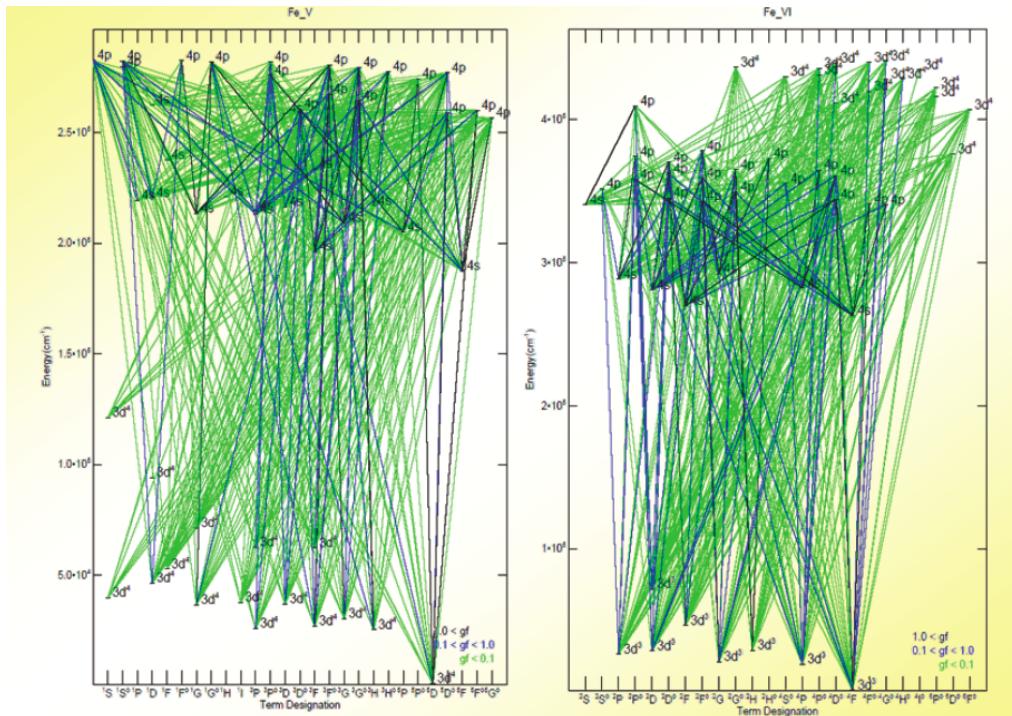
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Line Force

Contribution from an **ensemble** of lines

Currently: ~Mega lines, ~150 ionisation stages (H –Zn),



	I	II	III	IV	V	VI	VII	VIII
1	H_I							
2	He_I	He_II						
6	C_I	C_II	C_III	C_IV	C_V			
7	N_I	N_II	N_III	N_IV	N_V	N_VI		
8	O_I	O_II	O_III	O_IV	O_V	O_VI		
9	F_I	F_II	F_III	F_IV	F_V	F_VI		
10	Ne_I	Ne_II	Ne_III	Ne_IV	Ne_V	Ne_VI		
11	Na_I	Na_II	Na_III	Na_IV	Na_V	Na_VI		
12	Mg_I	Mg_II	Mg_III	Mg_IV	Mg_V	Mg_VI		
13	Al_I	Al_II	Al_III	Al_IV	Al_V	Al_VI		
14	Si_I	Si_II	Si_III	Si_IV	Si_V	Si_VI		
15	P_I	P_II	P_III	P_IV	P_V	P_VI		
16	S_I	S_II	S_III	S_IV	S_V	S_VI	S_VII	
17	Cl_I	Cl_II	Cl_III	Cl_IV	Cl_V	Cl_VI		
18	Ar_I	Ar_II	Ar_III	Ar_IV	Ar_V	Ar_VI	Ar_VII	Ar_VIII
19	K_I	K_II	K_III	K_IV	K_V	K_VI		
20	Ca_I	Ca_II	Ca_III	Ca_IV	Ca_V	Ca_VI		
22	Tl_I	Tl_II	Tl_III	Tl_IV	Tl_V			
23	V_I	V_II	V_III	V_IV	V_V			
24	Cr_I	Cr_II	Cr_III	Cr_IV	Cr_V	Cr_VI		
25	Mn_I	Mn_II	Mn_III	Mn_IV	Mn_V	Mn_VI		
26	Fe_I	Fe_II	Fe_III	Fe_IV	Fe_V	Fe_VI	Fe_VII	Fe_VIII
27	Co_I	Co_II	Co_III	Co_IV	Co_V	Co_VI	Co_VII	
28	Ni_I	Ni_II	Ni_III	Ni_IV	Ni_V	Ni_VI	Ni_VII	Ni_VIII
29	Cu_I	Cu_II	Cu_III	Cu_IV	Cu_V	Cu_VI		
30	Zn_I	Zn_II	Zn_III					

atomic data status:

excellent	good	poor	bad
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Radiation Driven Wind Hydrodynamics



Assumptions

Stationary - Low viscosity - Spherical symmetry - No Mag. Fields.

Mass Conservation

$$\dot{M} = 4 \pi r^2 \rho v$$

Momentum Conservation

$$v \frac{dv}{dr} = - \frac{1}{\rho} \frac{dP}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{line} \left(\rho, \frac{dv}{dr}, n_e \right)$$



Inertia

Rio de Janeiro, Jun. 2024



Gas Pressure



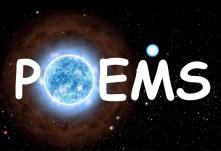
Gravity



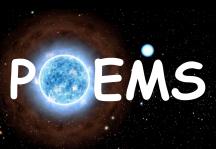
Line Force

Universidad de Valparaíso

Radiation Driven Winds



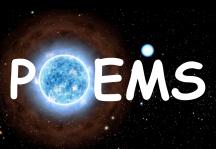
Radiation Driven Winds



$$g^{line} = \frac{C(k, \dot{M})}{r^2} CF\left(r, v, \frac{dv}{dr}\right) \left(r^2 v \frac{dv}{dr}\right)^\alpha \left(\frac{n_e}{W(r)}\right)^\delta$$



Radiation Driven Winds



Eigenvalue

$$g^{line} = \frac{C(k, \dot{M})}{r^2} CF\left(r, v, \frac{dv}{dr}\right) \left(r^2 v \frac{dv}{dr}\right)^\alpha \left(\frac{n_e}{W(r)}\right)^\delta$$



Radiation Driven Winds



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Finite Disk
Correction Factor



Radiation Driven Winds



$$g^{line} = \frac{C(k, \dot{M})}{r^2} CF \left(r, v, \frac{dv}{dr} \right) \left(r^2 v \frac{dv}{dr} \right)^\alpha \left(\frac{n_e}{W(r)} \right)^\delta$$

Eigenvalue

Ensemble of lines

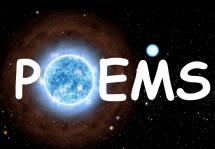
Finite Disk
Correction Factor



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y
Astronomía



Radiation Driven Winds



$$g^{line} = \frac{C(k, \dot{M})}{r^2} CF \left(r, v, \frac{dv}{dr} \right) \left(r^2 v \frac{dv}{dr} \right)^\alpha \left(\frac{n_e}{W(r)} \right)^\delta$$

Eigenvalue

Ensemble of lines

Finite Disk
Correction Factor

Changes in
Ionisation



Radiation Driven Winds



$$g^{line} = \frac{C(k, \dot{M})}{r^2} CF \left(r, v, \frac{dv}{dr} \right) \left(r^2 v \frac{dv}{dr} \right)^{\alpha} \left(\frac{n_e}{W(r)} \right)^{\delta}$$

Eigenvalue

Ensemble of lines

Finite Disk
Correction Factor

Changes in
Ionisation



Radiation Driven Winds

Change of variables

$$\begin{aligned} u &= \frac{-R_*}{r}, \\ w &= \frac{v}{a}, \\ w' &= \frac{dw}{du}, \end{aligned}$$

Equation of Motion

$$F(u, w, w') = 0$$

$$\begin{aligned} F(u, w, w') \equiv & \left(1 - \frac{1}{w^2}\right) w \frac{dw}{du} + A + \frac{2}{u} + a_{\text{rot}}^2 u \\ & - C' C F g(u) (w)^{-\delta} \left(w \frac{dw}{du}\right)^\alpha = 0 \end{aligned}$$



