Spectral numerical methodology to describe the line broadening effect produce by the stellar rotation

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Physics of Extreme Massive Stars

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Motivation

The challenge of estimating the physical properties and chemical composition of massive stars.

Context

Quantitative Stellar Spectroscopy

- Technique by which observed line profiles are compared with a set of synthetic ones.

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Issues

- Intrinsic shape of the non-rotating line profile cannot be measured.
- The observed line profile is affected by various phenomena that modify its shape.

Main goal

The aim consist in finding the non-rotational line profile $I(\zeta)$ from the observed data $\mathcal{O}(\zeta)$.

with

$$\zeta = \frac{\lambda - \lambda_m}{\lambda_m}$$

for λ wave length and λ_m wave length of the center of the line.

Based on the work in

THE SPECTROSCOPIC DETERMINATION OF STELLAR ROTATION AND ITS EFFECTS ON LINE PROFILES.

J. A. Carroll, M.A., Ph.D., Professor of Natural Philosophy in the University of Aberdeen.

where the following integral equation is addressed

2. *The Integral Equation for a Spectrum Line in a Rotating Star.*—It is unnecessary to repeat the algebra of the first paper * leading to the equation

$$O(\zeta) = \frac{2}{\pi}\int_{-1}^{+1} I(\zeta + \beta t)\sqrt{1 - t^2}\,dt, \qquad (2.1)$$

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* *M.N.*, **88**, 553, 1928.

We consider the following model

Model

$$\mathcal{O}(\zeta) = \kappa \int_{-1}^{1} I(\zeta + \beta x)\, g_{\epsilon,w}(x)\, dx$$

where:

- V is the equatorial velocity of the star
- θ inclination of its axis of rotation to the normal
- c speed of light
- $\beta = \dfrac{V \sin(\theta)}{c}$ rotational velocity parameter.

$g_{\epsilon,w}(x)$ function that depends on the limb-darkening law used in the model.

The expression for $g_{\epsilon,w}(x)$ has the following structure:

$$g_{e,w}(w) = C_1(1-x^2)^{1/2} + C_2(1-x^2) + C_3(1-x^2)^{3/2} + C_4(1-x^2)^{3/4}$$

where:

Model	Parameters			
	C_1	C_2	C_3	C_4
Linear	$2(1-\epsilon)$	$\frac{\epsilon\pi}{2}$	0	0
Quadratic	$2(1-\epsilon-w)$	$\frac{(\epsilon+2w)\pi}{2}$	$-\frac{4w}{3}$	0
Square root	$2(1-\epsilon-w)$	$\frac{(\epsilon+2w)\pi}{2}$	0	$\frac{\sqrt{\pi}\Gamma(5/4)w}{\Gamma(7/4)}$

Main Result

Formulation of a numerical methodology based on Fourier-Gegenbauer orthogonal decomposition to estimate the non-rotational line profile obtained from observational data.

- Galerkin Type approach.
- Linear system and regularization analysis

Proposal

$$I(z) = \sum_n \widehat{I}_n \varphi_n(z)$$

Galerkin type approach for certain shape functions $\varphi_0(z),\ \varphi_1(z), \ldots$

- The problem reduces to obtaining approximations of the coefficients $\widehat{I}_n$.

For each ζ_i and $\mathcal{O}(\zeta_i)$, we have

$$\mathcal{O}(\zeta_i) = \sum_{n=1}^{N} \widehat{I}_n \left(\int_{-1}^{1} \varphi_n(\zeta_i + \beta x)\, g_{\epsilon,w}(x) dx \right)$$

obtaining a linear system in terms of the coeficients $\widehat{I}_i$

- Appropriate selection of shape functions $\varphi_0(z),\ \varphi_1(z), \ldots$.
- Appropriate number of shape functions for the approximation $n = 0, 1, \ldots, N$
- Strategy for obtaining the coefficients $\widehat{I}_0, \ldots \widehat{I}_N$

We have

$$g_{\epsilon,w}(x) \quad \sim \quad (1-x^2)^{\frac{\alpha}{2}} \qquad\qquad \alpha = 1,\ \tfrac{3}{2},\ 2,\ 3.$$

