Structure formation in the envelopes of massive stars

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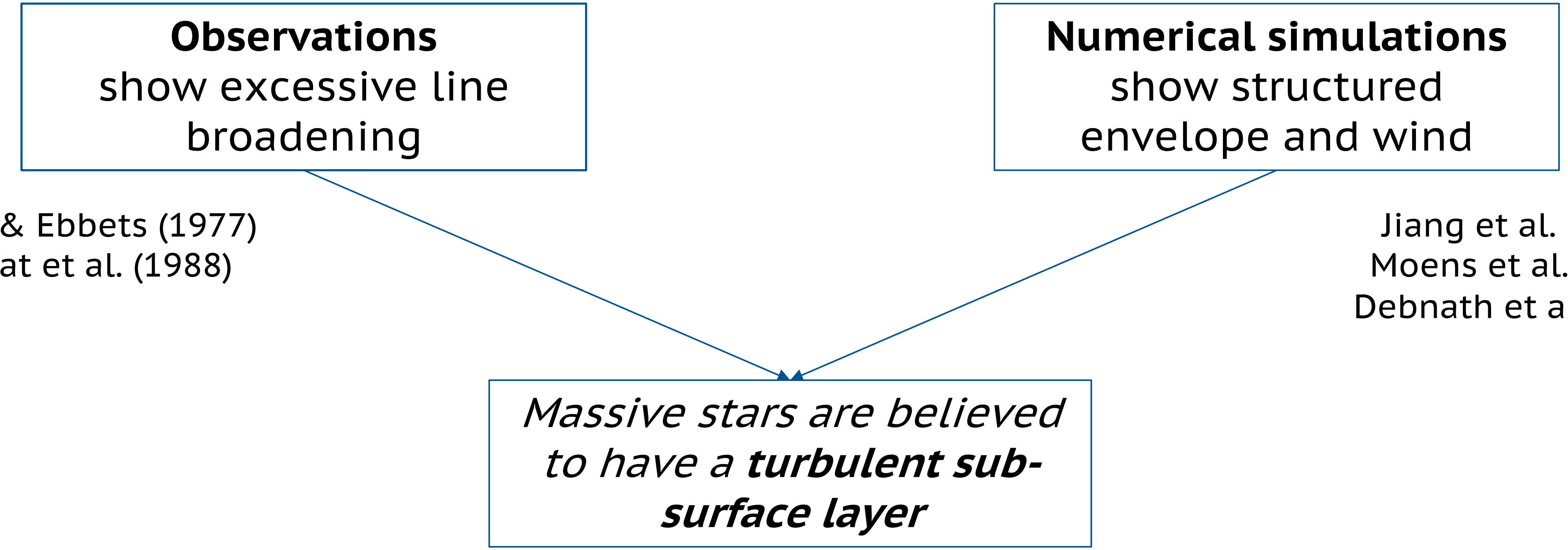
Turbulent layer



Observations
show excessive line broadening

Conti & Ebbets (1977)
Moffat et al. (1988)

Numerical simulations
show structured envelope and wind

Jiang et al. (2015)
Moens et al. (2022)
Debnath et al. (2024)

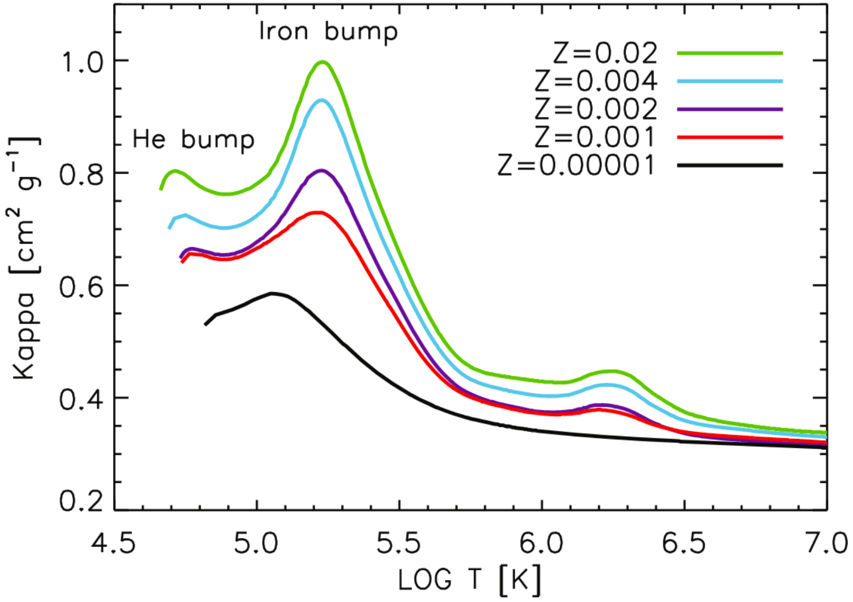
Massive stars are believed to have a ***turbulent sub-surface layer***

Sub-surface convection?

Iron opacity peak
("iron bump")

→

Sub-surface convection zone



Cantiello et al. (2009)

But *convection not the only instability triggered by iron bump and energy transport not always efficient…*

Sub-surface convection?