First Topological Analysis

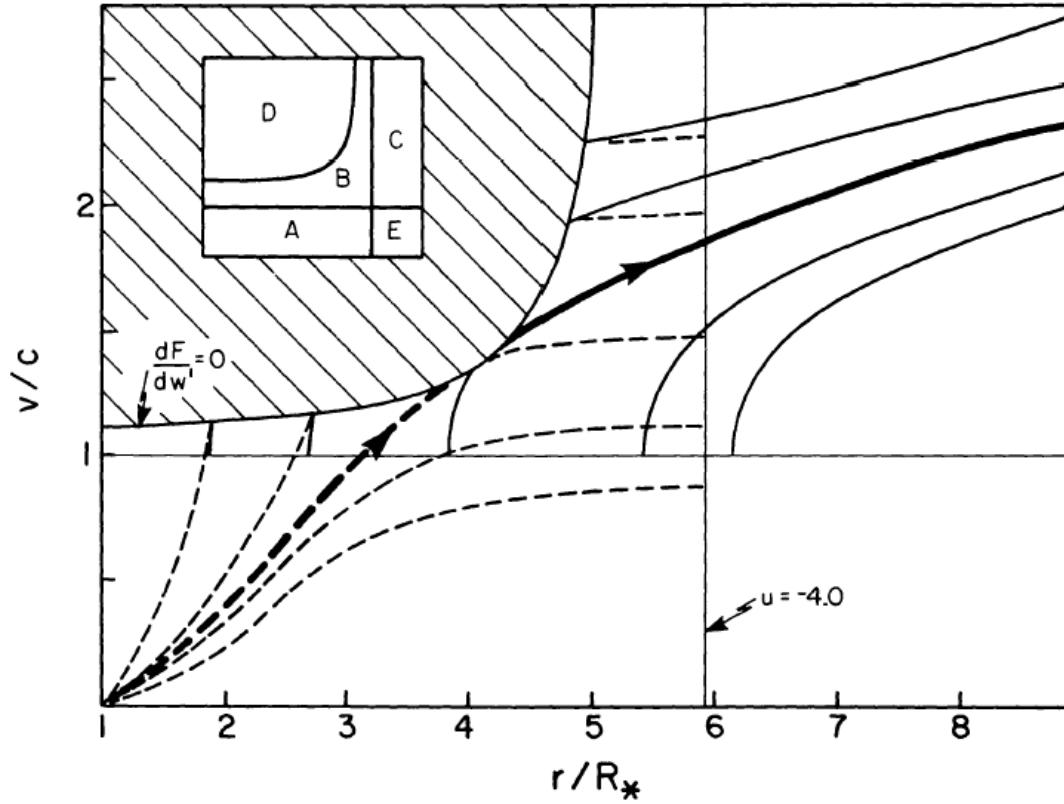


Non-Rotating Solution Schema



First Topological Analysis

Non-Rotating Solution Schema

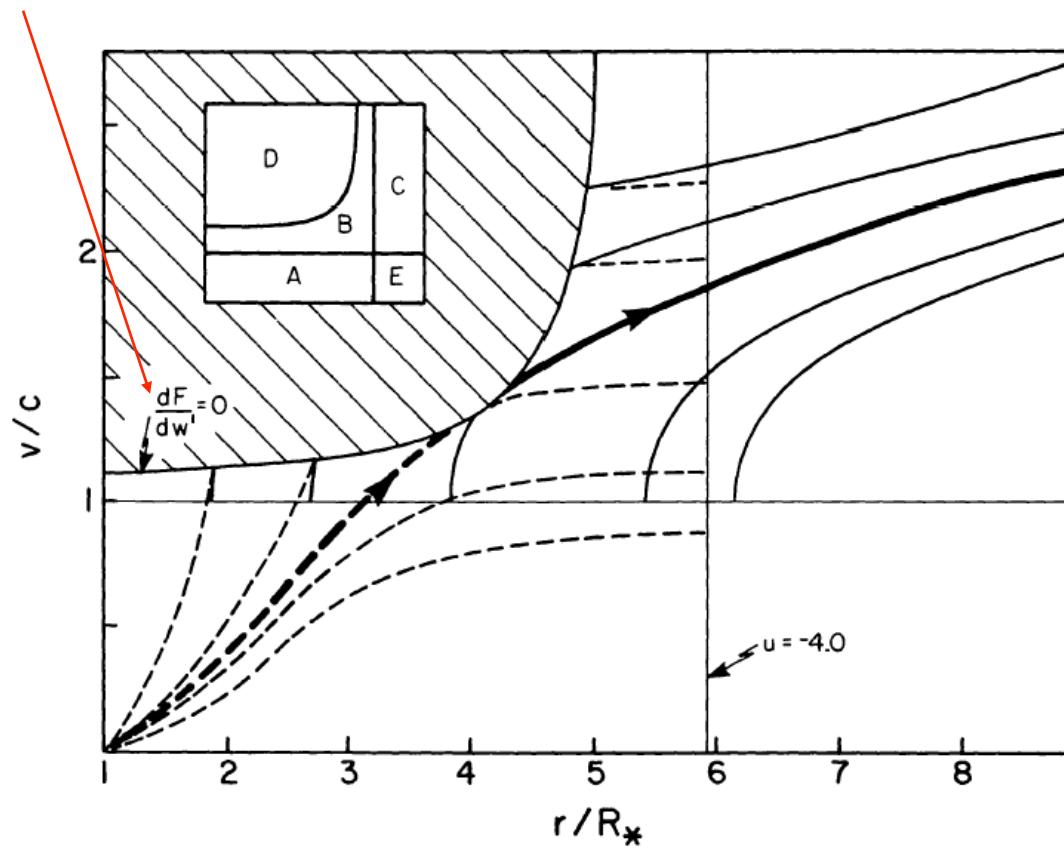


First Topological Analysis

Non-Rotating Solution Schema

Singularity Condition

$$\frac{\partial F(u, w, w')}{\partial w'} = 0$$

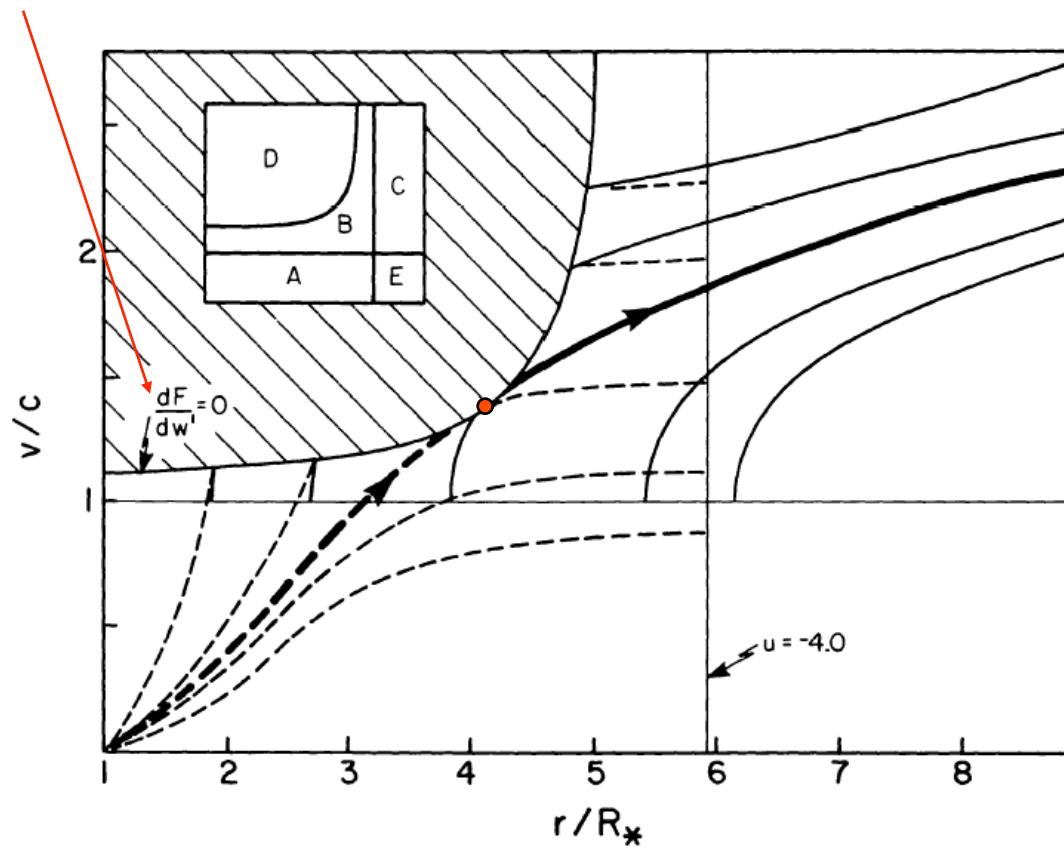


First Topological Analysis

Non-Rotating Solution Schema

Singularity Condition

$$\frac{\partial F(u, w, w')}{\partial w'} = 0$$



First Topological Analysis

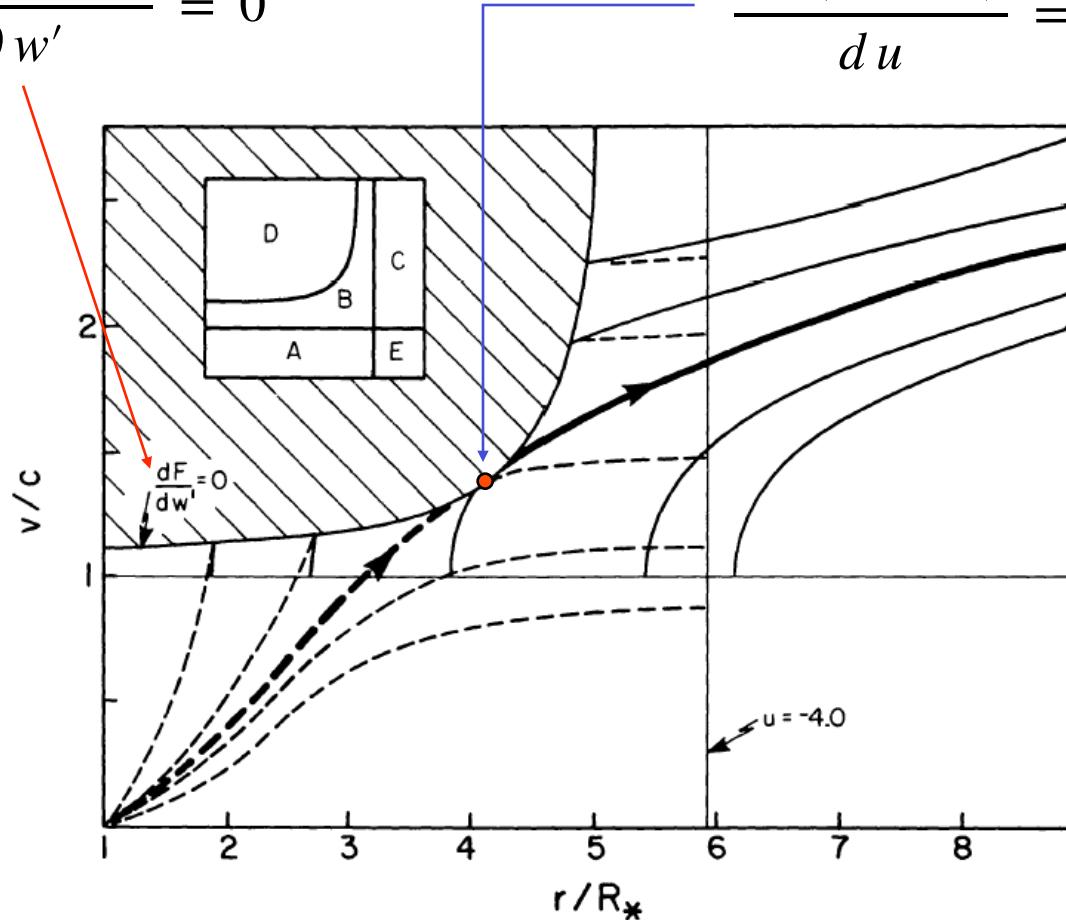
Non-Rotating Solution Schema

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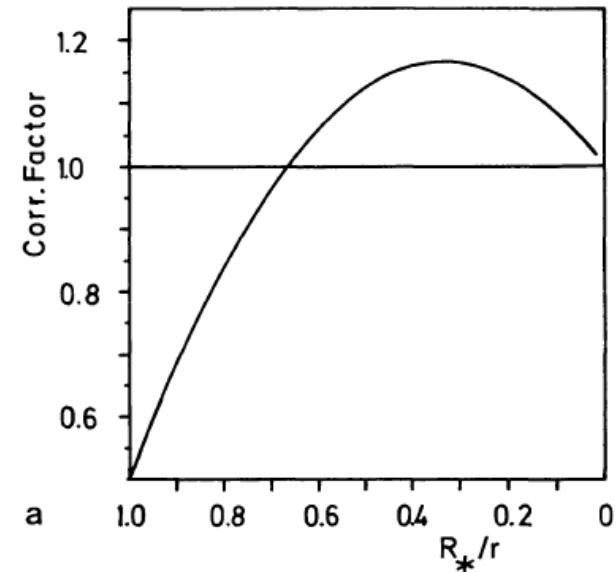
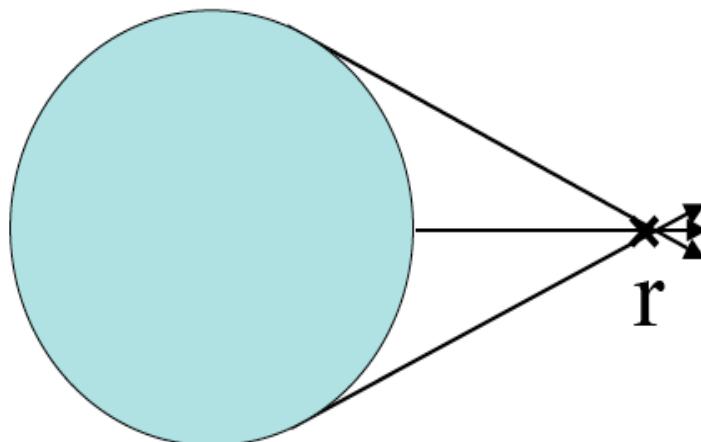
Regularity Condition

$$\frac{d F(u, w, w')}{d u} = \frac{\partial F}{\partial u} + \frac{\partial F}{\partial w} w' = 0$$



m-CAK Model

modified-CAK Theory:
Finite Disk Correction Factor



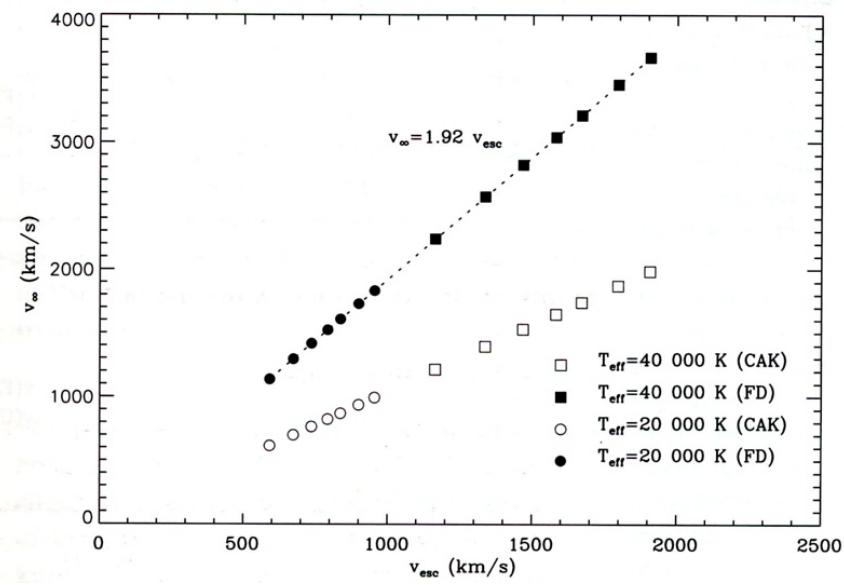
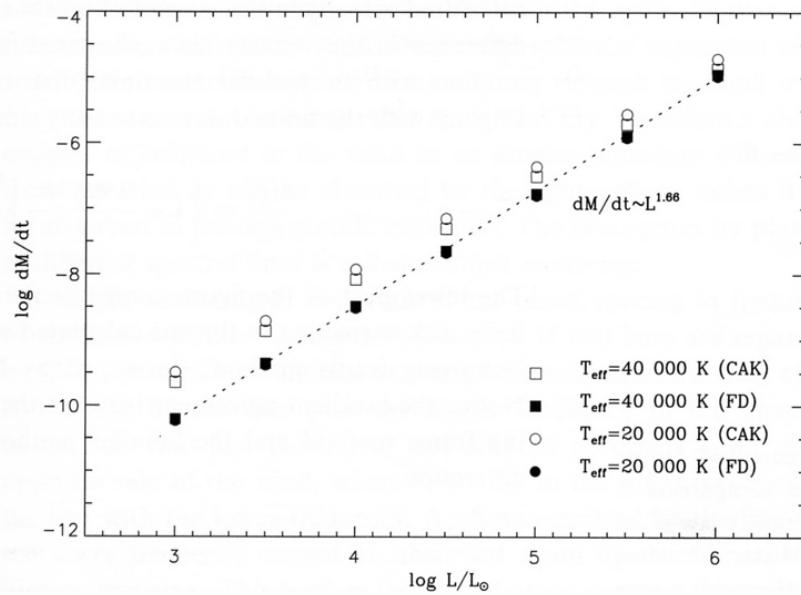
Friend & Abbott
ApJ, 311, 701, 1986

Pauldrach, Puls & Kudritzki
A&A, 164, 86, 1986

m-CAK Model



m-CAK: better agreement with observations



The β Law

$$v(r) = v_\infty (1 - R_*/r)^\beta$$

Slow solutions



Ω -Slow Solution



THE ASTROPHYSICAL JOURNAL, 614:929–941, 2004 October 20
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THE INFLUENCE OF ROTATION IN RADIATION-DRIVEN WIND FROM HOT STARS: NEW SOLUTIONS AND DISK FORMATION IN Be STARS

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Received 1999 December 28; accepted 2004 June 18

ABSTRACT

The theory of radiation-driven wind including stellar rotation is reexamined. After a suitable change of variables, a new equation for the mass-loss rate is derived analytically. The solution of this equation remains within 1% confidence when compared with numerical solutions. In addition, a nonlinear equation for the position of the critical (singular) point is obtained. This equation shows the existence of an additional critical point besides the standard m-CAK critical point. For a stellar rotation velocity larger than ~ 0.7 – $0.8 V_{\text{bkup}}$, there exists only one critical point, located away from the star's surface. Numerical solutions crossing through this new critical point are attained. In these cases, the wind has a very low terminal velocity and therefore a higher density. Disk formation in Be stars is discussed in the framework of this new line-driven stellar wind solution.

Subject headings: hydrodynamics — methods: analytical — stars: early-type — stars: mass loss — stars: rotation — stars: winds, outflows



Ω -Slow Solution

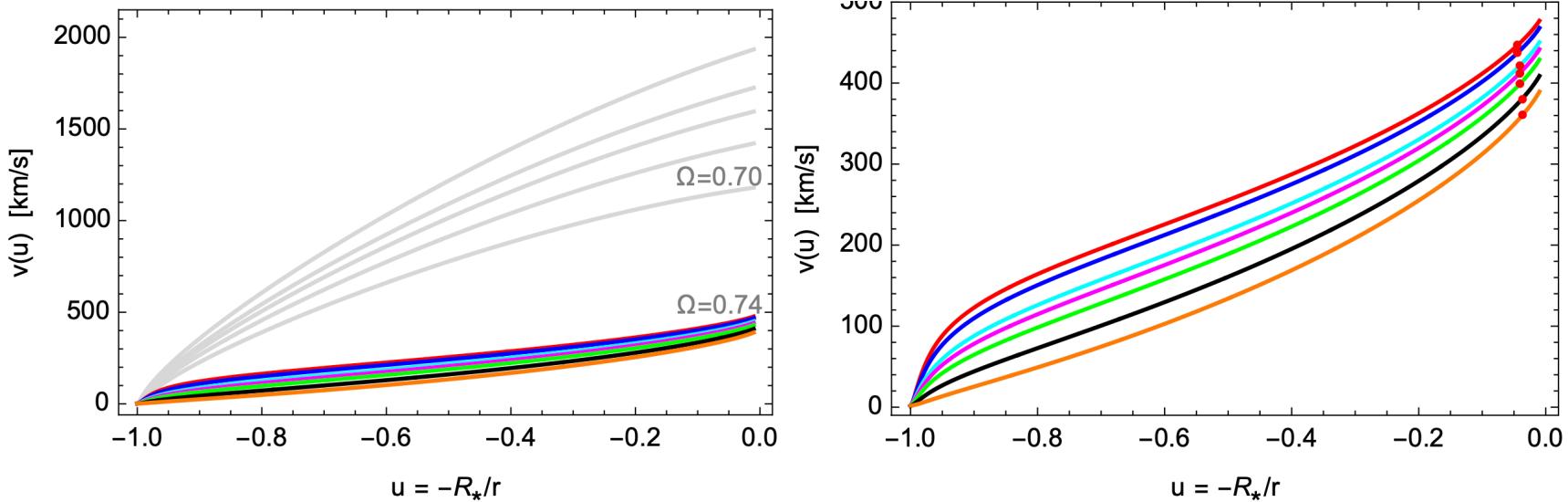


Figure 8. Velocity profiles, $v(u)$, as function of the rotational rate Ω . Left panel: fast solutions ($\Omega \lesssim 0.74$) are plotted in grey lines, while the Ω -slow solutions are in coloured lines: $\Omega = 0.74$ (red line), $\Omega = 0.76$ (blue line), $\Omega = 0.80$ (cyan line), $\Omega = 0.82$ (magenta line), $\Omega = 0.85$ (green line), $\Omega = 0.90$ (black line), and $\Omega = 0.95$ (orange line). Right panel: The same Ω -slow solutions, but zoomed and including the location of the singular points (red dots).

δ -Slow Solution



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doi:[10.1088/0004-637X/737/1/18](https://doi.org/10.1088/0004-637X/737/1/18)

SLOW RADIATION-DRIVEN WIND SOLUTIONS OF A-TYPE SUPERGIANTS

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Received 2011 February 24; accepted 2011 May 20; published 2011 July 25



δ -Slow Solution

To present the topological analysis of this type of solution, we adopt the model T19 from Venero et al. [53]. The stellar and line-force parameters are $T_{\text{eff}} = 19000\text{K}$, $\log g = 2.5$, $R/R_{\odot} = 40$, $k = 0.32$, and $\alpha = 0.5$. We use $\tau_* = 2/3$ as a boundary condition at the stellar surface.

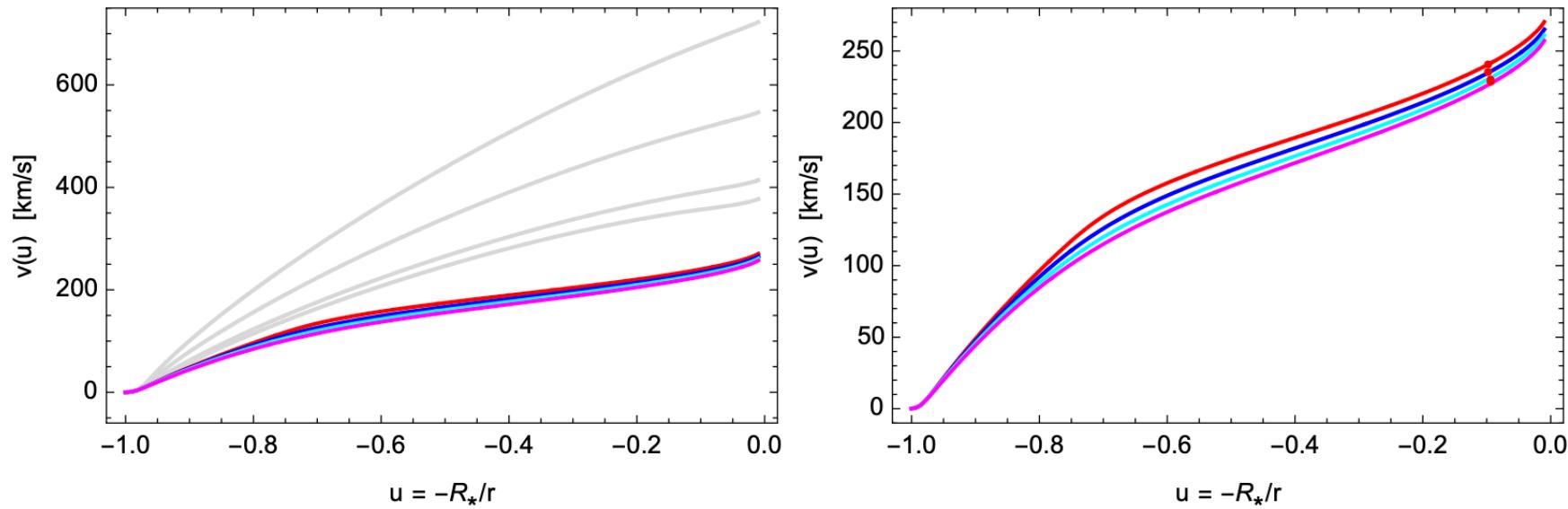


Figure 11. Velocity profiles, $v(u)$, for different values of the line-force parameter δ . Left panel: fast solutions are plotted in grey lines for $\delta = 0.0, 0.1, 0.2, 0.24$, while δ -slow solutions are in coloured lines. Red line corresponds to $\delta = 0.3$, blue line to $\delta = 0.31$, cyan line to $\delta = 0.32$, and magenta line to $\delta = 0.33$. The right panel shows only δ -slow solutions, and the location of the singular points for each solution is shown with a red dot.

Detailed CAK topological analysis



A&A 428, 545–554 (2004)
 DOI: 10.1051/0004-6361:20040325
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**Astronomy
&
Astrophysics**

The influence of rotation in radiation driven winds from hot stars

II. CAK topological analysis

M. Curé¹ and D. F. Rial²

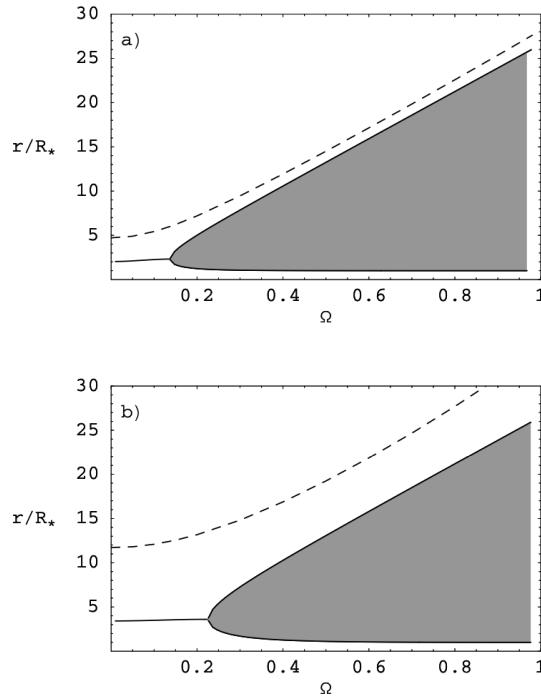


Fig. 6. The curves r_{\min} and r_{\pm} are shown with continuous lines as function of Ω . These curves represent the boundaries for the type of the singular point topology. The numerical results for the location of the singular point is also shown in dashed-line. **a)** is for $\delta = 0.02$ and **b)** for $\delta = 0.1$. See text for details.