From *Generating function* for the ultraspherical polynomial

$$\frac{1}{(1-2xz+z^2)^{\nu}} = \sum_{n=0}^{\infty} C_n^{(\nu)}(x) z^n$$

with $C_n^{(\nu)}(x)$ denoting the Gegenbauer polynomial which are defined as

$$C_n^{(\nu)}(z) = \frac{n!}{2^n\Gamma(n+\nu)} \sum_{m=0}^{[n/2]} \frac{(-1)^m \Gamma(n-m+\nu)}{m!(n-m)!}(2z)^{n-2m}$$

Formulation

$$[A^{(\alpha)}]_{i,j} = \int_{-1}^{1} C_j^{(3/2)}(z_i + \beta x)(1 - x^2)^{\frac{\alpha}{2}} dx$$

with

- $i = 1, 2, \ldots, M$ number of observations (data size)
- $j = 0, 1, \ldots, N$ number of Gegenbauer polynomials (shape functions).
- $\alpha = 1,\ \frac{3}{2},\ 2,\ 3$ term in the model

Linear system

Let

$$[\mathcal{A}] = \tilde{c}_1[A^{(1)}] + \tilde{c}_2[A^{(\frac{3}{2})}] + \tilde{c}_3[A^{(2)}] + \tilde{c}_4[A^{(3)}]$$

Thus, the linear system take the form

$$[\mathcal{A}] \begin{bmatrix} \widehat{I}_0 \\ \vdots \\ \widehat{I}_N \end{bmatrix} = \begin{bmatrix} \mathcal{O}_1 \\ \vdots \\ \mathcal{O}_M \end{bmatrix}$$

Questions about the linear system $[\mathcal{A}][\mathcal{I}] = [\mathcal{O}]$

- Appropriate Dimension $(\textit{data-size/normal modes})$
- Spatial mesh associate with the observational data $\mathcal{O}_i$ $(\textit{interpolation procedure})$.
- Condition number of each $[A^{(\alpha)}]$ $(\textit{model selection})$
- Control on Stability of solutions $(\textit{Resolution technique})$

Proposal for the formulation and solution of $[\mathcal{A}][\mathcal{I}] = [\mathcal{O}]$:

- A uniformly spaced grid interior to the data boundaries to Gibbs-like oscillation effects.

- Tikhonov regularization approach

$$\min_{\gamma} \; \| [\mathcal{A}][\mathcal{I}] - [\mathcal{O}] \|^2 + \gamma \| [\mathcal{I}] \|^2$$

Procedure

- **Step 1** Data selection such that

$$\|\mathcal{O}\|_{data} \leq 0.95\,\|\mathcal{O}\|_{total}$$

- **Step 2** Interpolation procedure on $[\mathcal{O}]_{data}$ the data-information $(size\ M)$ and construction of $[\mathcal{O}]_{num}$ from a uniformly spaced grid $(size\ M_{num})$.

- **Step 3** Given the approximation of β, construction of each $[A^{(\alpha)}]$ $(size\ M_{num} \times M_{num})$.

- **Step 4** Solve

$$\min_{\gamma}\ \|[\mathcal{A}][\mathcal{I}] - [\mathcal{O}]\|^2 + \gamma\|[\mathcal{I}]\|^2$$