Iron opacity peak
("iron bump")

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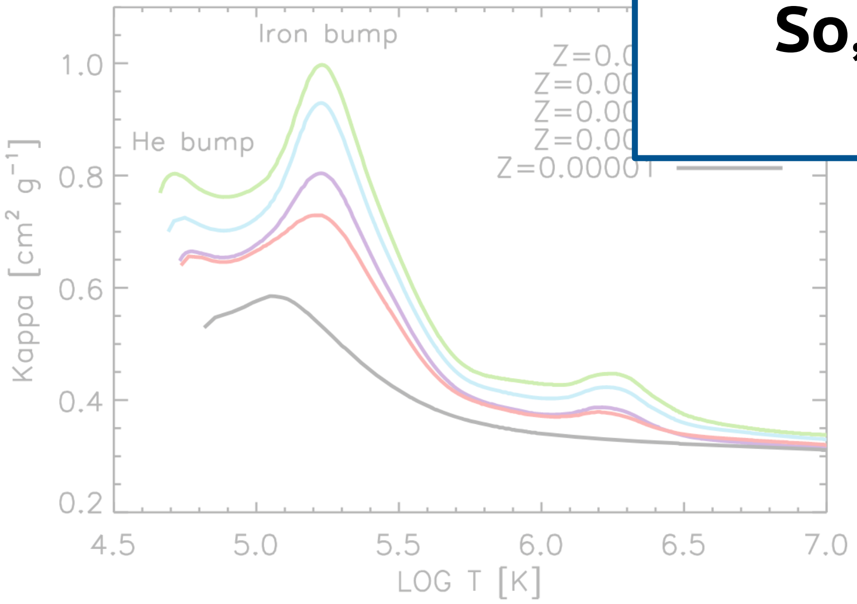
Sub-surface convection zone



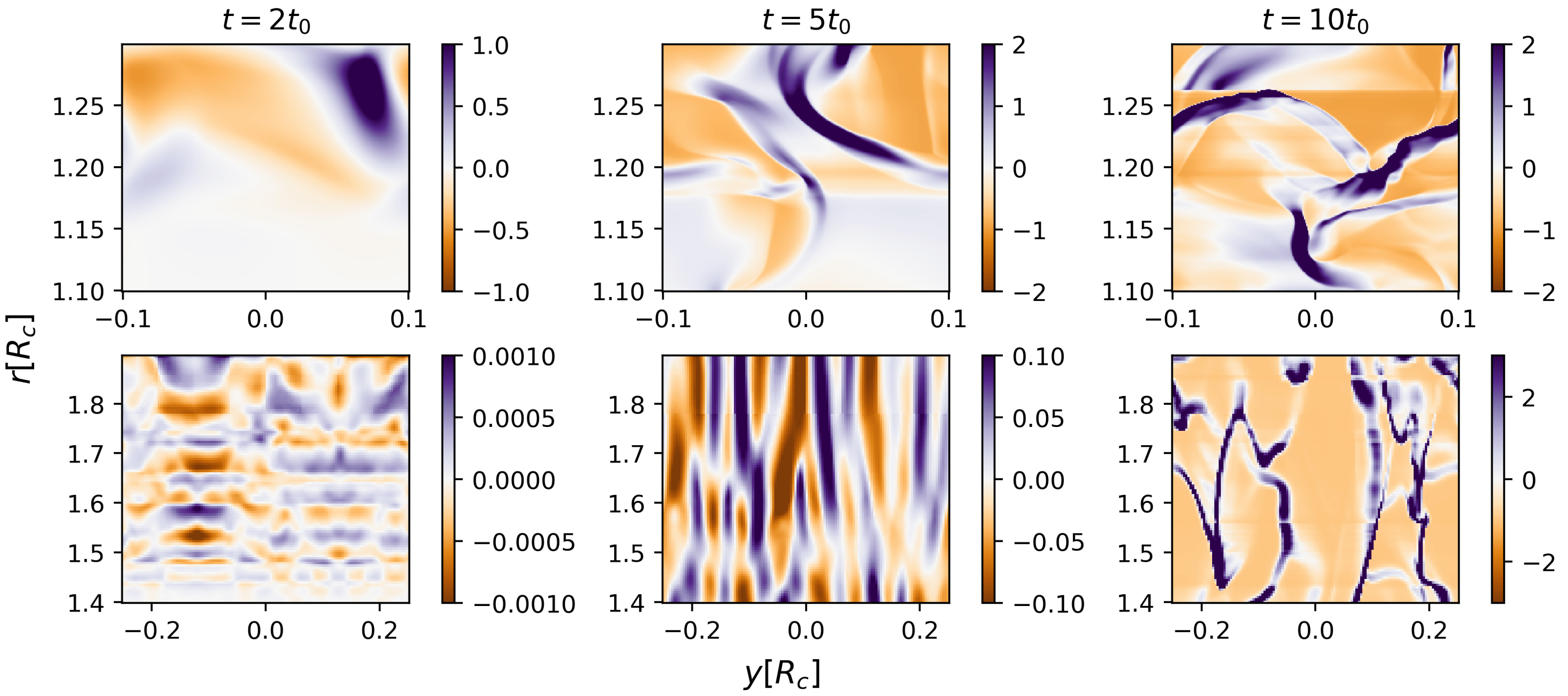
Cantiello et al. (2009)

So, is it really 'convection'?