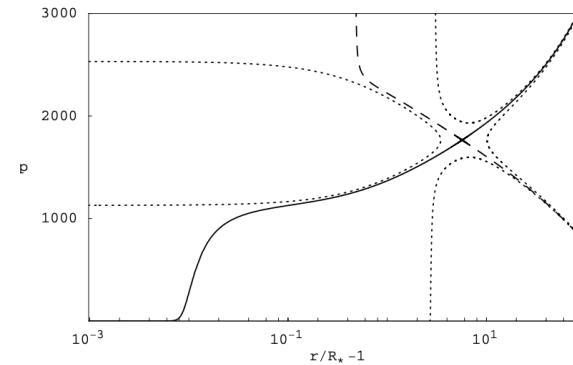
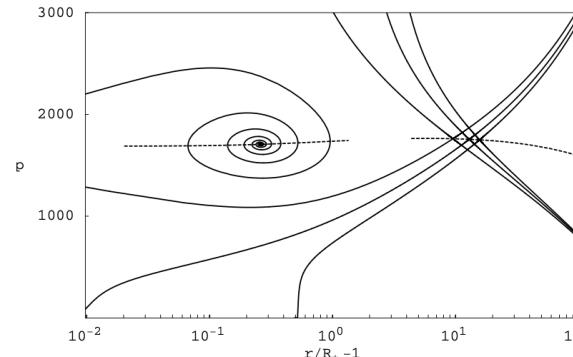


Fig. 7. The topology of the rotating-CAK model, p versus $r/R_\star - 1$ for $\Omega = 0.25$ and $\delta = 0.02$. The unique curve that starting at the stellar surface and reaches infinity is the CAK original solution (continuous-line).



Detailed m-CAK topological analysis

Astron. Nachr. / AN 328, No. 6, 513–520 (2007) / DOI 10.1002/asna.200610748

A new numerical method for solving radiation driven winds from hot stars

M. Curé^{1,*} and D.F. Rial^{2,1}

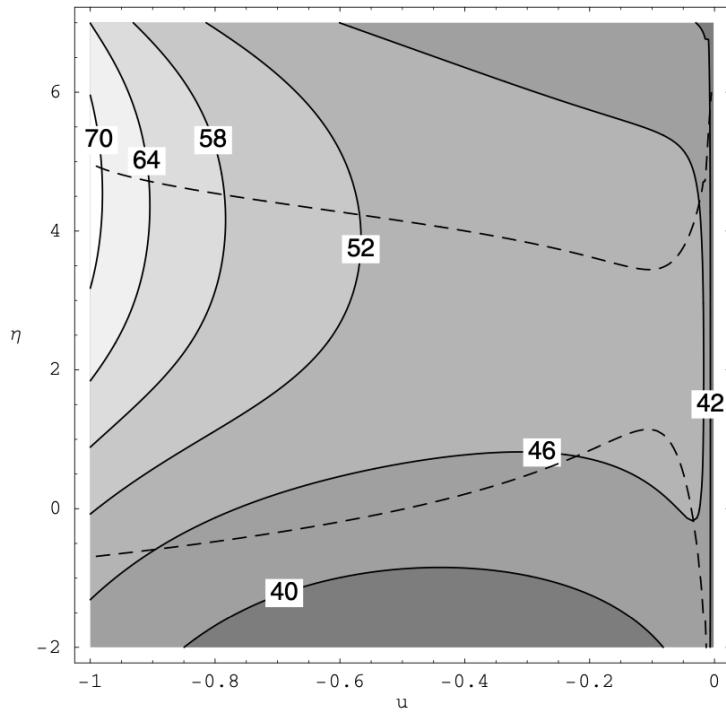


Fig. 1 The iso-contours $R(u, \eta) = 0$ (dashed line) and iso-contours $H(u, \eta, \bar{C}) = 0$ (solid-lines) for different labelled values of \bar{C} as function of the variables u and η . See text for details.

Table 3 Singular points coordinates.

Label	\bar{C}	r_c	u_c	η_c	ζ_c
A	70	1.018	-0.982	+4.905	+7.468
B	46	1.116	-0.896	-0.587	+5.638
C	46	4.505	-0.222	+0.753	+7.253
D	46	29.41	-0.034	-0.176	+7.344
E	46	40.32	-0.020	+4.605	+7.439

	$\det(B)$	Topology
A	—	X-type
B	—	X-type
C	+	Spiral
D	—	X-type
E	—	X-type

Detailed m-CAK topological analysis



Astron. Nachr. / AN 328, No. 6, 513–520 (2007) / DOI 10.1002/asna.200610748

A new numerical method for solving radiation driven winds from hot stars

M. Curé^{1,*} and D.F. Rial^{2,1}

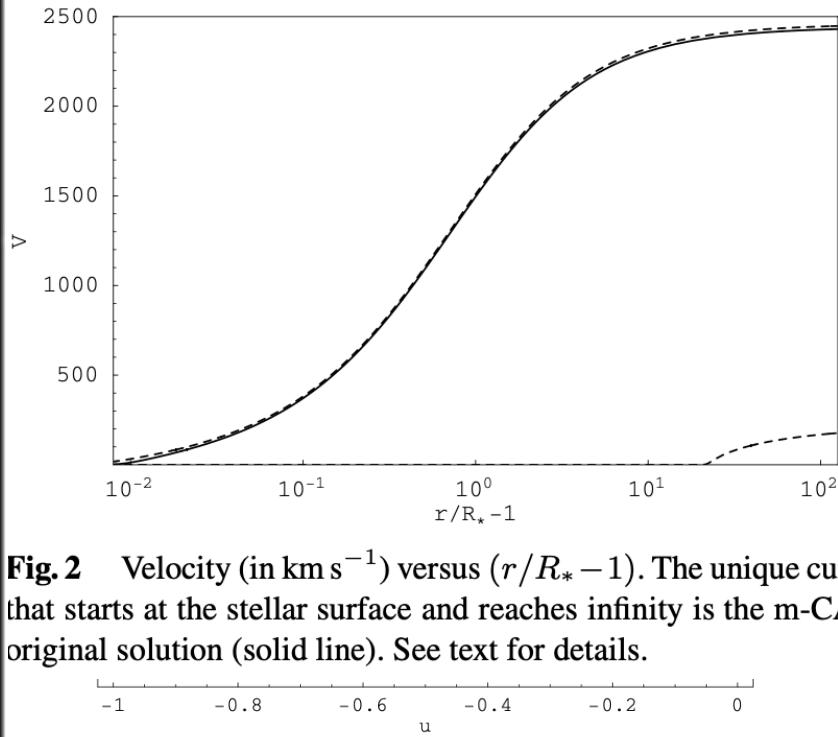


Fig. 2 Velocity (in km s^{-1}) versus $(r/R_* - 1)$. The unique curve that starts at the stellar surface and reaches infinity is the m-CAK original solution (solid line). See text for details.

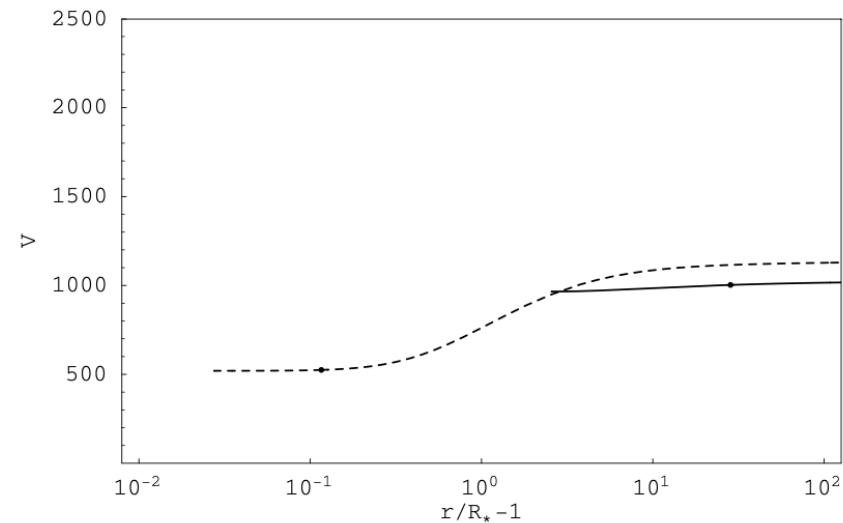


Fig. 3 Velocity (in km s^{-1}) versus $(r/R_* - 1)$. The solution that passes through singular point B (dashed-line) reaches the photosphere with a too high value of the velocity. While the solution passing through the singular point D (solid-line) does not reach the photosphere. See text for details.



Review

Radiation-Driven Wind Hydrodynamics of Massive Stars: A Review

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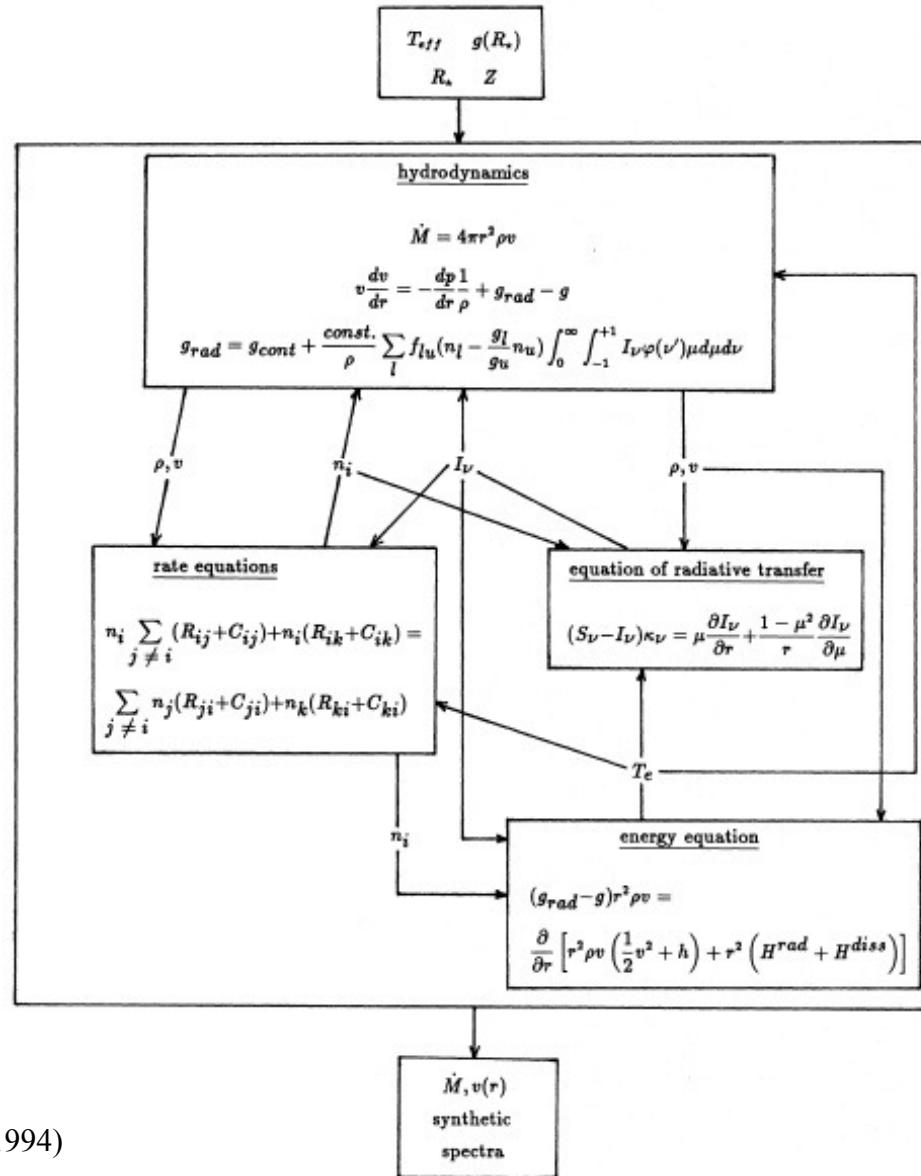


Spherical solutions

Fast and δ -Slow Wind Solutions



Quantitative Spectroscopy



From Pauldrach et al. (1994)

Atmospherical Models

	Detail/Surf. (Butler)	TLUSTY (Hubeny)	Fastwind (Puls)	WM-basic (Pauldrach)	CMFGEN (Hillier)	PoWR (Hamann)	Phoenix (Hauschildt)
geometry							
blanketing							
radiative transfer							
temperature structure							
photosphere							
diagnostic range							
major application							
comments							
execution time							

color coding of following Table

optimum treatment
(at present state of the art)

less than optimum
(but usually faster)

Atmospherical Models

	Detail/Surf. (Butler)	TLUSTY (Hubeny)	Fastwind (Puls)	WM-basic (Pauldrach)	CMFGEN (Hillier)	PoWR (Hamann)	Phoenix (Hauschildt)
geometry	plane-parallel	plane-parallel	spherical	spherical	spherical	spherical	spherical/ plane-parallel
blanketing	LTE	yes	approx.	yes	yes	yes	yes
radiative transfer	observer's frame	observer's frame	CMF/ Sobolev	Sobolev	CMF	CMF	CMF/ obs.frame
temperature structure	radiative equilibrium	radiative equilibrium	e ⁻ therm. balance	e ⁻ therm. balance	radiative equilibrium	radiative equilibrium	radiative equilibrium
photosphere	yes	yes	yes	approx.	from TLUSTY	yes	yes
diagnostic range	no limitation	no limitation	optical/IR	UV	no limitation	no limitation	no limitation
major application	hot stars with negl. winds	hot stars with negl. winds	OB-stars, early A-sgs	hot stars with dense winds, ion. fluxes, SNe	OB(A)-stars, WRs, SNe	WRs	stars below 10kK, SNe
comments	no wind	no wind	expl./backgr. elements	no clumping	start model required		molecules incl. no clumping
execution time	few minutes	hour(s)	few minutes to 0.5 h	1 to 2 h	hours	hours	hours

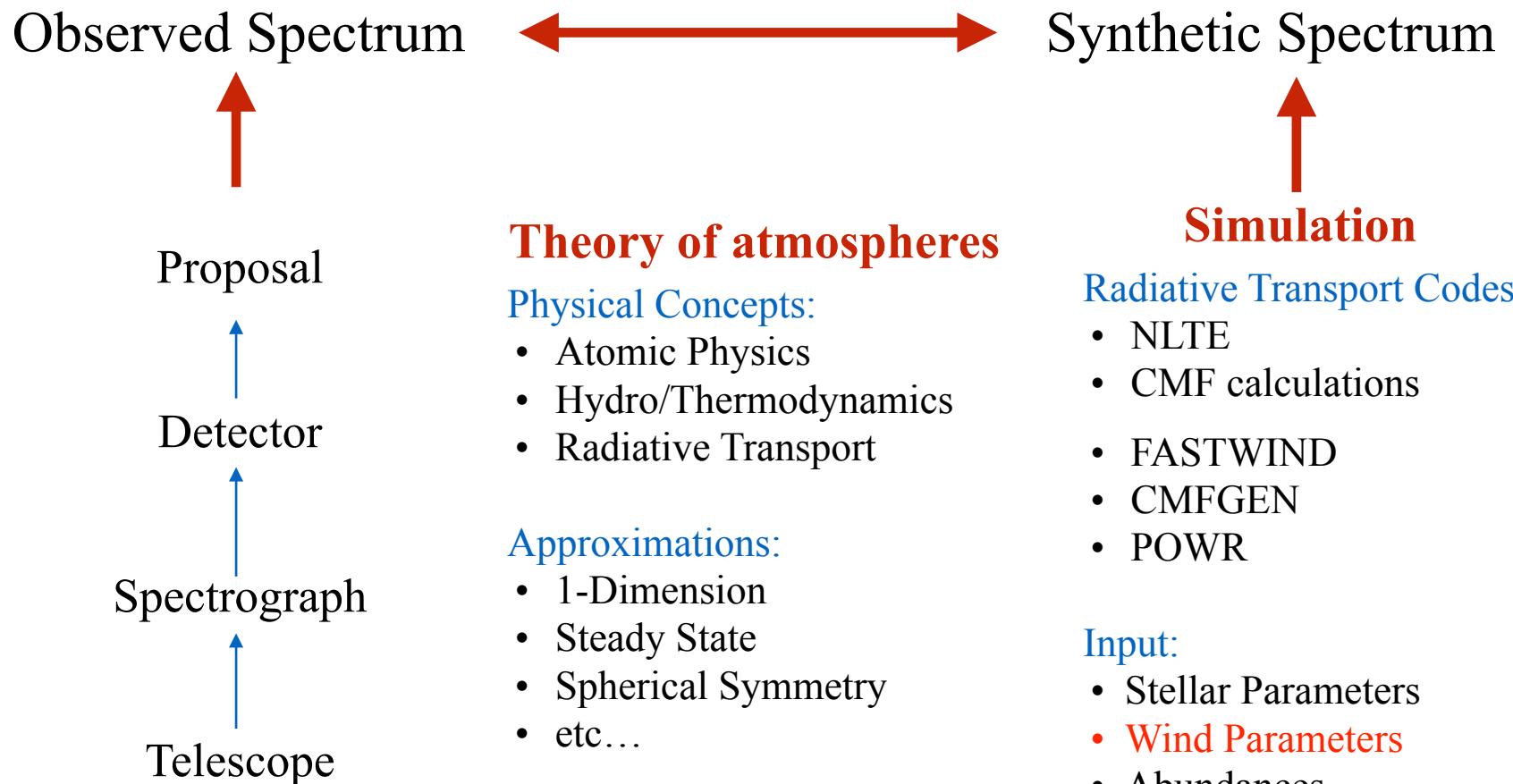
From J. Puls Lectures

https://www.usm.uni-muenchen.de/people/puls/stellar_at/teneriffa_lectures.pdf

Pages 124-125

Quantitative Spectroscopy

Spectral Analysis



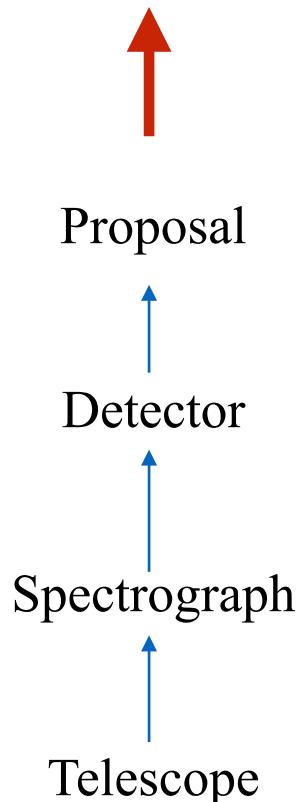
Quantitative Spectroscopy

Spectral Analysis

Comparison by
“eye inspection”

Observed Spectrum

Synthetic Spectrum



Theory of atmospheres

Physical Concepts:

- Atomic Physics
- Hydro/Thermodynamics
- Radiative Transport

Approximations:

- 1-Dimension
- Steady State
- Spherical Symmetry
- etc...

