Where the stellar wind impinges the interstellar medium

Physics of Extreme Massive Stars

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Abstract

Massive stars have strong radiation-driven winds, losing part of their mass. Where these winds encounter the interstellar medium, a separatrix surface can form, called astropause. The astropause separates the stellar wind region, the astrosphere, from the outer streaming interstellar medium. At the apex of the astropause the stagnation point is located, where the streaming velocity of both flows, namely the flow of the interstellar medium and the stellar wind flow, vanishes. The velocity fields in the astropause regions can be described by potential theory, while the m-CAK theory is used to describe the radiation-driven winds, in the proximity of the star. In this work, we aim to connect these two theories to describe the interaction between the stellar wind and their surroundings. We propose a criterion for delimiting validity regions for both theories, and we found a relation between the stellar wind parameters (mass-loss rate and terminal velocity), the interstellar medium conditions, and the distance between the star and the stagnation point, called the stagnation distance.

Potential theory

Potential flows are found by solving the Laplace equation $\Delta\psi = 0$ following, for example, Nickeler et al. (2014). We concentrate on 2D theory to describe approximately the outflow in the equatorial plane, which is often the region that contributes more to the mass flow than the higher latitudes of the wind. We can use conformed mappings or more general (anti-)holomorphic functions to solve the Laplace equation. We apply the Ψ function that can be written as

$$\Psi(u := x + iy) = \phi(x, y) + i\psi(x, y) = W_\infty u + C_0 \ln(u), \tag{1}$$

with the complex velocity field

$$W(u) = \frac{d\Psi}{du} = W_\infty + C_0\,\frac{1}{u}\,. \tag{2}$$

Here, the linear term guarantees the asymptotic boundary condition, and the monopole term is chosen to ensure a decreasing flow field from the origin toward infinity. In addition, the monopole term generates a radial flow, emulating the radial flow in the proximity of the star. Here, the source of the stellar wind is located at the origin. The constant C_0 is the imaginary circulation, determined for the stagnation point u_{sp}, in which the velocity vanishes, while the value W_∞ is proportional to the square root of the kinetic pressure at infinity. Purely imaginary circulation guarantees radial flow lines in the vicinity of the origin. We chose the stagnation point to be real and located between the outer flow and the star. Then, taking the imaginary part of the complex velocity field, the stream function is

$$\psi(x, y) = W_\infty[y - u_{sp}\arctan(y/x)]. \tag{3}$$

This stream function leads to the following flow lines, where is set the value $u_{sp} = -1$.

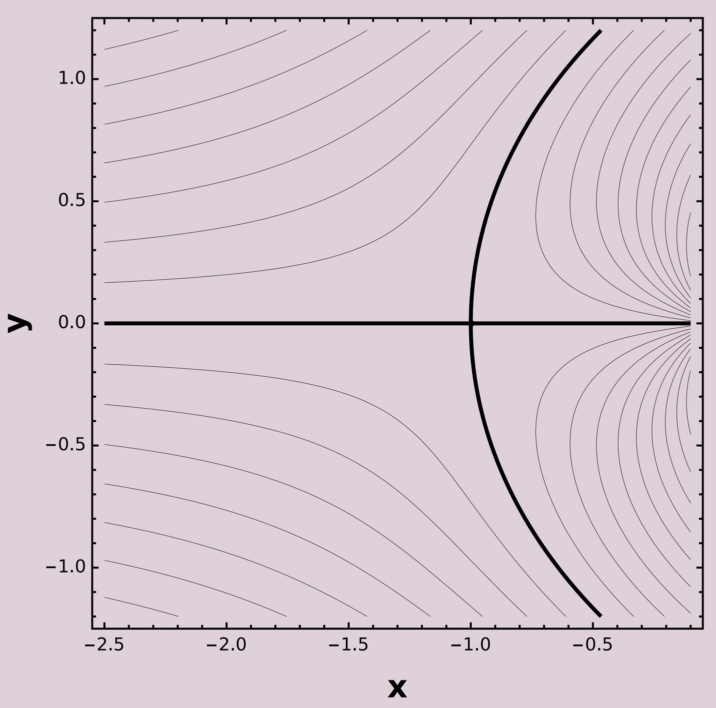

Figure 1: Flow velocity lines for the astrosphere region. Thick lines represent the astropause and the stagnation line (horizontal line).

m-CAK theory

This theory, developed by Pauldrach et al. (1986) and Friend & Abbot (1986), describes the radiation-driven winds of massive stars. Under the assumptions of spherical symmetry and stationary state, the radiative force produced by the lines is modeled through the parameters α, δ, and k. To describe the wind's velocity law, there are the mass-continuity equation,

$$\dot{M} = 4\pi r^2 \rho v = \text{constant}, \tag{4}$$

and the momentum conservation,

$$\left(1 - \frac{a^2}{v^2}\right) v\frac{dv}{dr} = \frac{2a^2}{r} - \frac{GM_*(1-\Gamma)}{r^2}\left(1 - \Omega^2\frac{R_*}{r}\right) + g^L_{\text{rad}}, \text{ with} \tag{5}$$

$$g^L_{\text{rad}} = \frac{C}{r^2}\,CF\left(\frac{n_e}{W(r)}\right)^\delta \left(r^2 v\frac{dv}{dr}\right)^\alpha \text{ and } C = kGM_*\Gamma\left(\frac{4\pi}{\sigma_e \dot{M} v_{th}}\right)^\alpha. \tag{6}$$

Ω represents the stellar rotation ($\Omega = V_{rot}/V_{max}$), CF is the correction factor introduced by the finite disk of the star, $W(r)$ is the dilution factor, n_e is the electron density, and $\Gamma = L_*/L_{Edd}$.