But *convection not the only instability triggered by iron bump and energy transport not always efficient…*

Simulations

Relative density $\delta = (\rho - \langle\rho\rangle)/\langle\rho\rangle$



O-star simulations by Debnath et al. (2024)

WR star simulations by Moens et al. (2022)

Local linear analysis

Linearize RHD equations + assume **WKB Ansatz** $\delta \sim \exp[i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)]$

→ Result: dispersion relation with two possible instabilities

Convective instability

Strange mode instability

Blaes & Socrates (2003)

Growth rates

Blaes & Socrates (2003)
Owocki (2014)
Van der Sijpt et al. (in prep.)

Convective instability

$$\omega_g = -\frac{3\kappa\rho}{ck^2}(1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{r}})\left(1 + \frac{4E}{3p}\right)\left(\frac{\gamma p}{4(\gamma - 1)E} N_g^2 + \frac{1}{3} N_r^2\right) = -\overbrace{\boxed{\frac{1}{Dk^2}}}^{\text{Diffusion time}}(1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{r}})\overbrace{\boxed{N_{tot}^2}}^{\text{Brunt-Vaisala frequency}}$$

Strange mode instability

$$\omega_a = -\frac{\kappa}{2cc_i}\left(1 + \frac{3p}{4E}\right)\left[\left(\frac{4E}{3} + p\right)c_i - \left(\hat{\boldsymbol{k}} \cdot \boldsymbol{F}_{\text{diff}}\right)\Theta_\rho\right] \approx \frac{\Gamma g}{2c_i}\left(\underbrace{\boxed{\Theta_\rho}}_{\text{Logarithmic derivative of opacity w.r.t. density}} - \underbrace{\boxed{\tilde{D}}}_{\text{Radiation drag}}\right)$$

with $\tilde{D} = \left(\frac{4E}{3} + p\right)\frac{c_i}{F}$

Instability if $\omega > 0$

Growth rates

Blaes & Socrates (2003)
Owocki (2014)
Van der Sijpt et al. (in prep.)

Diffusion time

Brunt-Vaisala frequency

Convective instability

$$\omega_g = -\frac{3\kappa\rho}{ck^2}(1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{r}})\left(1 + \frac{4E}{3p}\right)\left(\frac{\gamma p}{4(\gamma - 1)E} N_g^2 + \frac{1}{3} N_r^2\right) = -\boxed{\frac{1}{Dk^2}}(1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{r}})\boxed{N_{tot}^2}$$

$$\boxed{\sim k^{-2}}$$

Strange mode instability

$$\omega_a = -\frac{\kappa}{2cc_i}\left(1 + \frac{3p}{4E}\right)\left[\left(\frac{4E}{3} + p\right)c_i - \left(\hat{\boldsymbol{k}} \cdot \boldsymbol{F}_{\text{diff}}\right)\Theta_\rho\right] \approx \frac{\Gamma g}{2c_i}\left(\boxed{\Theta_\rho} - \boxed{\tilde{D}}\right) \quad \text{with } \tilde{D} = \left(\frac{4E}{3} + p\right)\frac{c_i}{F}$$

$$\boxed{\sim k^0}$$

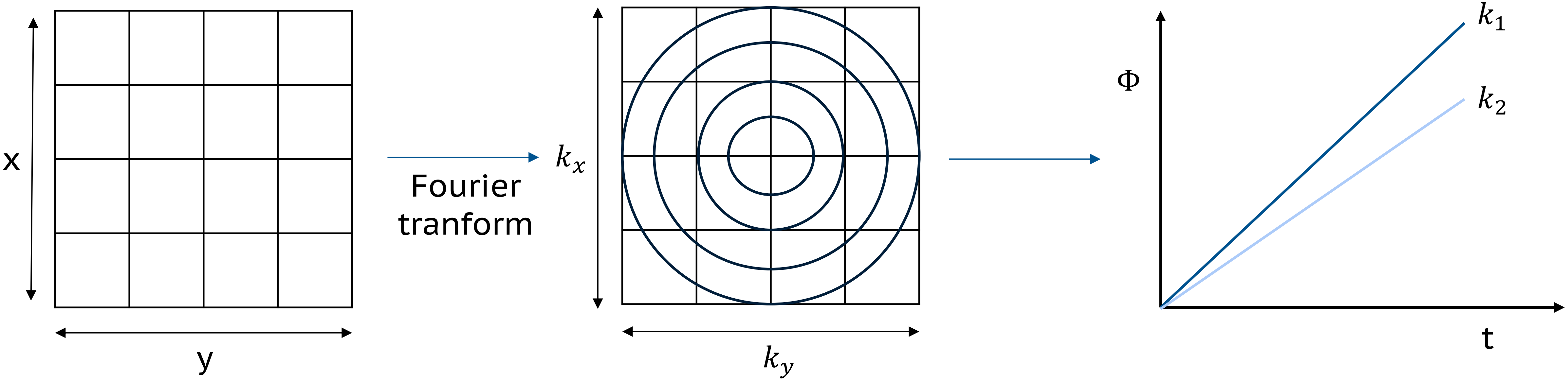
Logarithmic derivative of opacity w.r.t. density

Radiation drag

Instability if $\omega > 0$

Power spectra

$$\delta = (\rho - \langle\rho\rangle)/\langle\rho\rangle \longrightarrow \Phi(k) = \sqrt{\langle \hat{\delta}^*(\boldsymbol{k})\hat{\delta}(\boldsymbol{k})\rangle}$$