Simulation

Radiative Transport Codes

- NLTE
- CMF calculations
- FASTWIND
- CMFGEN
- POWR

Input:

- Stellar Parameters
- Wind Parameters
- Abundances

B Supergiant winds

“Eye inspection”

Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY



MNRAS **527**, 93–111 (2024)
Advance Access publication 2023 October 31

<https://doi.org/10.1093/mnras/stad3030>

The wind of rotating B supergiants – II. The δ -slow hydrodynamic regime

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ISOSCELES: B Supergiant winds

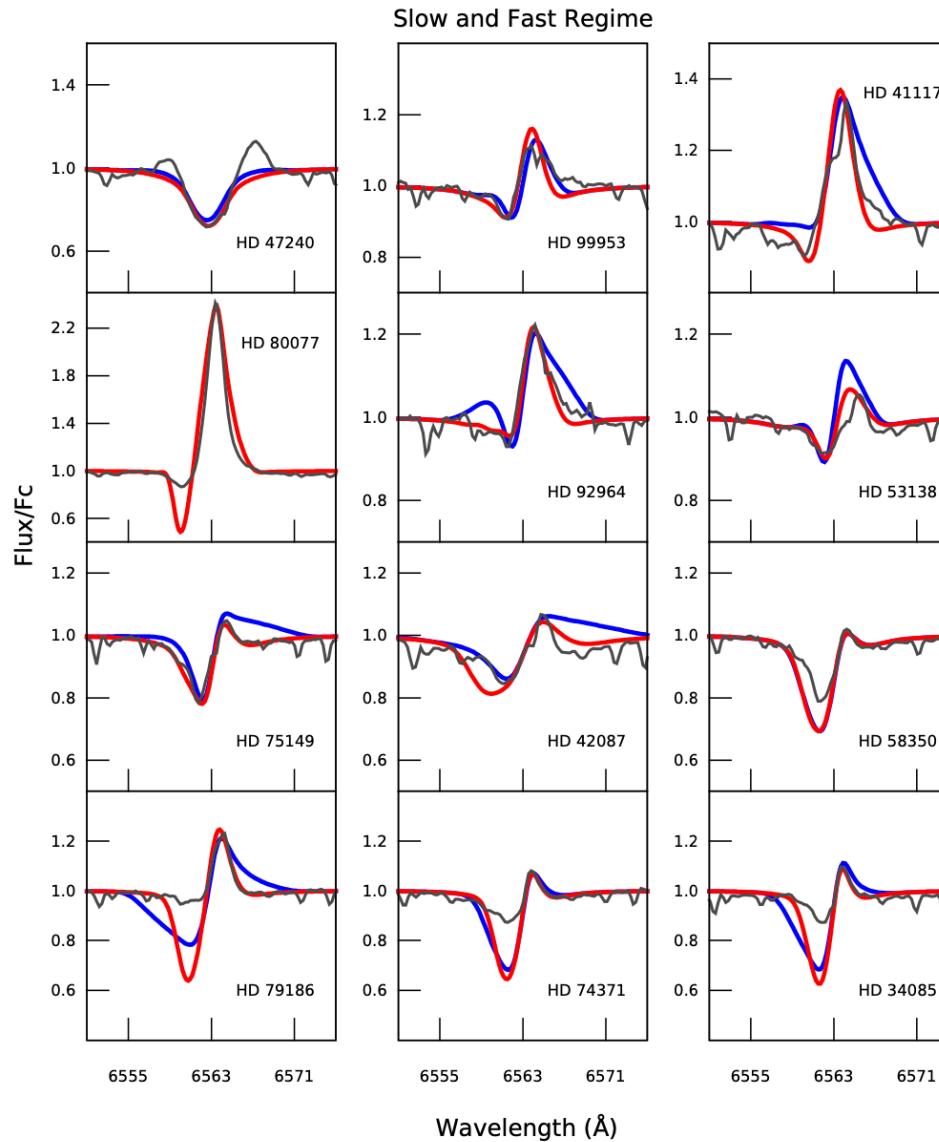


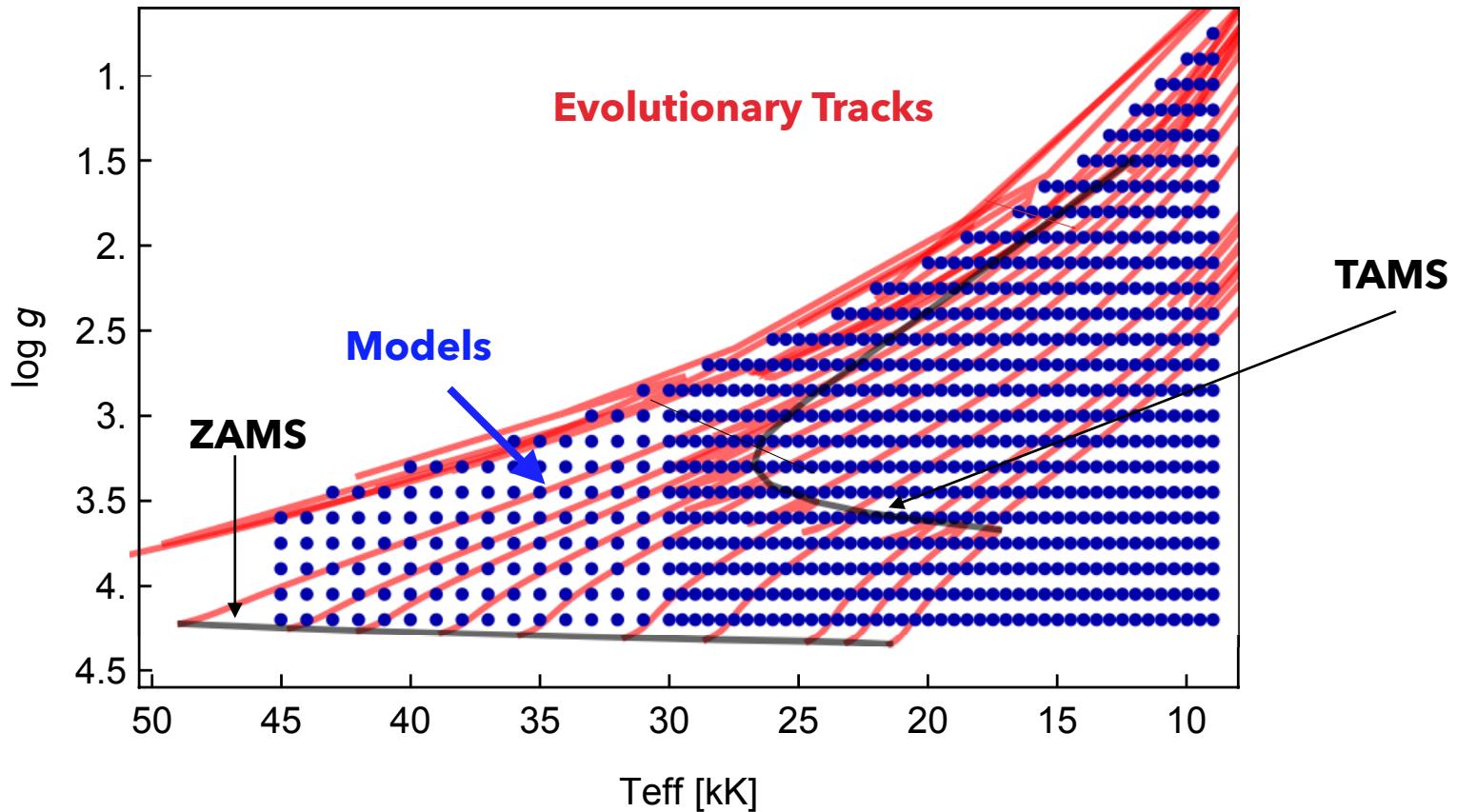
Figure 10. $\text{H}\alpha$ line profile fits for selected B supergiants using δ -slow (red) and fast (blue) hydrodynamical models. The corresponding stellar and wind parameters are given in Tables 2 and 3, respectively. In the case of HD 80077, we could not find a fast solution. The line profiles calculated for the fast and slow regimes for the star HD 58350 exactly overlap.

GrId of Stellar atmOSphere and hydrodynamiC modELs of massivE Stars



Araya et al. 2023 *Winds of Stars and Exoplanets*.
Proceedings of the International Astronomical Union, Volume 370, pp. 180-184

ISOSCELES: HYDWIND

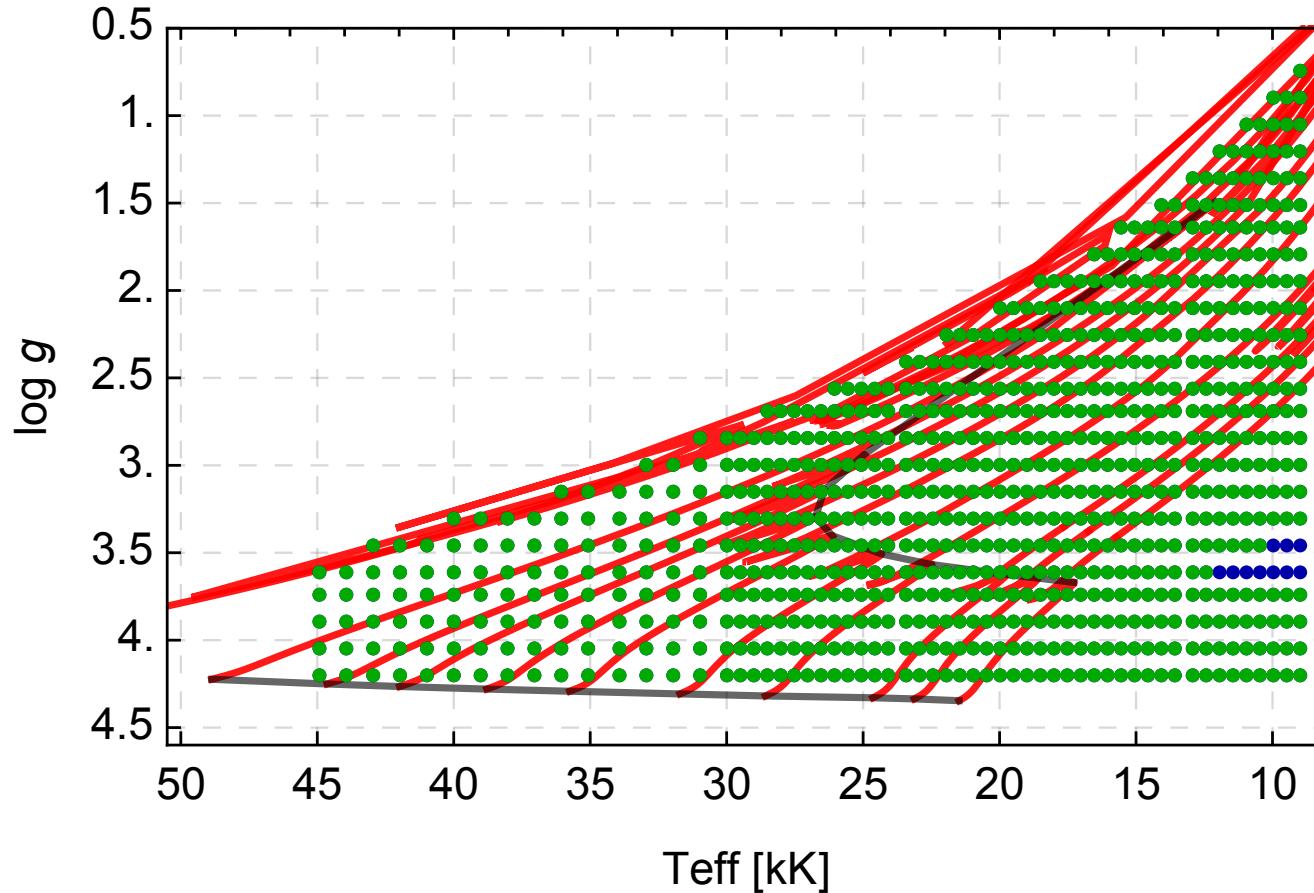


4 056 755 converged models

ISOSCELES: FASTWIND



Fastwind grid (without rotation)



Total = 573 423 converged models

Temperature

9 kK - 30 kK (steps of 500 k)

31 kK - 45 kK (steps of 1000 k)

log g

4.2 - 0.75 (steps of 0.15)

Line force parameters

α	0.45	0.47	0.51	0.53	0.55	0.57	0.61	0.65					
k	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	
δ	0.	0.04	0.1	0.14	0.2	0.24	0.3	0.31	0.32	0.33	0.34	0.35	

Si abundance log ϵ_{Si}

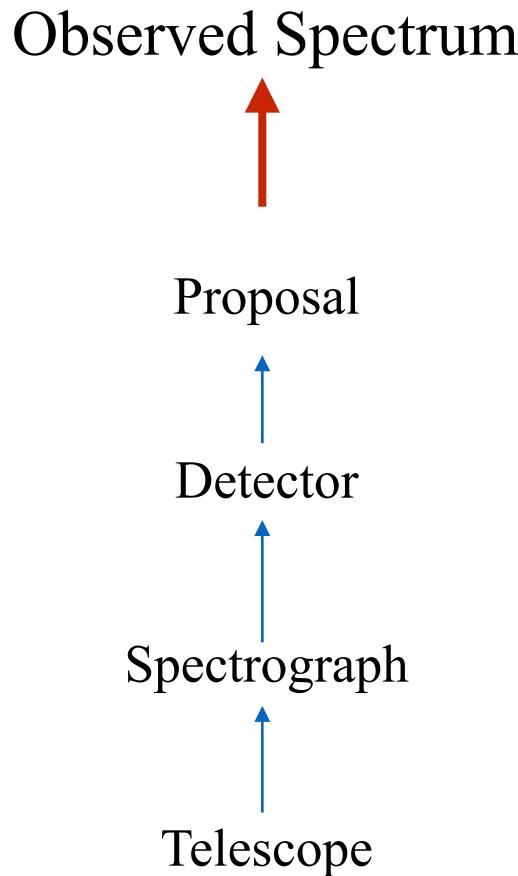
7.21, 7.36, 7.51 (solar), 7.66, and 7.81 dex.

Micro-turbulent velocity

1, 5 , 10, 15, 20, 25 km/s

“New” Quantitative Spectroscopy

Spectral Analysis via Machine Learning



Best Synthetic Spectrum
Stellar and wind parameters

Machine Learning

- Genetic Algorithm
- Grid Search (Brute Force)
- Classification via Gaussian Mixture Models
- Neural Networks
- Chi-Square
- Bayesian Neural Networks

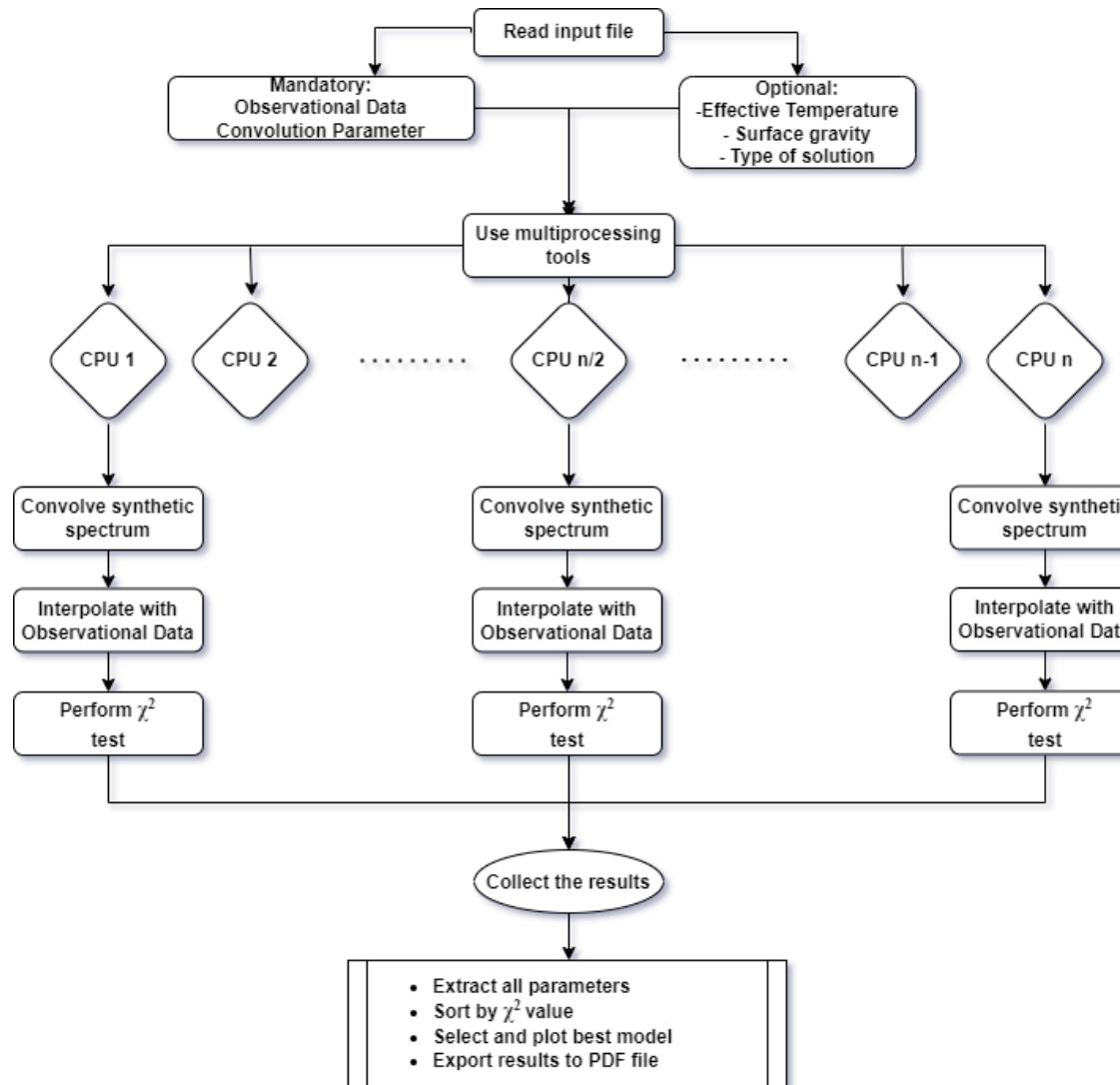
Synthetic Spectra Database
e.g., ISOSCELES