Depending on the values of the parameters α, δ, and k, there are different types of solutions, namely fast, δ-slow, and Ω-slow solutions (Curé & Araya, 2023).

Aims

We aim to connect these two descriptions of the flow field and search for a criterion to match the radial flow of the m-CAK theory and the potential flow which also has azimuthal components of the flow.

Results

The velocity field can be obtained as

$$\vec{v}(x, y) = \frac{1}{\sqrt{\rho(x,y)}}\vec{\nabla}\psi(x, y) \times \vec{e_z} = \frac{1}{\sqrt{\rho(x,y)}}\left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}\right). \tag{7}$$

In addition, the radial and tangential components of the velocity field are

$$\begin{cases} v_r &= v_x \cos\varphi + v_y \sin\varphi \\ v_\varphi &= -v_x \sin\varphi + v_y \cos\varphi, \end{cases} \tag{8}$$

where $\varphi = \arctan(y/x)$. To define a region around the star where the flow is almost purely radial, we search for a distance R_0, where

$$v_\varphi(R_0) = \varepsilon v_r(R_0), \text{ with } |\varepsilon| \ll 1. \tag{9}$$

We found that this condition is fulfilled between the lines given by

$$y = \pm\varepsilon(x - u_{sp}), \text{ where } u_{sp} \in \mathbb{R}. \tag{10}$$

By searching for the minimum radial distance from the star to the relation found, we obtained

$$R_0 = \frac{|\varepsilon|}{\sqrt{1+\varepsilon^2}}|u_{sp}|. \tag{11}$$

This represents the radial distance from the star in which the flow is approximately radial, so the m-CAK theory can be used. Then, at this distance R_0 we can assume that the radial velocity is known, as the terminal velocity of the wind v_∞,

$$v_r(R_0) = \frac{W_\infty}{\sqrt{\rho(R_0)}}\,\frac{1}{|\varepsilon|\sqrt{1+\varepsilon^2}} = v_\infty. \tag{12}$$

Applying the mass-continuity equation at the distance R_0, the density is given by

$$\rho(R_0) = \frac{\dot{M}}{4\pi R_0^2 v_\infty}. \tag{13}$$

Inserting this form of the mass conservation into the equation for the terminal velocity, Eq. (12), the following equation for the stagnation distance is found

$$|u_{sp}| = \sqrt{\frac{v_\infty \dot{M}(1+\varepsilon^2)}{4\pi W_\infty^2}}. \tag{14}$$

Conclusions

- It's possible to define a limiting radius for a small value of $|\varepsilon|$, as $R_0 = |\varepsilon u_{sp}|$, in which the m-CAK model describes the radial flow. For larger distances from the star, the transition to a corresponding potential stagnation point flow can be applied.
- We found a relation between the stagnation distance $|u_{sp}|$, the stellar wind's parameters (the terminal velocity of the wind v_∞ and the mass-loss rate $\dot{M}$), and the kinetic pressure of the interstellar medium $W_\infty^2/2$, given for a small value of $|\varepsilon|$ by
$$|u_{sp}| = \sqrt{\frac{v_\infty \dot{M}}{4\pi W_\infty^2}}. \tag{15}$$
- For representative values corresponding to a B supergiant star, $v_\infty = 500$ km/s and $\dot{M} = 1 \times 10^{-7}$ $M_\odot$/yr, and for the interstellar medium, $W_\infty^2 = \rho_{ISM} V_{ISM}^2$ with $\rho_{ISM} = 5 \times 10^{-23}$ g/cm^3 and $V_{ISM} = 30$ km/s, the stagnation distance results in $u_{sp} \approx 135\,000 R_*$ (with $R_* = 25 R_\odot$). In addition, the m-CAK theory is valid in the radial distance $R_0 \approx 135 R_*$ from the star (with $\varepsilon = 0.001$).

Open questions – Future work

- Higher multipole order of the Ψ function can be taken into account (e.g. dipole moment).
- Considering a high rotational rate for the star, u_{sp} can be complex, as one has to take into consideration the real part of the circulation, implying complex coefficient C_0, i.e. an additional real circulation. This can produce asymmetries of the astropause.
- A similar method to calculate 3D potential flow via complex function theory exists (Whittaker, 1902), and can also describe the outer non-radial parts of wind and interstellar medium flow.
- For the presented cases, different criteria for matching radial models to these potential flows must be found.
- Further the models can be used to calculate the pressure distribution, and as the density for the incompressible models is a free function of the stream function ψ, the emission measure can be calculated, where $EM \propto Tn^2 \propto P\rho(\psi)$.
- An extension to MHD models is possible, via non-canonical transformations for field-aligned ideal MHD flows (Nickeler, 2014).

References

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