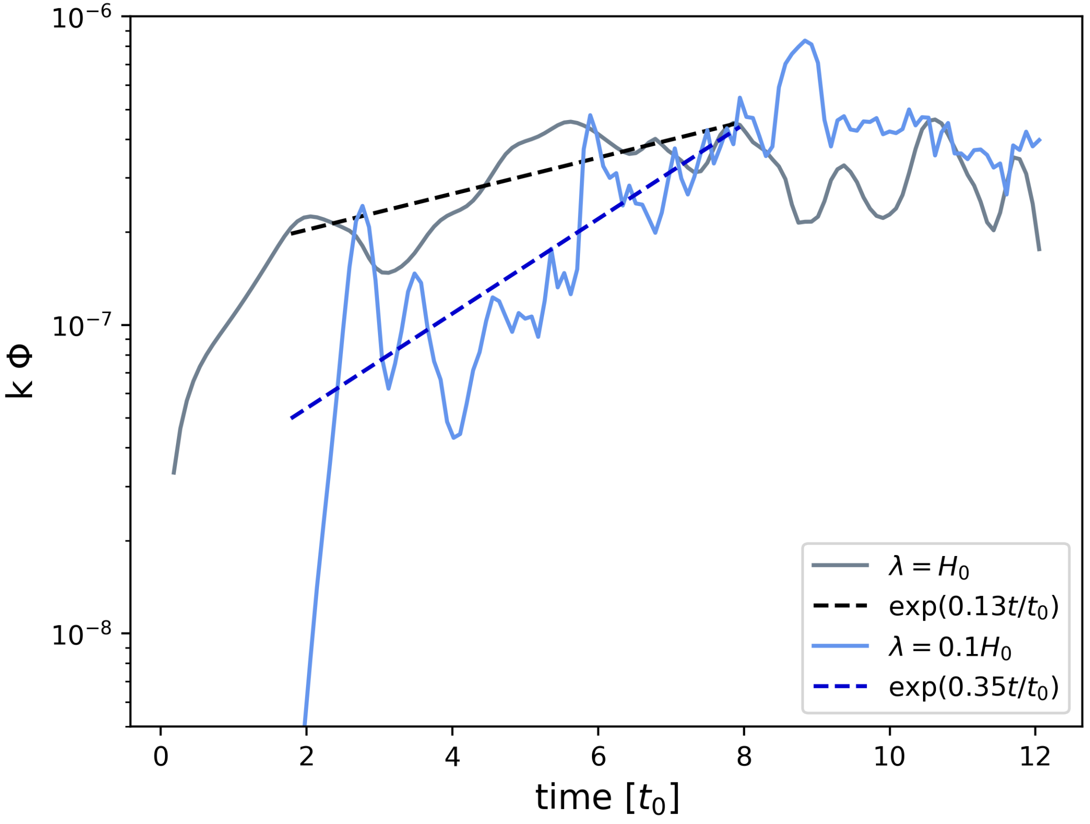


Fourier tranform

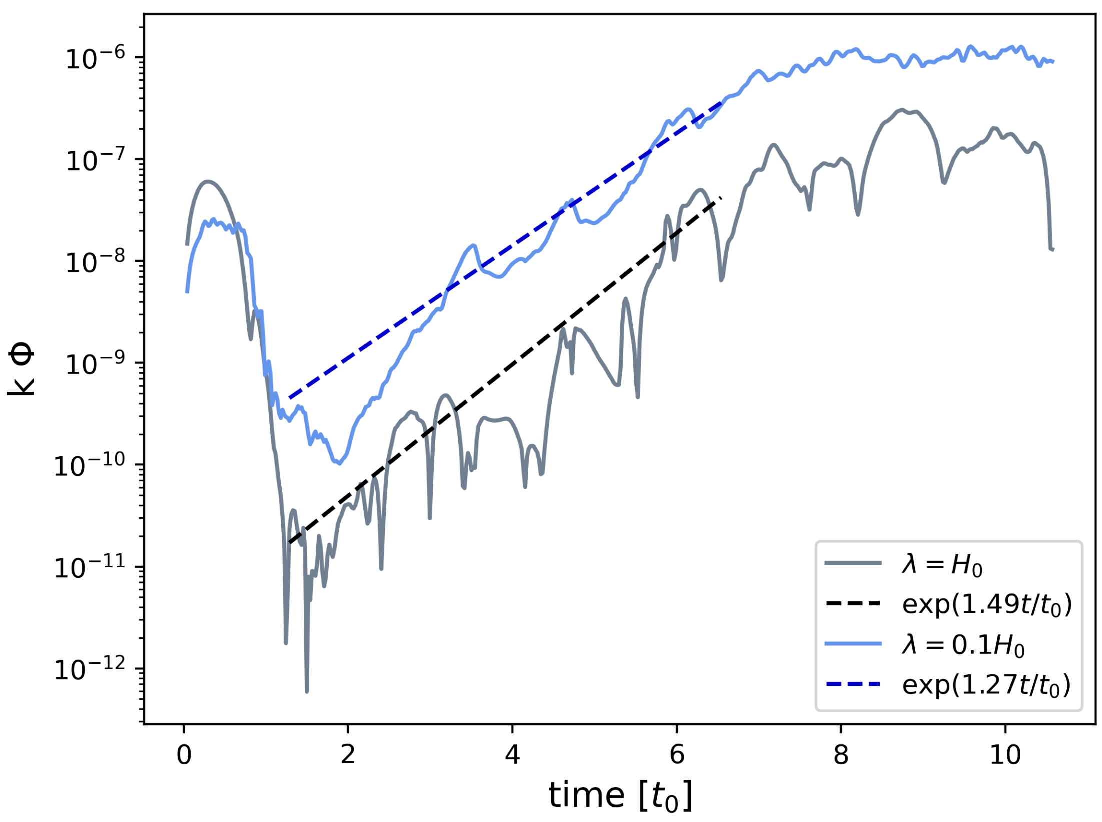
Power spectra

Van der Sijpt et al. (in prep.)