Michel Curé
Universidad de Valparaíso

ISOSCELES: Grid Search

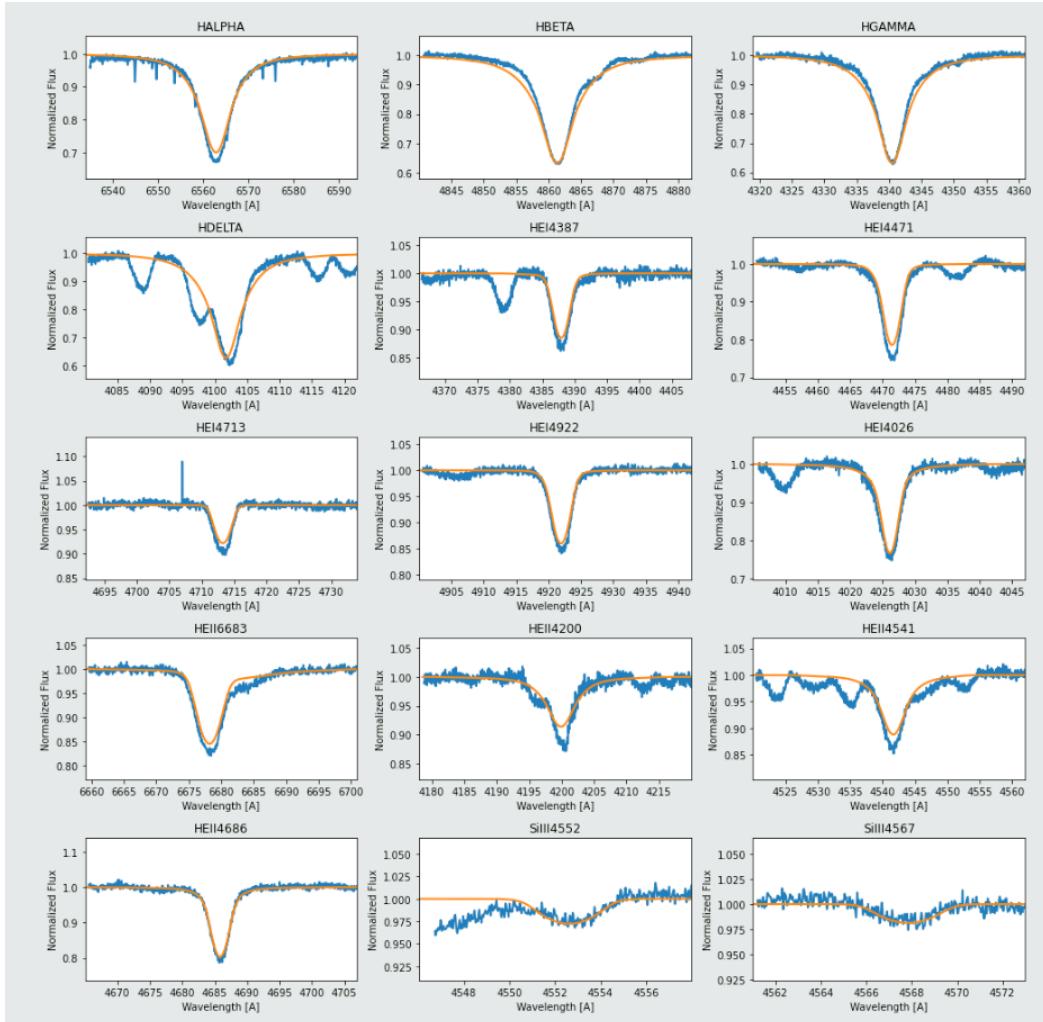


Natalia Machuca PhD thesis UV



ISOSCELES: First Results

HD 146333 (O8.5 V). Fast solution



$$T_{\text{eff}} = 36000 \text{ K}$$

$$\log g = 4.05$$

$$\alpha = 0.51$$

$$k = 0.30$$

$$\delta = 0.10$$

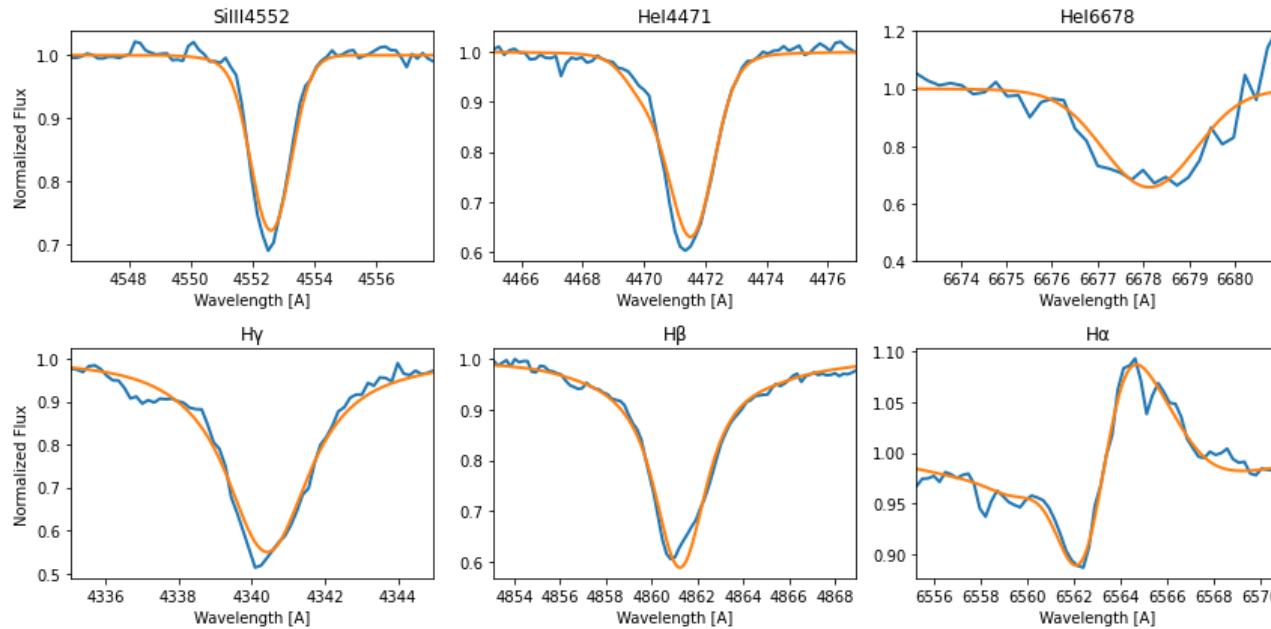
$$\dot{M} = 3.08 \times 10^{-8} M_{\odot}/\text{yr}$$

$$v_{\infty} = 597.41 \text{ km/s}$$

ISOSCELES: First Results



HD 99953



	Haucke et al. 2018	This work
T _{eff} [K]	19000	18500
log g	2.30	2.40
α	–	0.53
k	–	0.15
δ	–	0.34
β	2.0	–
\dot{M} [$10^{-6} M_{\odot} /$ yr]	0.13	0.24
v _∞ [km/s]	500	254

Observations from Haucke et al. 2018

WML relationship

Since the stellar winds of massive stars are driven by radiation, it is reasonable to expect that the mechanical momentum of the flowing material is related to the momentum of the radiation field.

Wind Momentum Definition

$$D_{\text{mom}} = \dot{M} v_\infty \sqrt{R_*/R_\odot}$$

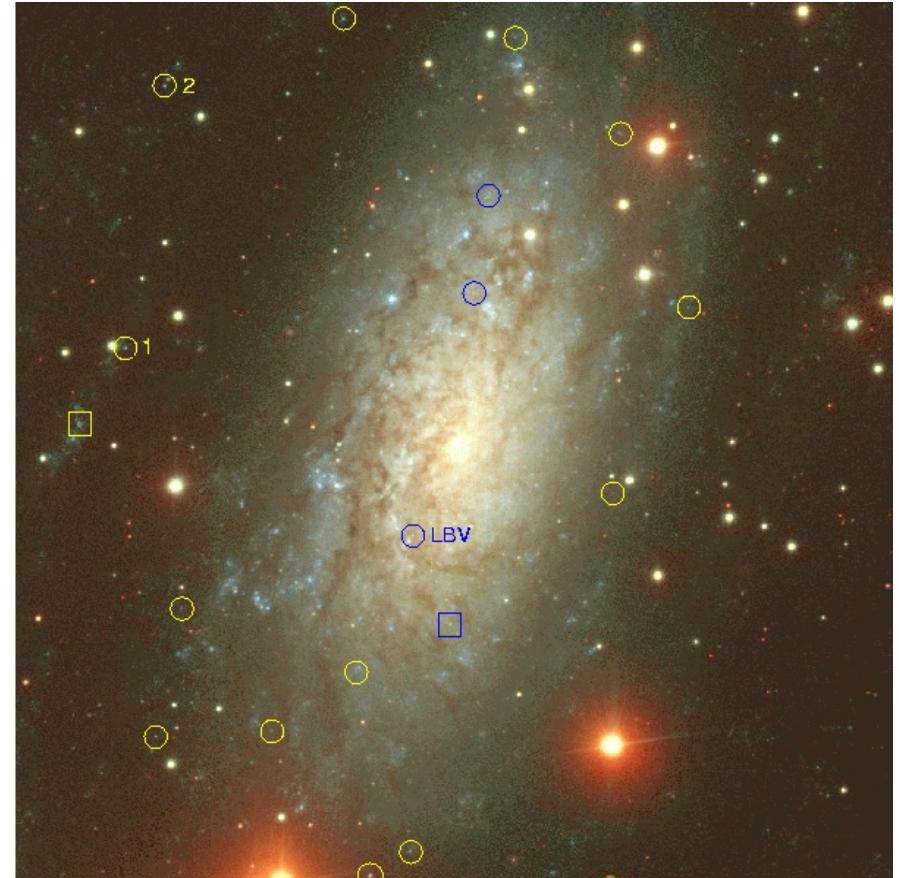
Wind momentum Luminosity Relationship

$$D_{\text{mom}} \propto \left(\frac{L}{L_\odot} \right)^x$$

WML relationship

Kudritzki et al. 1999

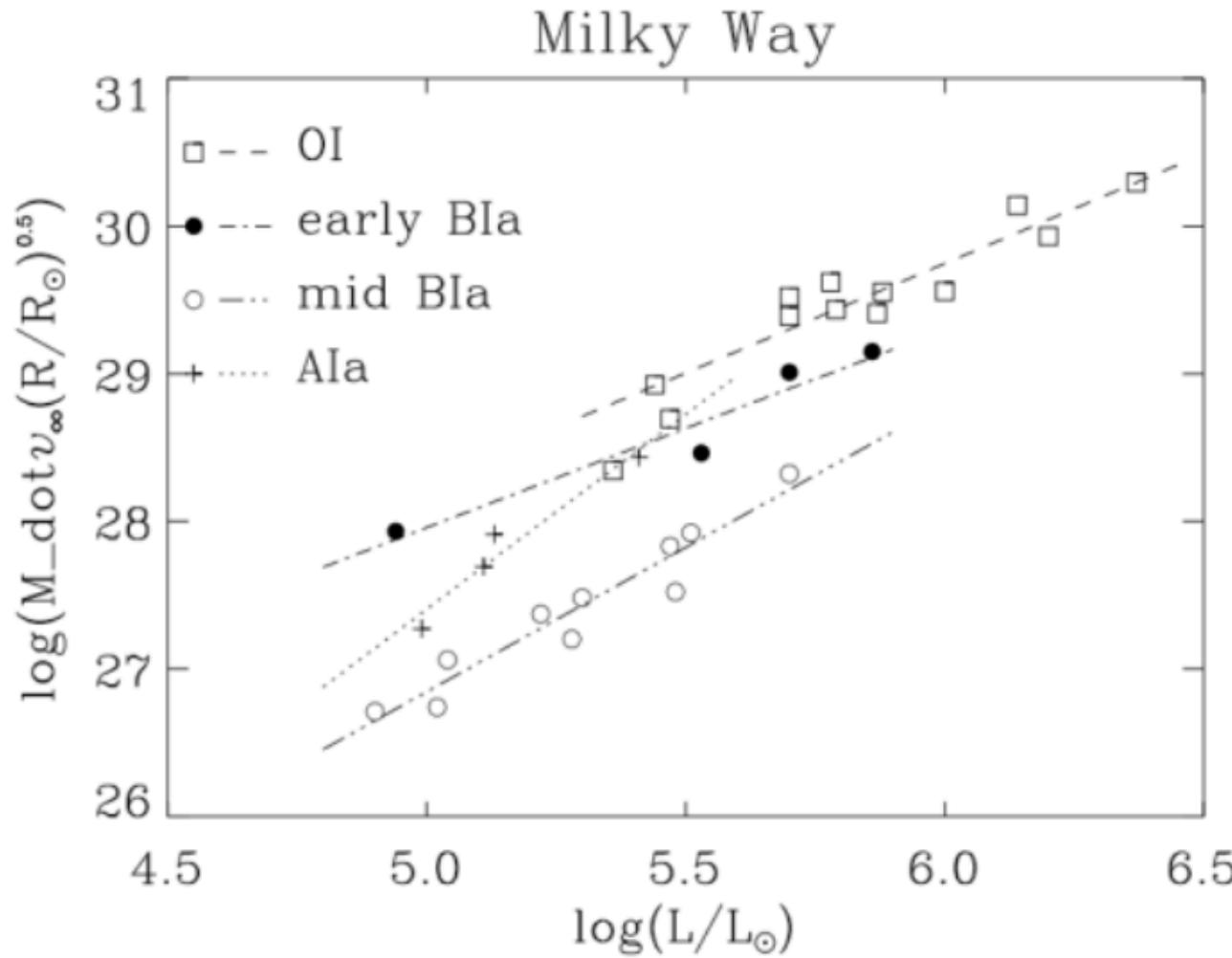
$$\dot{M} v_\infty \propto \frac{1}{\sqrt{R_*}} L^x$$



NGC3621

WML relationship

Kudritzki et al. 1999



ISOSCELES:WML relationship



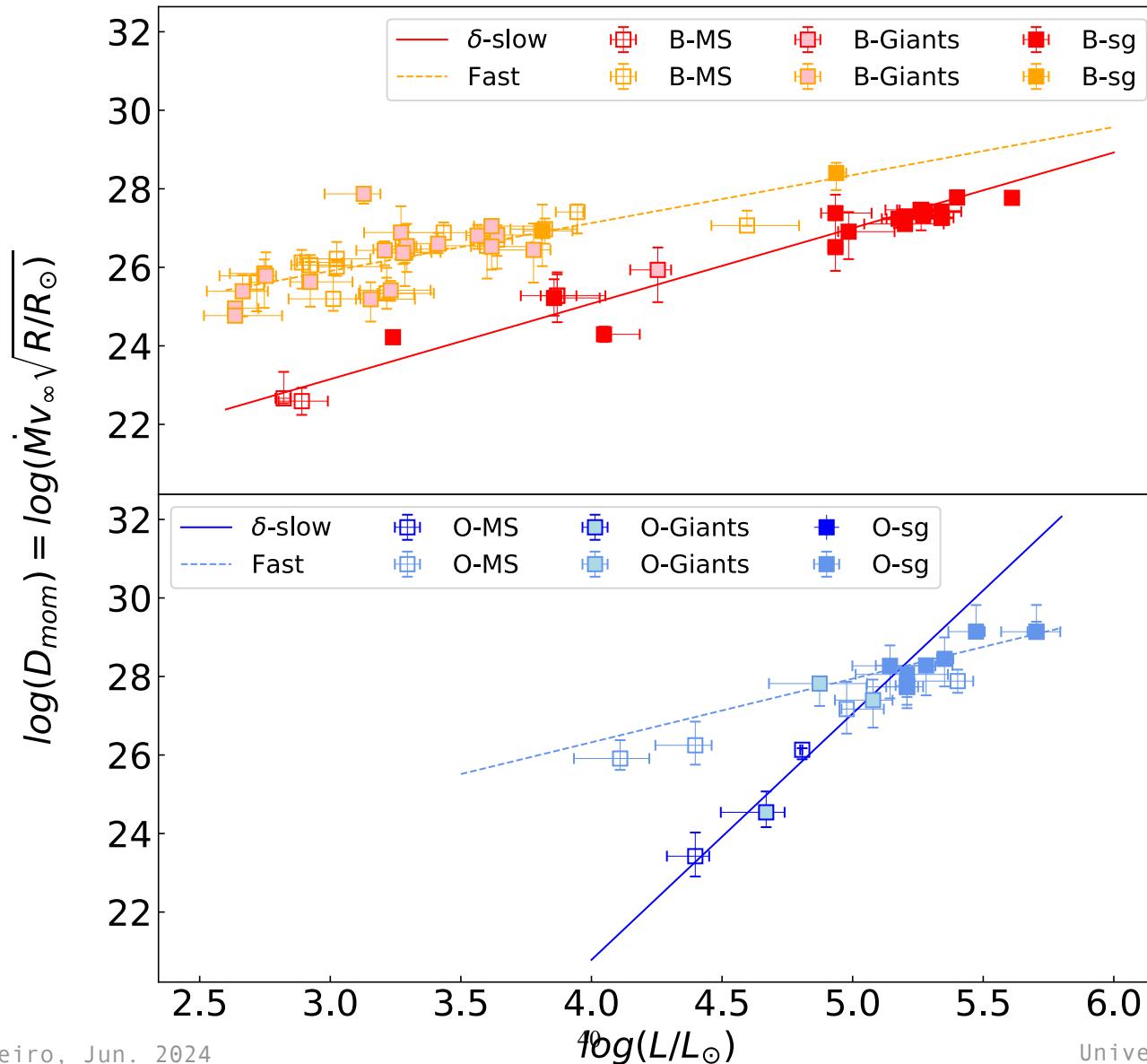
Natalia Machuca PhD thesis UV: work in progress, 100 OB stars analysed