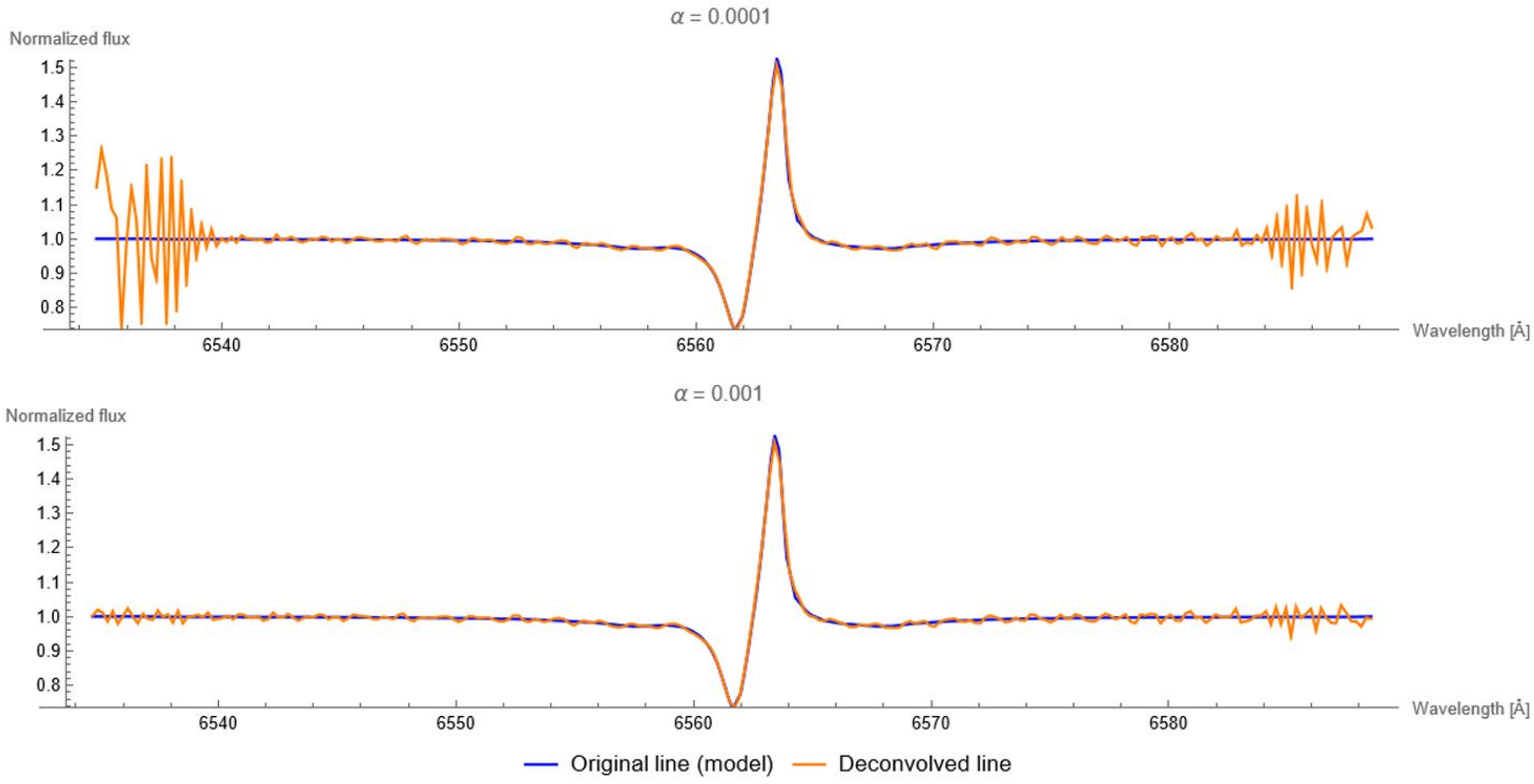
Case Study

Type	λ_{min}	λ_{max}	N°Coeff. $\widehat{I}_n$
P-Cygni	6534.71	6590.89	90
Emisión	6539.67	6585.93	90
Absorción	6535.74	6589.86	90