O-star



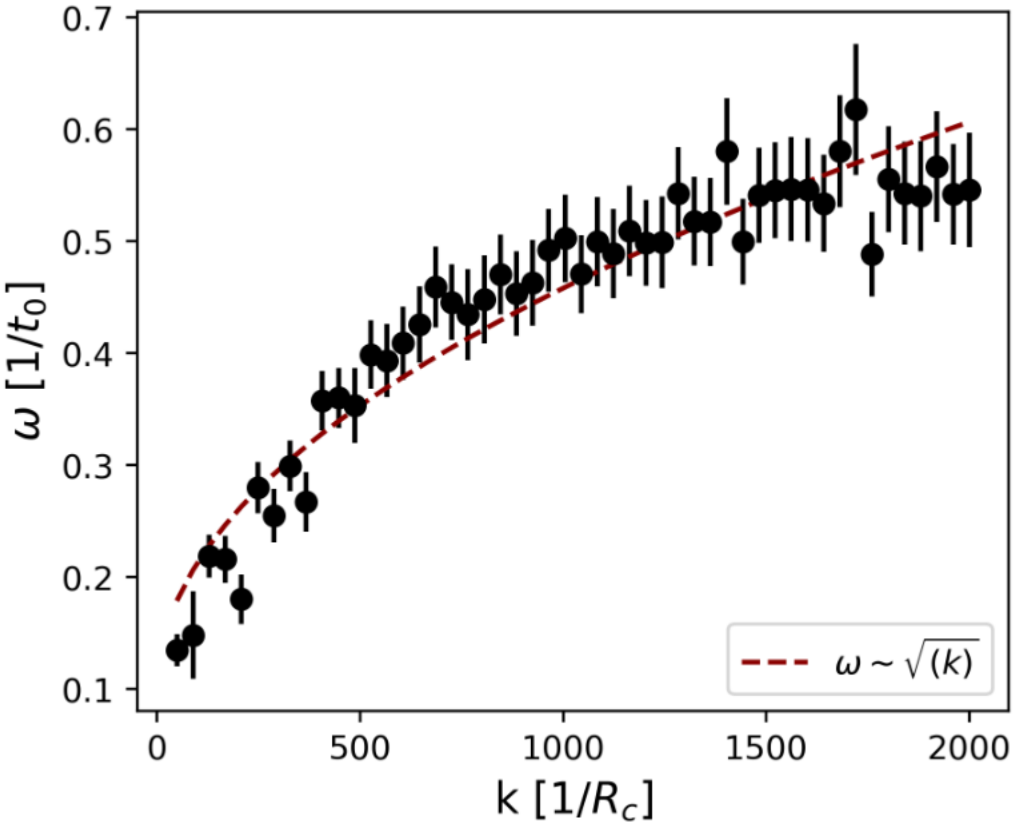
WR star



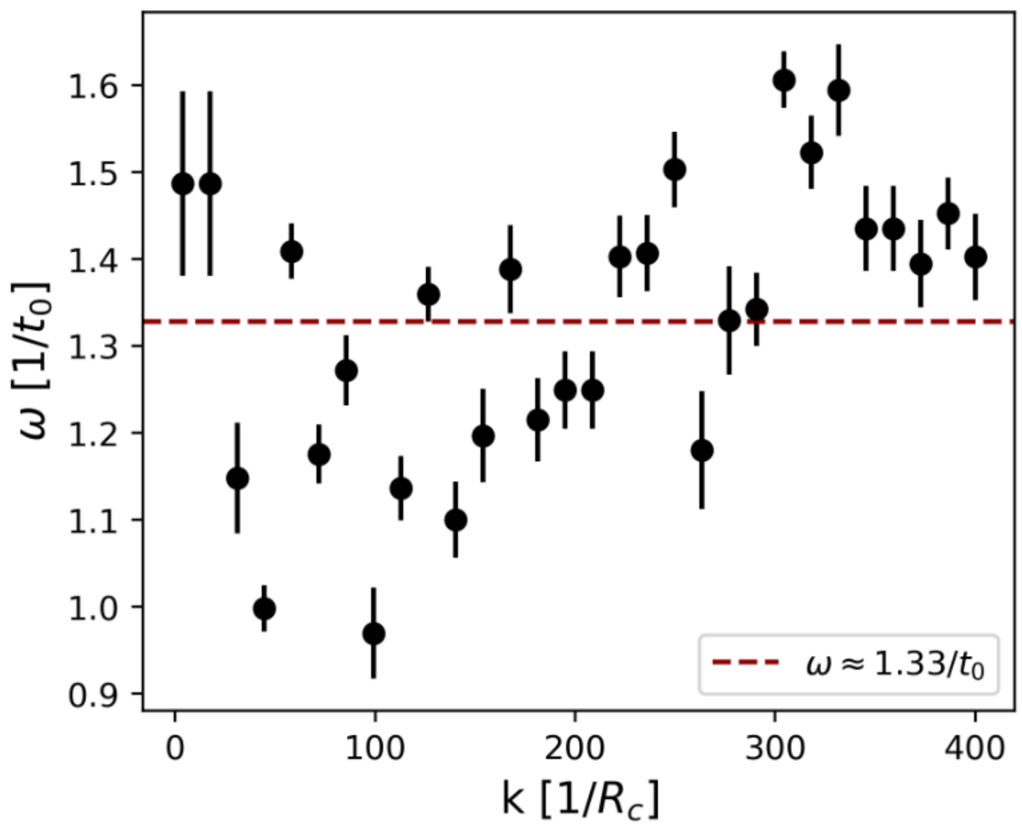
k-dependence

Van der Sijpt et al. (in prep.)

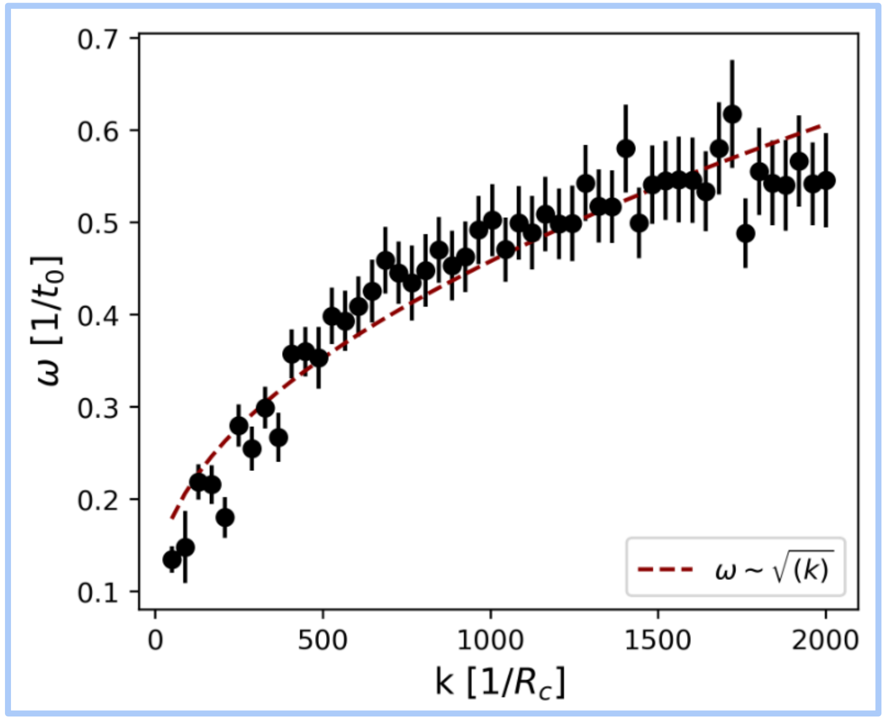
O-star



WR star



O-star



- **Convective instability** $\omega \sim k^{-2}$