ISOSCELES:WML relationship

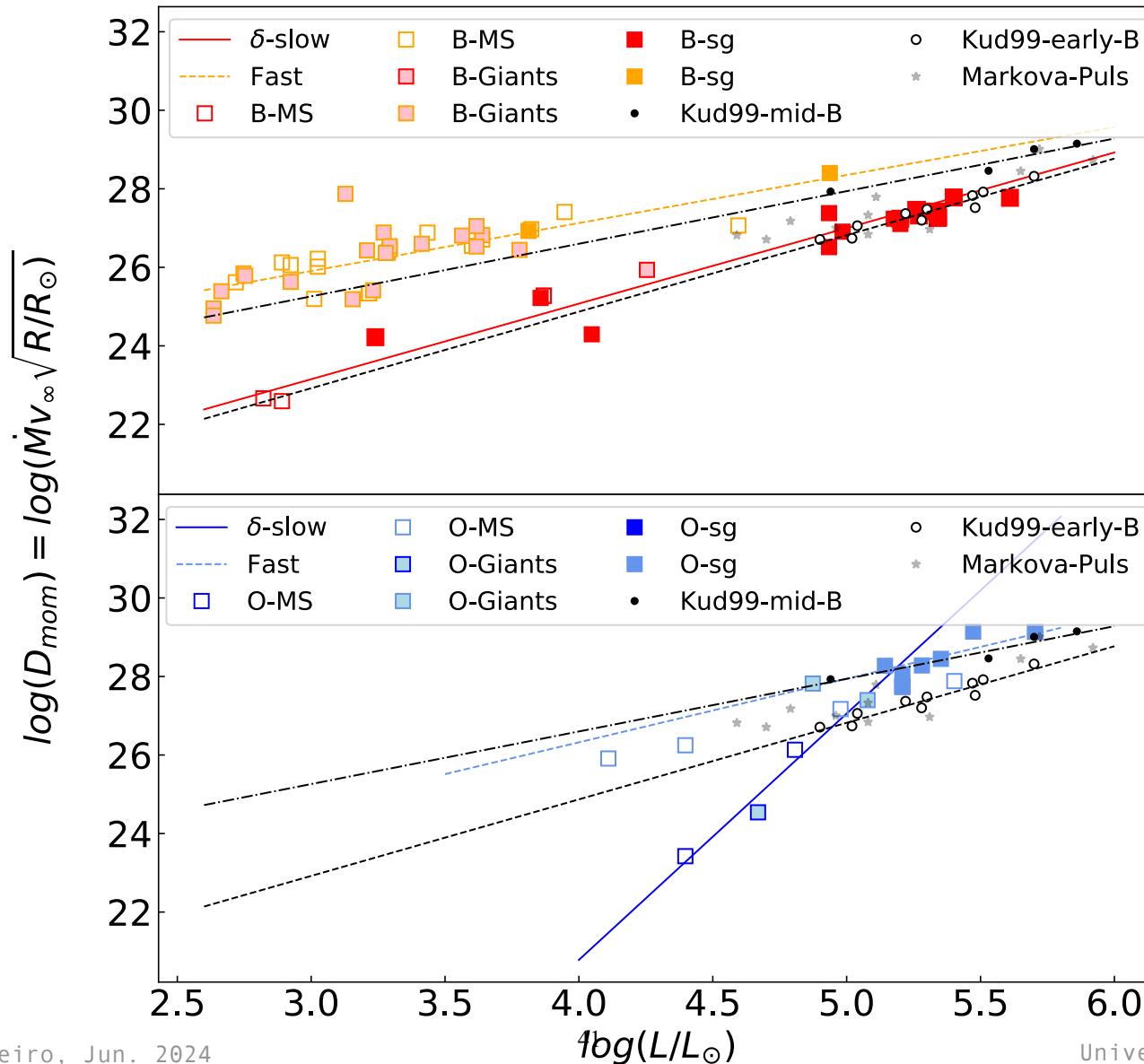


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ISOSCELES:WML relationship

Natalia Machuca PhD thesis UV: work in progress, 100 OB stars analysed



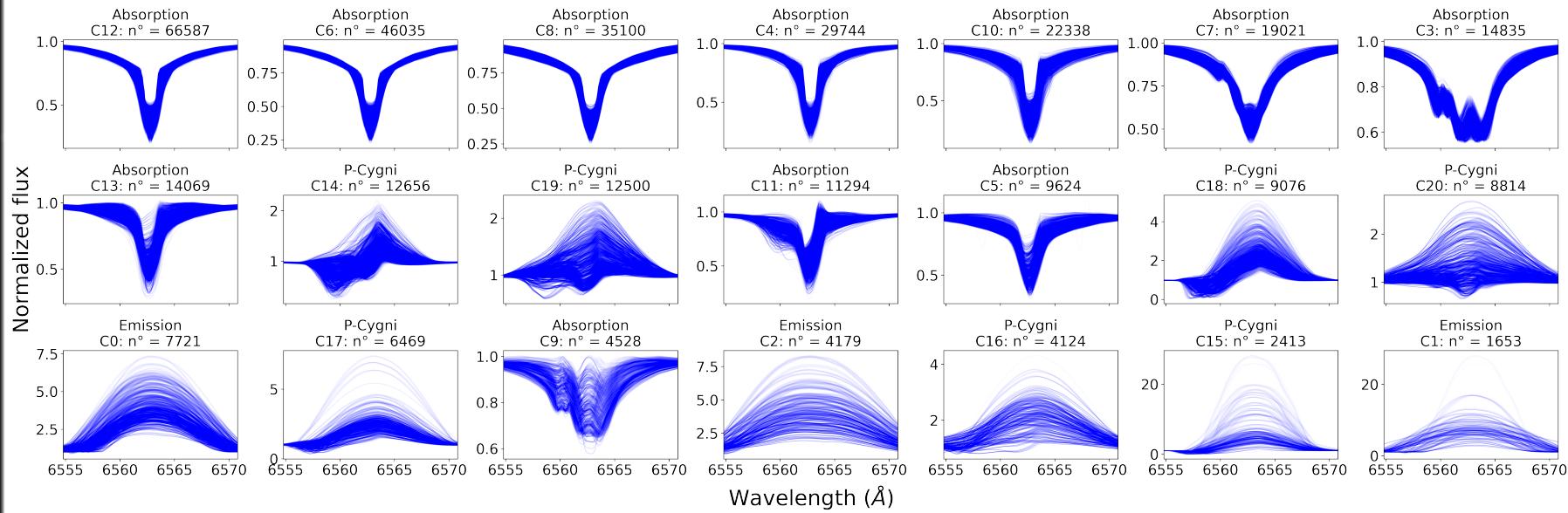
ISOSCELES Grid & Machine Learning



ISOSCELES: Machine Learning

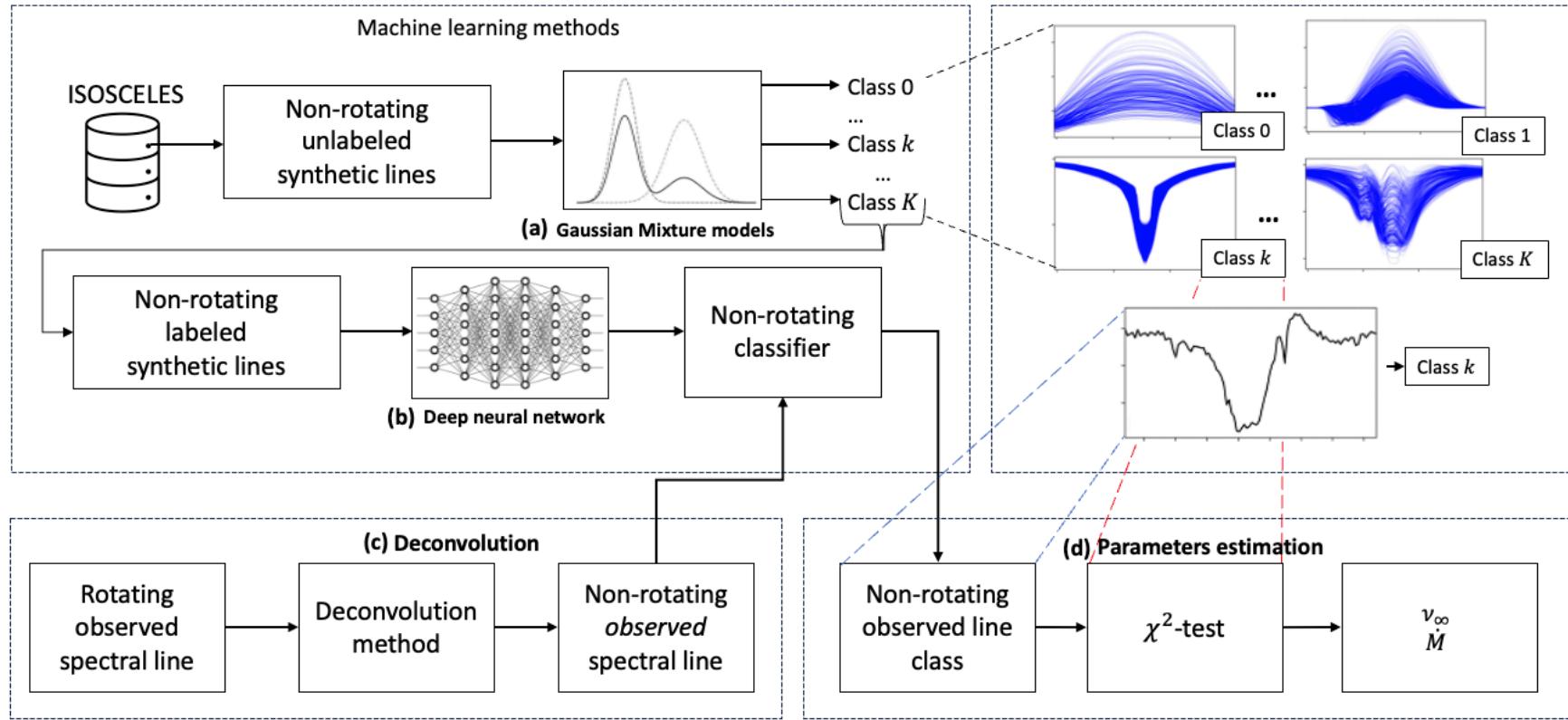


Gaussian Mixture Model Classification,
Neural Networks and “Brut” force:
ISOSCELES’ δ -Slow Wind Solutions (~ 330000 models)
H α spectral line