Linear model

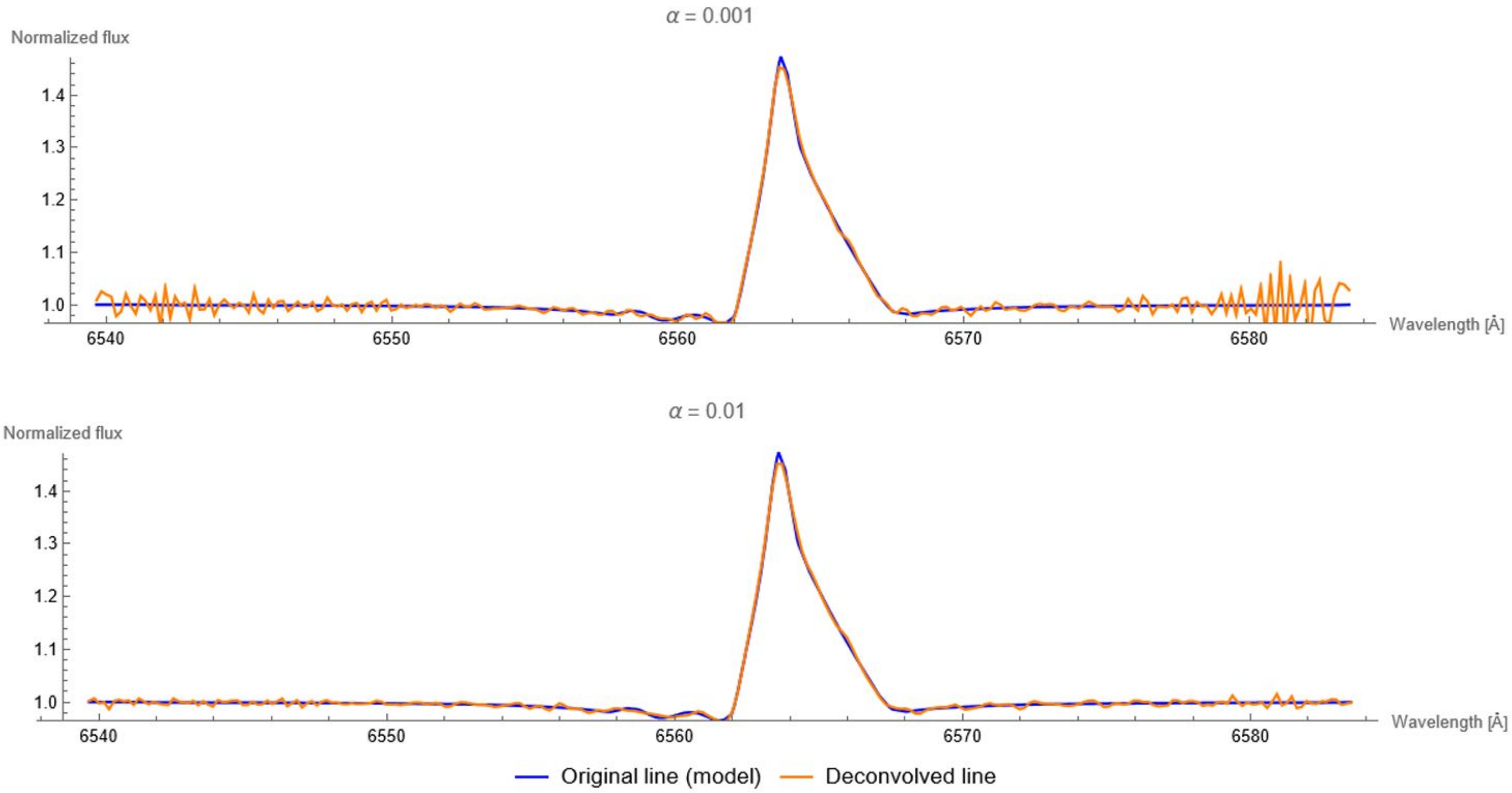
$$g_{\epsilon,w}(x) = 2(1-\epsilon)(1-x^2)^{1/2} + \frac{\epsilon\pi}{2}(1-x^2)$$