- **Strange mode instability** $\omega \sim k^{0}$

→ $\omega \sim \sqrt{k}$ **not compatible** with either

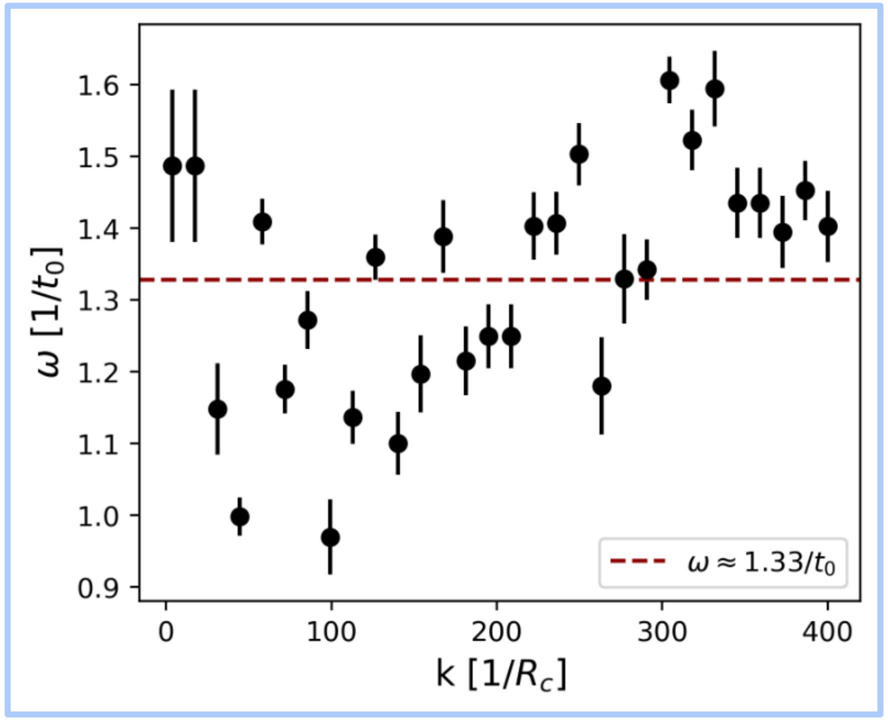
Alternative: Rayleigh-Taylor instabilities due to density inversion in initial conditions?