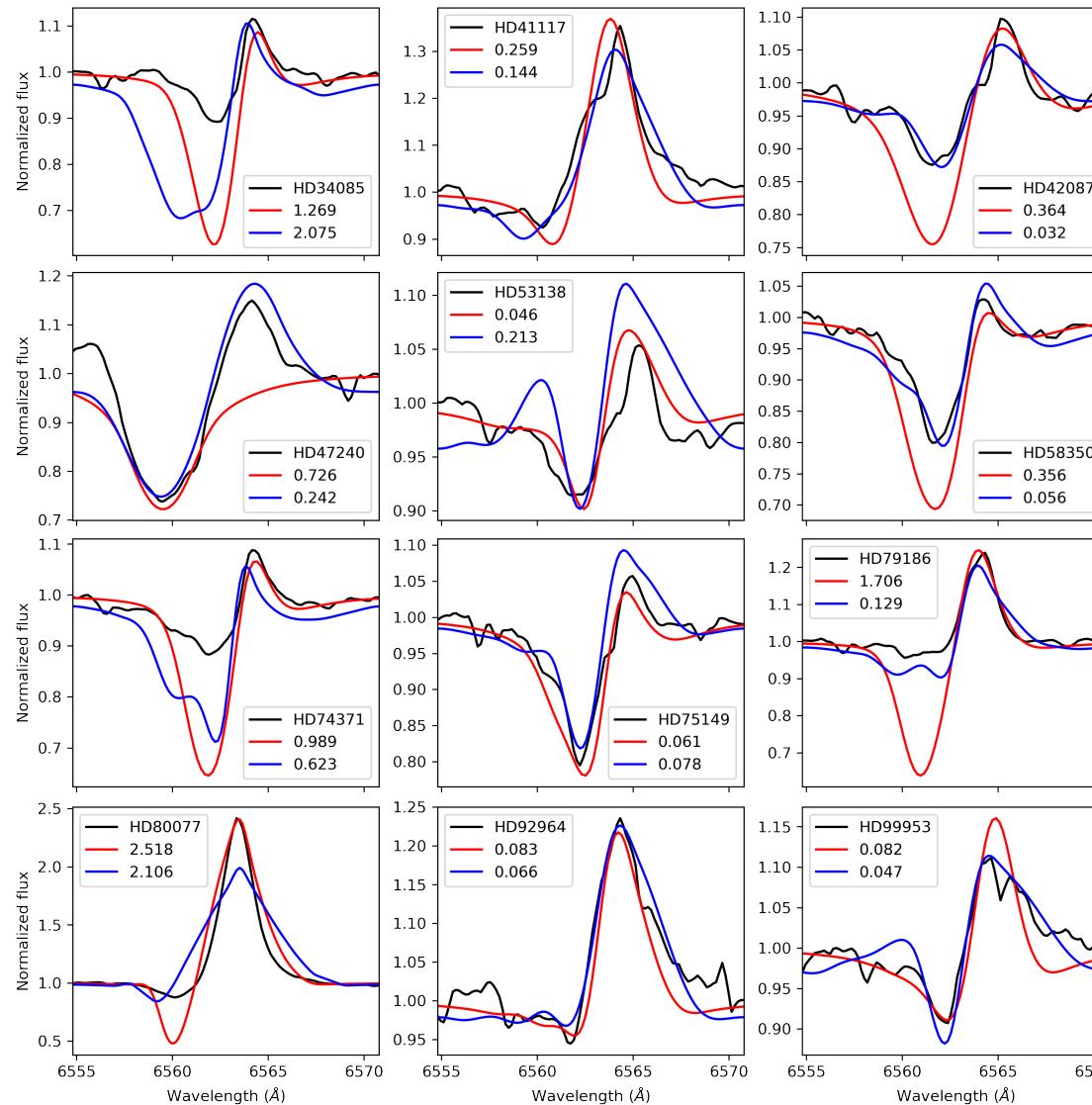


Felipe Ortiz Master's thesis UV

ISOSCELES: Machine Learning

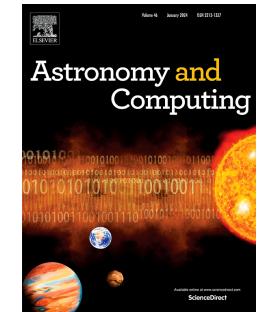


ISOSCELES: B Supergiant winds



Ortiz et al. 2024,
Astronomy and
Computing

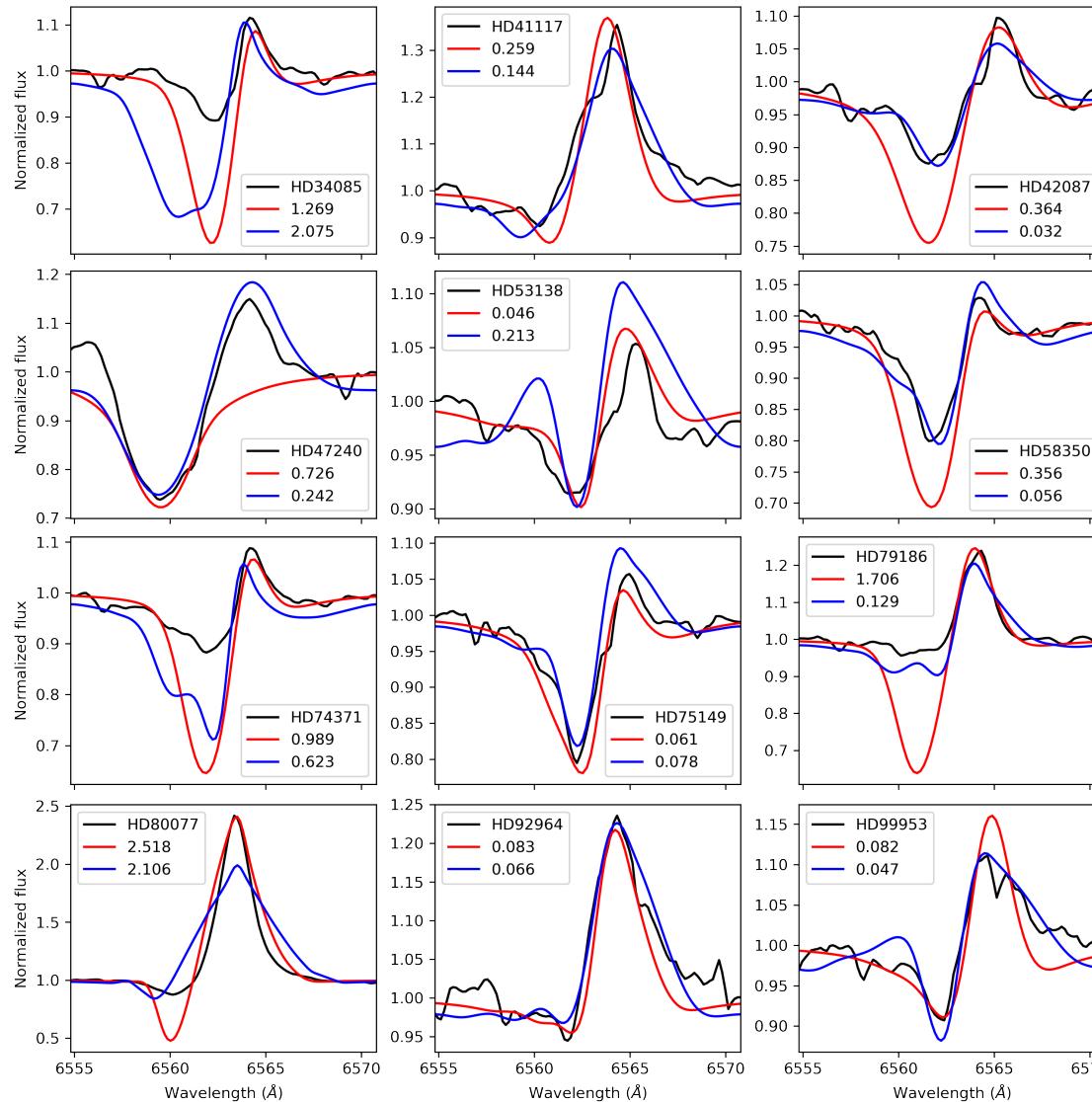
Submitted



ISOSCELES: B Supergiant winds

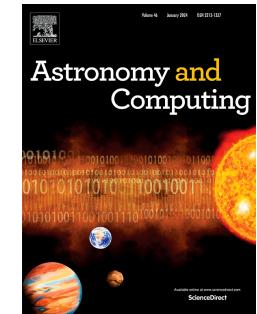


Improvements in 9 of the 12 B-supergiants



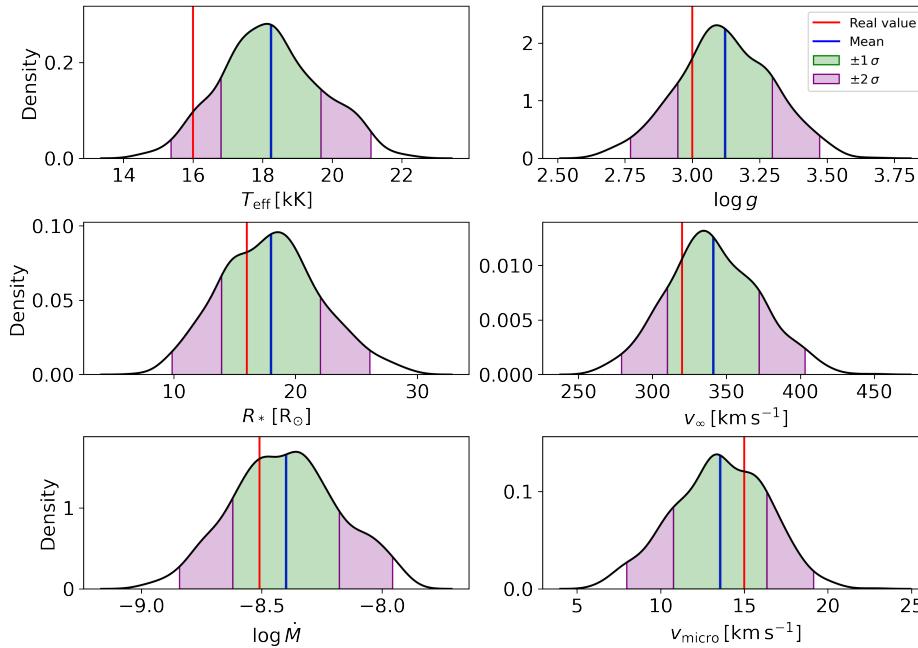
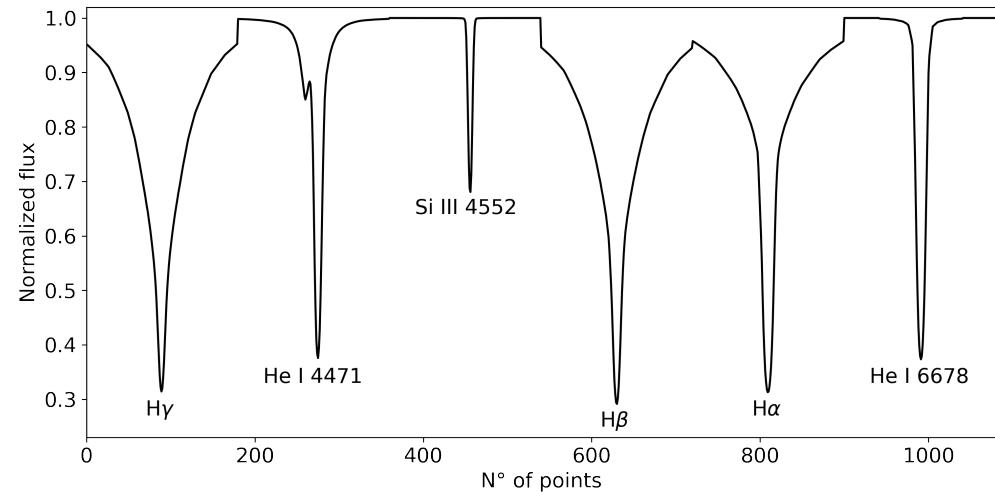
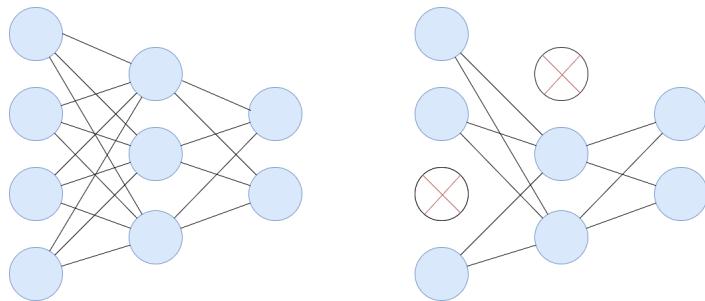
Ortiz et al. 2024,
Astronomy and
Computing

Submitted



Bayesian Neural Networks

Monte Carlo Dropout



Felipe Ortiz PhD thesis UV

ISOSCELES

- ISOSCELES is the **first** grid of synthetic data for massive stars that involves both, the **m-CAK** hydrodynamics and the NLTE radiative transport.
- Calculate new grids including UV range for different metallicities
- The results obtained from ISOSCELES should have a better justification than the currently and widely calculated models with a **β -law** in quantitative spectroscopic analyses, specially for high values of β
- We are analysing the x^2 distributions and then use them to compute the uncertainties for each derived parameter
- More fast and efficient “searching” algorithms (Machine Learning)
- Bayesian Neural Networks, etc.

Radiation Driven Stellar Winds



The Lambert-W procedure

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New Hydrodynamic Solutions for Line-driven Winds of Hot Massive Stars Using the Lambert W-function

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Abstract

Hot massive stars present strong stellar winds that are driven by absorption, scattering, and reemission of photons by the ions of the atmosphere (line-driven winds). A better comprehension of this phenomenon, and a more accurate calculation of hydrodynamics and radiative acceleration, is required to reduce the number of free parameters in spectral fitting and to determine accurate wind parameters such as mass-loss rates and velocity profiles. We use the non-LTE model-atmosphere code CMFGEN to numerically solve the radiative transfer equation in the stellar atmosphere and to calculate the radiative acceleration $g_{\text{rad}}(r)$. Under the assumption that the radiative acceleration depends only on the radial coordinate, we solve analytically the equation of motion by means of the Lambert W-function. An iterative procedure between the solution of the radiative transfer and the equation of motion is executed in order to obtain a final self-consistent velocity field that is no longer based on any β -law. We

Mass Conservation

$$\dot{M} = 4 \pi r^2 \rho v$$

Momentum Conservation

$$v \frac{dv}{dr} = - \frac{1}{\rho} \frac{dP}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{line} \left(\rho, \frac{dv}{dr}, n_e \right)$$



Mass Conservation

$$\dot{M} = 4 \pi r^2 \rho v$$

Momentum Conservation

$$v \frac{dv}{dr} = - \frac{1}{\rho} \frac{dP}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{line} \left(\rho, \frac{dv}{dr}, n_e \right)$$



Radiation Driven Wind Hydrodynamics



Mass Conservation

$$\dot{M} = 4 \pi r^2 \rho v$$

Momentum Conservation

$$g^{line}(r)$$

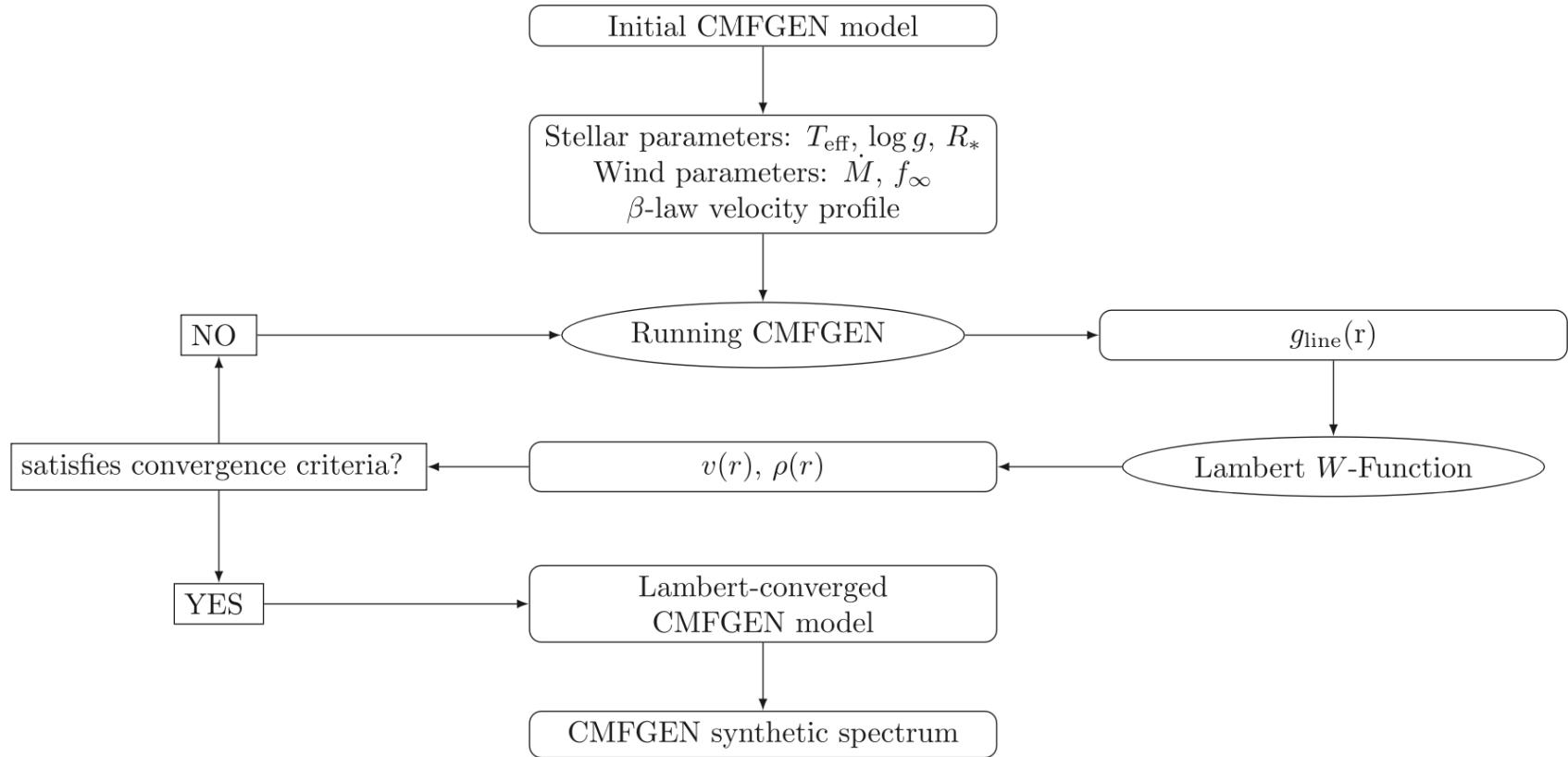
$$v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM(1-\Gamma)}{r^2} + g^{line} \left(\rho, \frac{dv}{dr}, n_e \right)$$



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y
Astronomía



The Lambert-W procedure



The Lambert-W procedure

ζ Puppis parameters

Table 2

Summary of Self-consistent Solutions Performed by Sander et al. (2017),
Gormaz-Matamala et al. (2019), and This Present Study (Table 1)

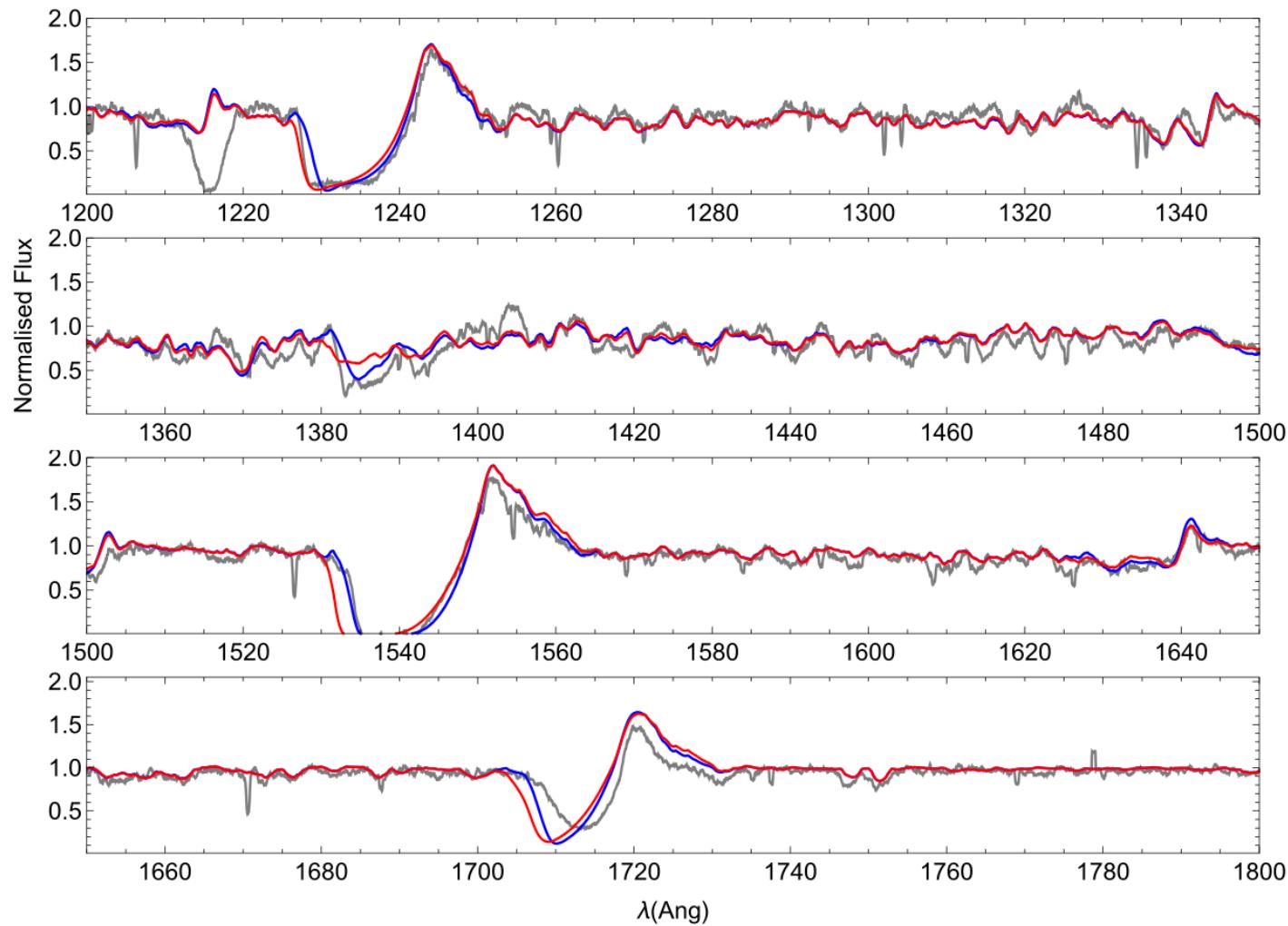
	Sander et al. (2017)	Gormaz-Mata- mala et al. (2019)	Björklund et al. (2021)	This work
RT code	POWR	FASTWIND	FASTWIND	CMFGEN
Hydro method	...	HYDWIND	...	Lambert W - function
T_{eff} [kK]	40.7	40	40.7	41
$\log g$	3.63	3.64	3.65	3.6
R_*/R_{\odot}	15.9	18.7	18.91	17.9
v_{∞} [km s ⁻¹]	2046	2700	2302	2740
\dot{M} [$10^{-6} M_{\odot}$ yr ⁻¹]	1.6	5.2	2.5	2.7
f_{∞} [1/ D_{∞}]	0.1	0.2	...	0.10

Note. These three models assume the same abundances as Bouret et al. (2012). The radiative transfer code used to perform the respective synthetic spectra is also marked, together with the code/methodology used to calculate the hydrodynamics. Equivalence between filling factor and clumping factor D_{∞} (used by POWR) was made assuming the interclump medium is void, following Sundqvist & Puls (2018). Additionally, we include from the grid of Björklund et al. (2021), the closest parameters to ζ Puppis.