P-Cygni

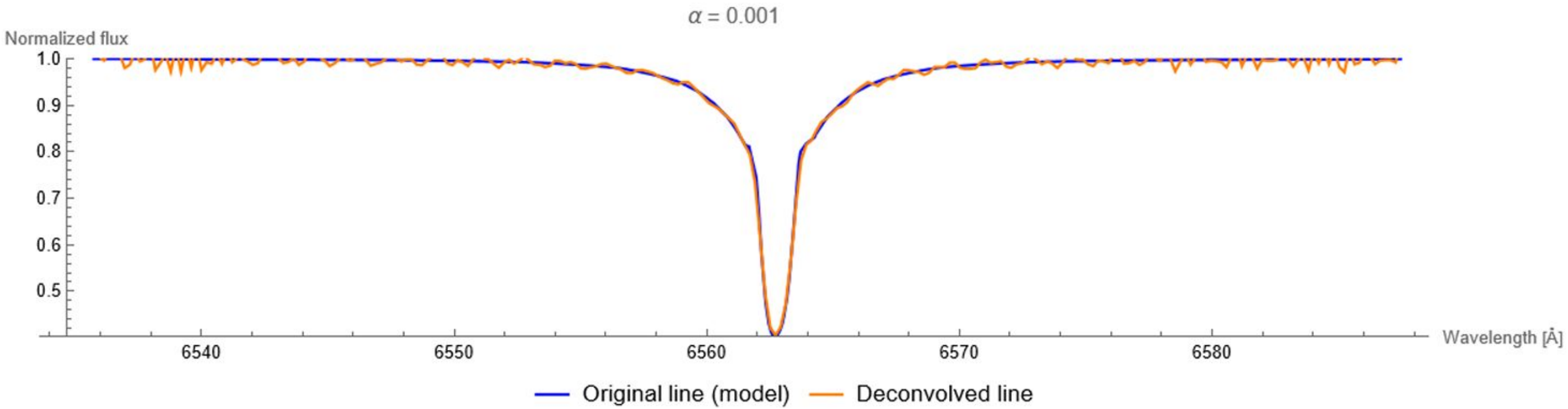


α = 0.0001
Normalized flux
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
6540
6550
6560
6570
6580
Wavelength [Å]
α = 0.001
Normalized flux
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
6540
6550
6560
6570
6580
Wavelength [Å]
Original line (model)
Deconvolved line

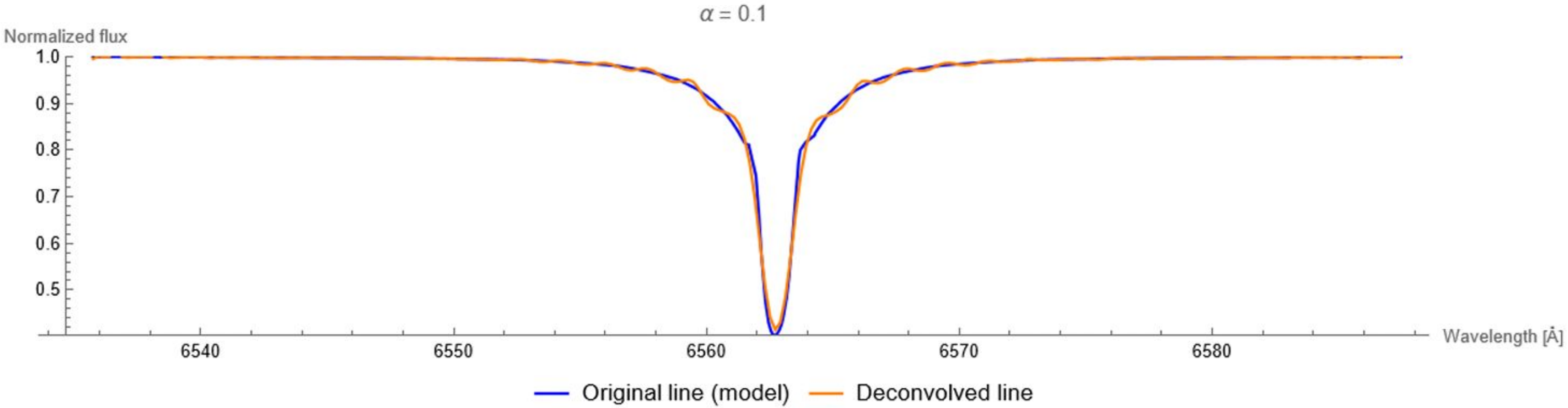
Emission


α = 0.001
Normalized flux
1.4
1.3
1.2
1.1
1.0
6540
6550
6560
6570
6580
Wavelength [Å]
α = 0.01
Normalized flux
1.4
1.3
1.2
1.1
1.0
6540
6550
6560
6570
6580
Wavelength [Å]
Original line (model)
Deconvolved line

Absorption



α = 0.001
Normalized flux
1.0
0.9
0.8
0.7
0.6
0.5
6540
6550
6560
6570
6580
Wavelength [Å]
Original line (model)
Deconvolved line



α = 0.1
Normalized flux
1.0
0.9
0.8
0.7
0.6
0.5
6540
6550
6560
6570
6580
Wavelength [Å]
Original line (model)
Deconvolved line

Conclusions and Future Directions

- Applying a Fourier-Gegenbauer spectral approach, a numerical scheme was obtained for the coefficients of the non-rotational line profile.
- Through the spectral coefficients $\widehat{I}_n$, a semi-analytical description of the non-rotational line profile $I(\zeta)$ is obtained.
- The ill-posed problem was addressed using the Tikhonov regularization method, yielding satisfactory results.
- We are working on developing a methodology to reduce the dimensions of linear systems.
- In a future work, we aim to develop a procedure that also addresses the issue concerning the calculation of the velocity parameter β.

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Thank You