→ $\omega_{RT} \sim \sqrt{k}$

WR star

- **Convective instability** $\omega{\sim}k^{-2}$
- **Strange mode instability** $\omega{\sim}k^{0}$

$\omega \sim k^{0}$ could be **compatible** with **strange mode**



Mean growth rate $\omega \sim 1.33/t_0$

Theoretical strange mode growth rate $\omega \sim 1.89/t_0$

O-star vs WR star

$$\omega_a \approx \frac{\Gamma g}{2c_i}\left(\Theta_\rho - \tilde{D}\right)$$

Radiation drag

$$\tilde{D} = \left(\frac{4E}{3} + p\right)\frac{c_i}{F}$$

Radiation drag **smaller in WR star** due to **larger flux**

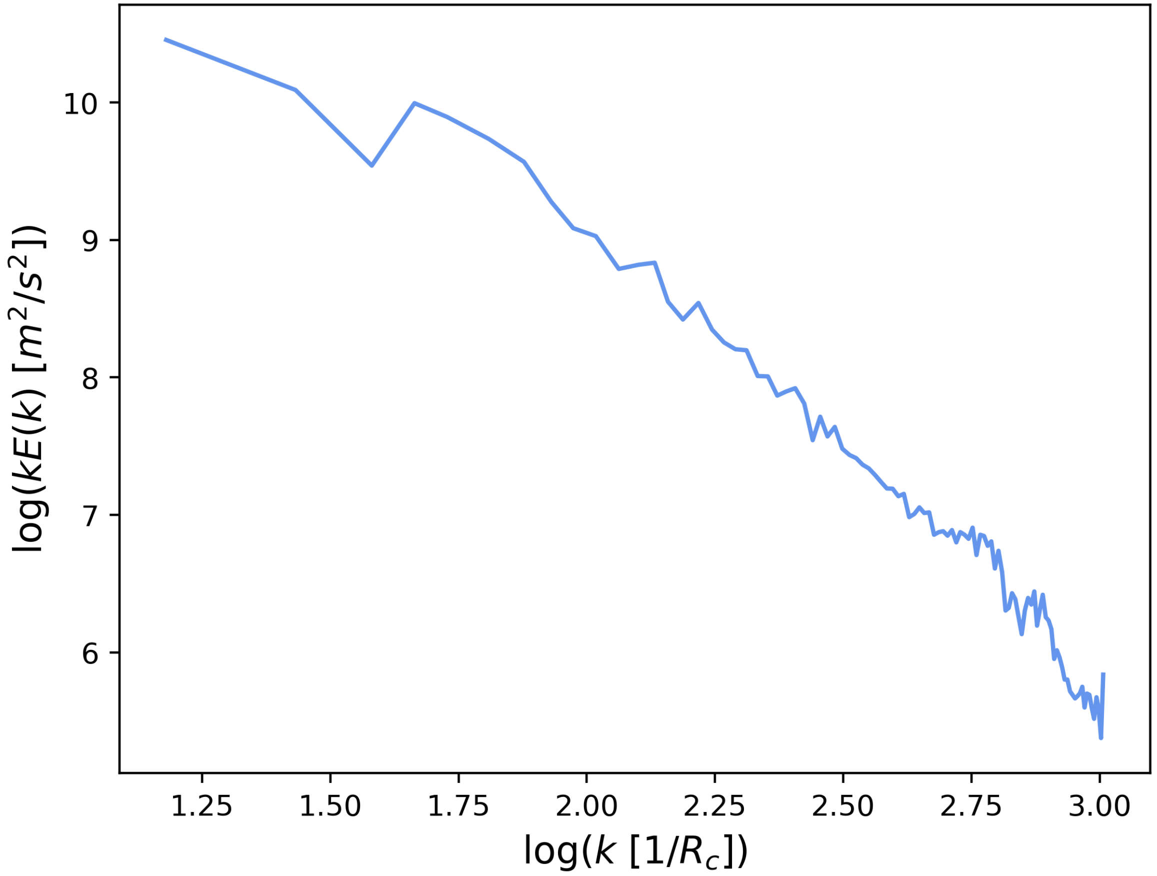
→ Strange mode instability more effective in WR stars than in O-stars

Density inversion

Density inversion present in O-star initial conditions due to hydrostatic equilbrium

→ Rayleigh-Taylor?

Ongoing work…



Energy power spectrum of **non-linear** structures

$$E(k) = \frac{1}{2} \langle \hat{v}^*(\boldsymbol{k}) \hat{v}(\boldsymbol{k}) \rangle$$

Compare to classical **Kolmogorov** turbulence theory

Summary

- Structure growth in “sub-surface convection zone” **does not behave as convective instability** for WR stars and O-stars
  → treatment of this layer needs rethinking