The Lambert-W procedure



ζ Puppis visual wavelength range



The Lambert-W procedure

ζ Puppis UV wavelength range

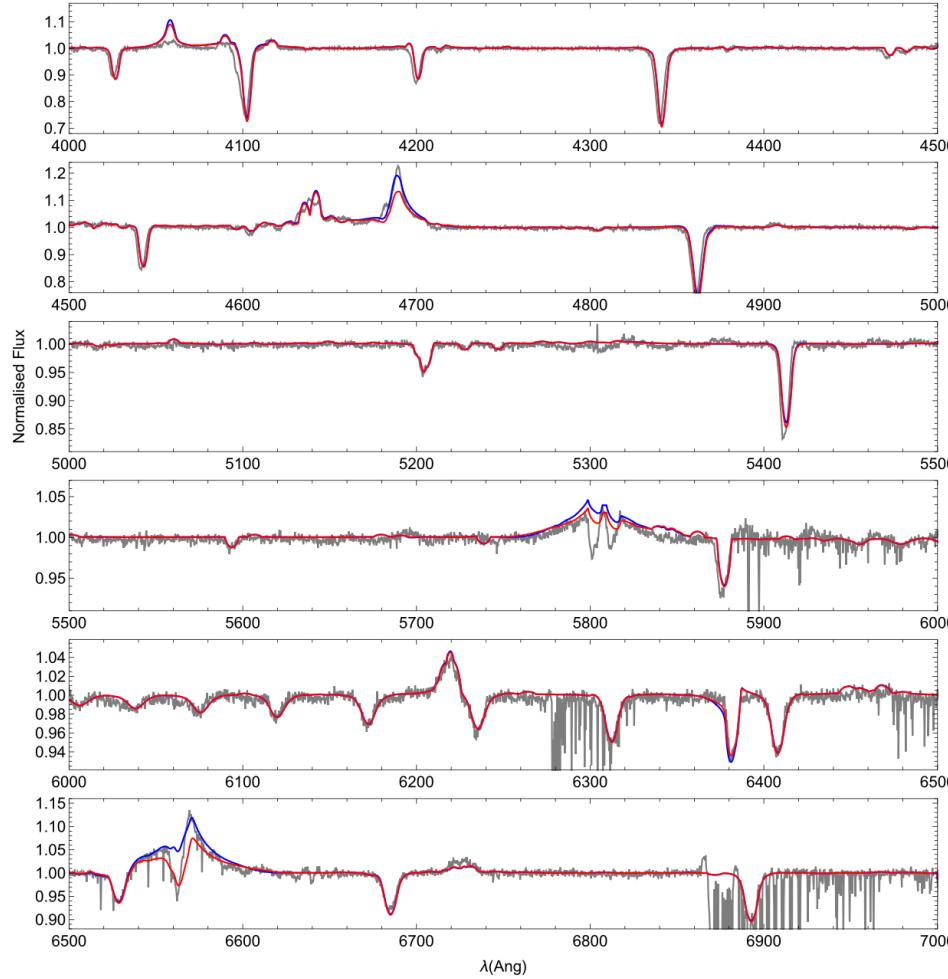
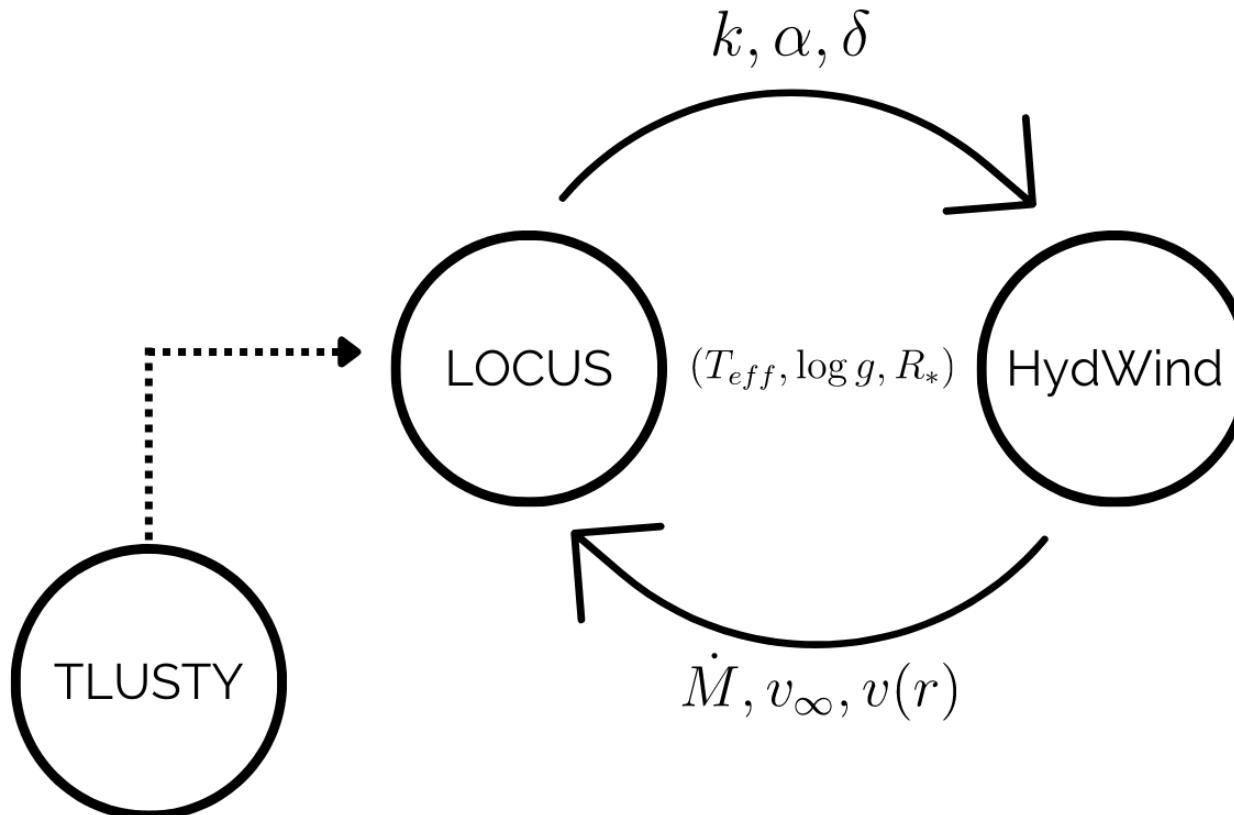


Figure 24. Comparison of the Lambert solution for ζ Puppis (red), compared with the initial CMFGEN spectrum (blue), with the observed FEROS spectrum for the visible range from 4000 to 7000 Å. These spectra were computed using CMF_FLUX (Busche & Hillier 2005), and rotational broadening was taken into account using standard convolution procedures. However, the H_{α} profile (and particularly the absorption component), for example, is modified when rotational broadening is explicitly calculated using a formal solution of the transfer equation (Hillier et al. 2012).

Self-consistent winds

LOCUS: Line fOrCe mUltiplieR parameterS



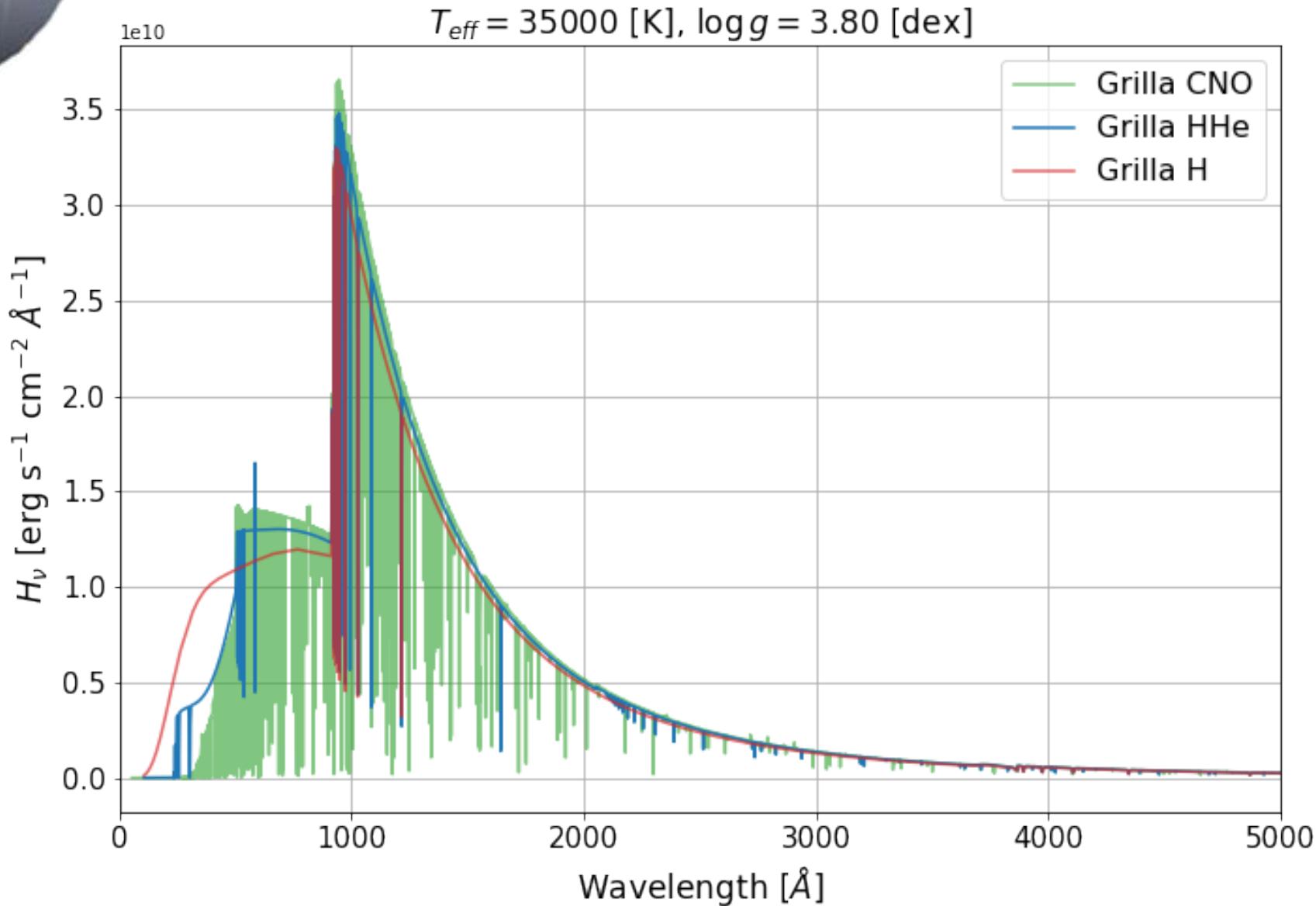
New Mass-loss rates

Grid Name	Elements
H	Hydrogen
HHe	Hydrogen
	Helium
CNO	Hydrogen
	Helium
	Carbon
	Nitrogen
	Oxygen

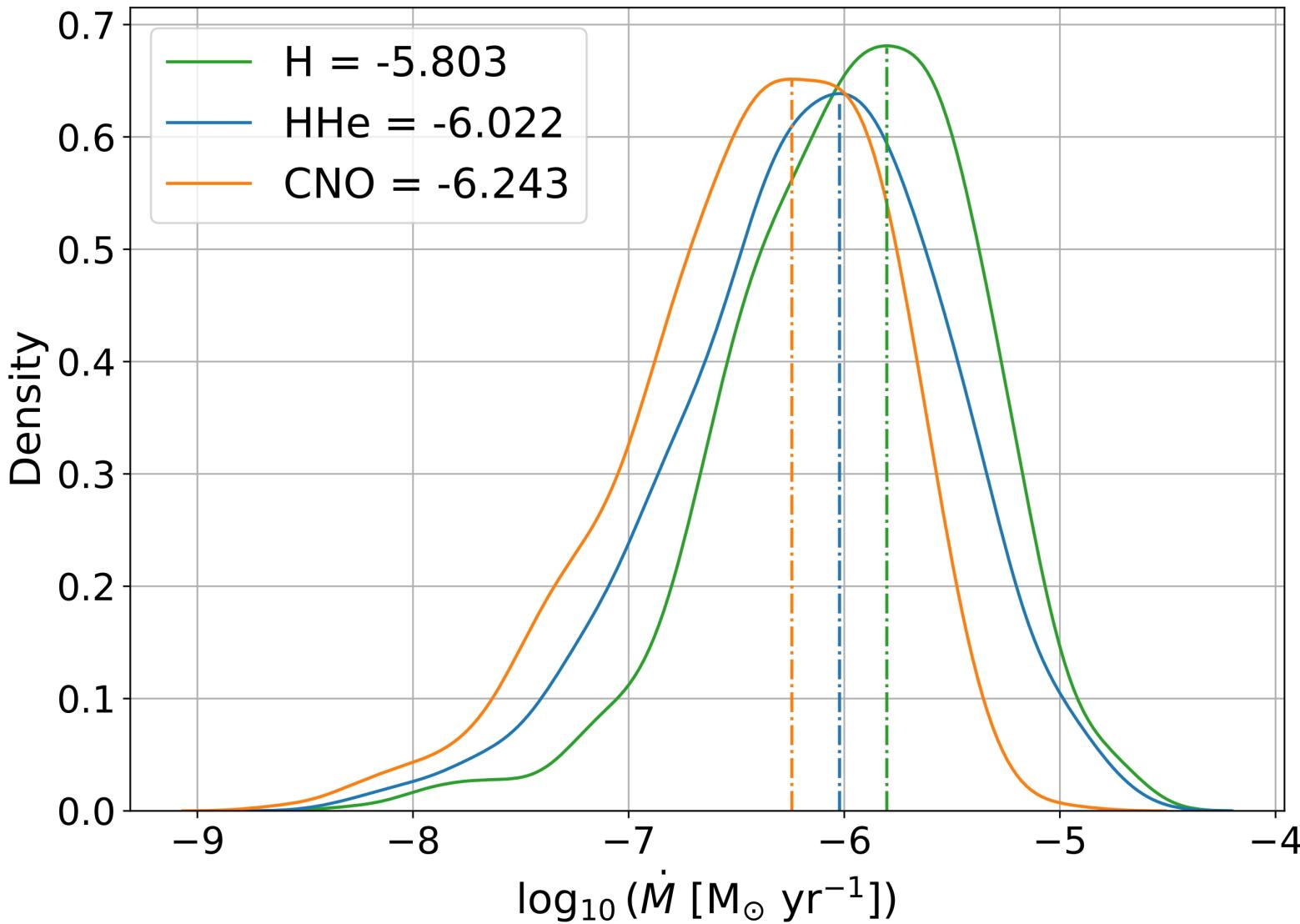
Parameter	Range	Step
T_{eff}	30000 – 50000 [K]	1000 [K]
$\log g$	2.8 – 4.3 [dex]	0.1 [dex]
R_*	7 – 70 [R_{\odot}]	1.5 [R_{\odot}]

Grid	Initial Models	Converged Models	Filtered Models
H	9374	6009	911
HHe	9718	6121	913
CNO	8557	4004	617

New Mass-loss rates



New Mass-loss rates

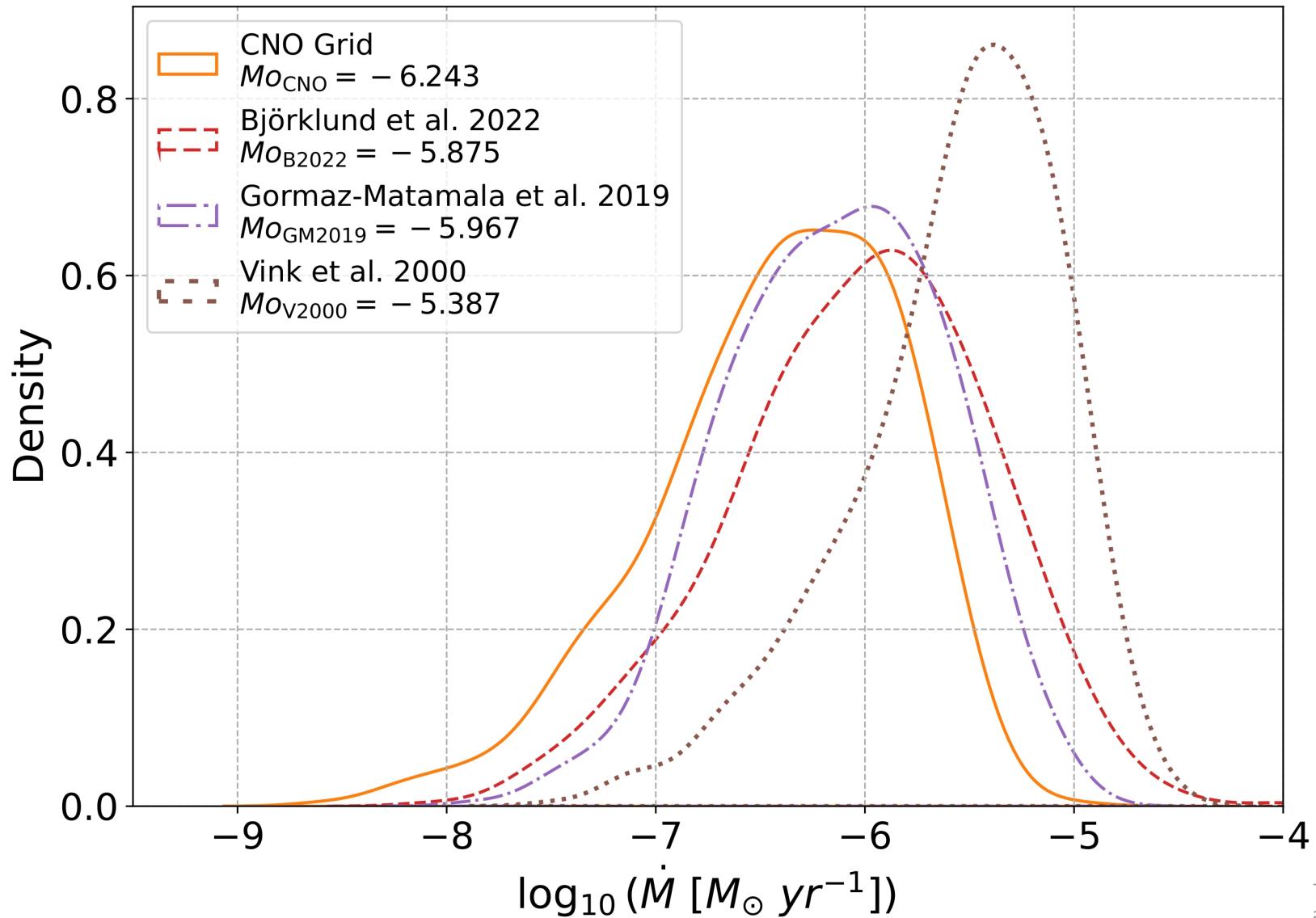


New Mass-loss rates

$$\log \dot{M} = A \cdot \log \left(\frac{T_{\text{eff}}}{1000 \text{ K}} \right) + B \cdot \log g + C \cdot \log \left(\frac{R_*}{R_\odot} \right) + D$$

Grid	A	B	C	D	R^2
H	8.86 (± 0.09)	-1.66 (± 0.05)	1.82 (± 0.05)	-16.2 (± 0.09)	0.9486
HHe	12.6 (± 0.17)	-2.23 (± 0.05)	1.75 (± 0.04)	-20.3 (± 0.18)	0.9734
CNO	13.2 (± 0.25)	-2.25 (± 0.07)	1.78 (± 0.05)	-21.3 (± 0.24)	0.8893

New Mass-loss rates



Quantitative Spectroscopy



requires

Plasma Physics, plasma is "normal" state of atmospheres and interstellar matter (plasma diagnostics, line broadening, influence of magnetic fields,...)

Atomic physics/Quantum mechanics, interaction light/matter (micro quantities)

Radiative transfer, interaction light/matter (macroscopic description)

Thermodynamics, thermodynamic equilibria: TE, LTE (local), NLTE (non-local)

Hydrodynamics, atmospheric structure, velocity fields, shockwaves,...

Machine Learning and Big Data

provides

Stellar properties, mass, radius, luminosity, energy production, chemical composition, properties of outflows

Properties of (inter) stellar plasmas, temperature, density, excitation, chemical comp., magnetic fields

Input for stellar, galactic and cosmological **evolution** and for stellar and galactic structure.

Massive stars binaries → Gravitational Waves



Conclusions



Slow wind solutions may solve some of the problems from massive stars:

- Winds from B[e] Supergiants (outflowing disk)
- Winds from BA Supergiants (WML relationship)
- Time-dependent Calculations (bifurcation, oscillation, clumping, instabilities,...)
- 2D-calculations
- Observations (constraints to theory)
- Grids for different Metallicities
- Different Machine Learning Algorithms



F I N
por ahora (for now)....



Gracias