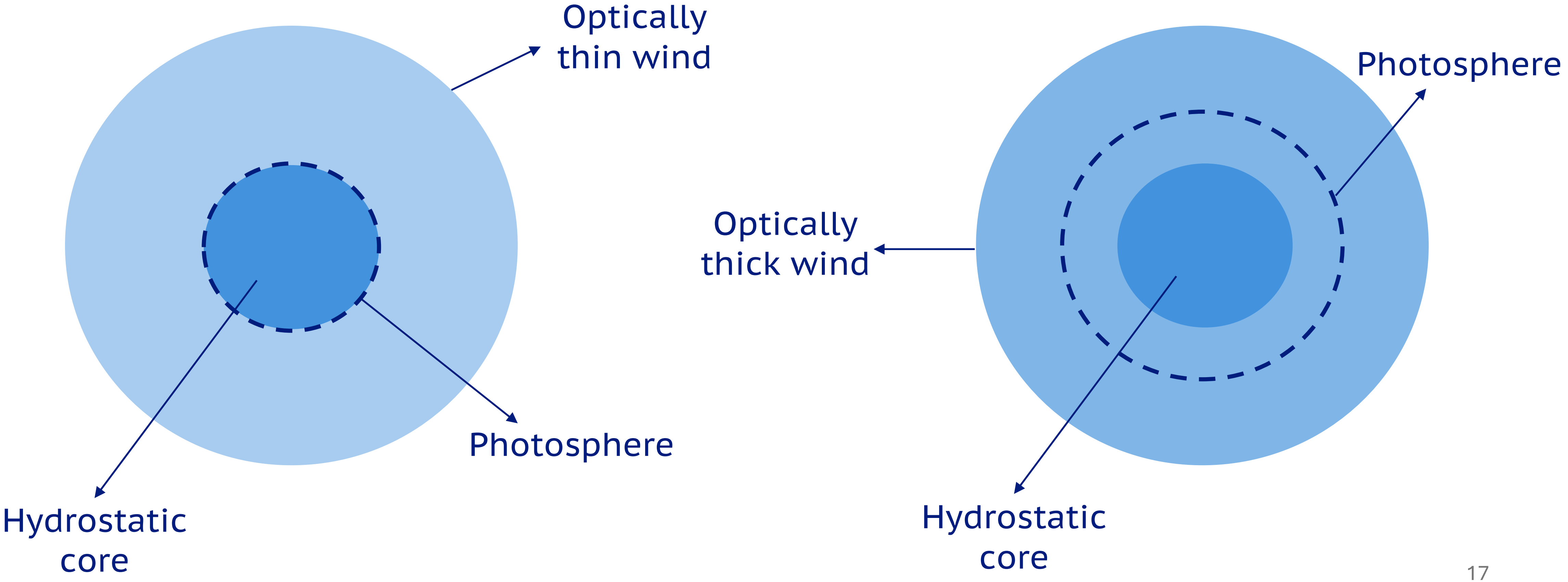
- Structure growth instead driven by:
  Rayleigh-Taylor instabilities for **O-stars**?
  strange mode instabilities for **WR stars**?

- Simulations show **long, finger-like structures** instead of isotropic clumps

O-star vs WR star

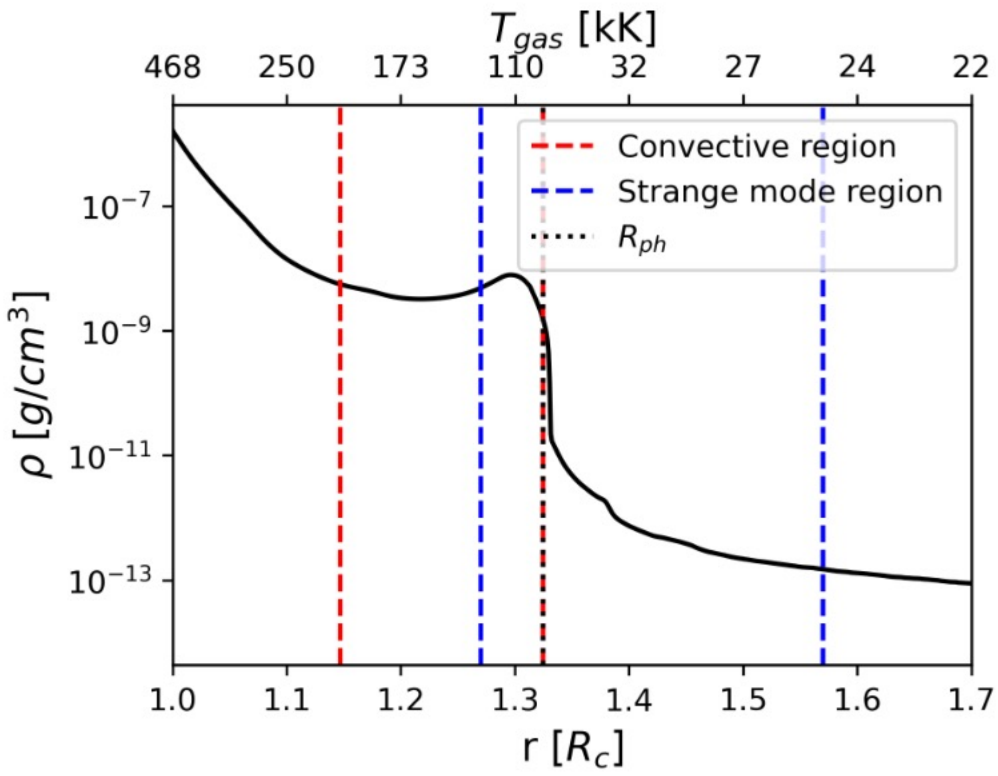
O-star

WR star

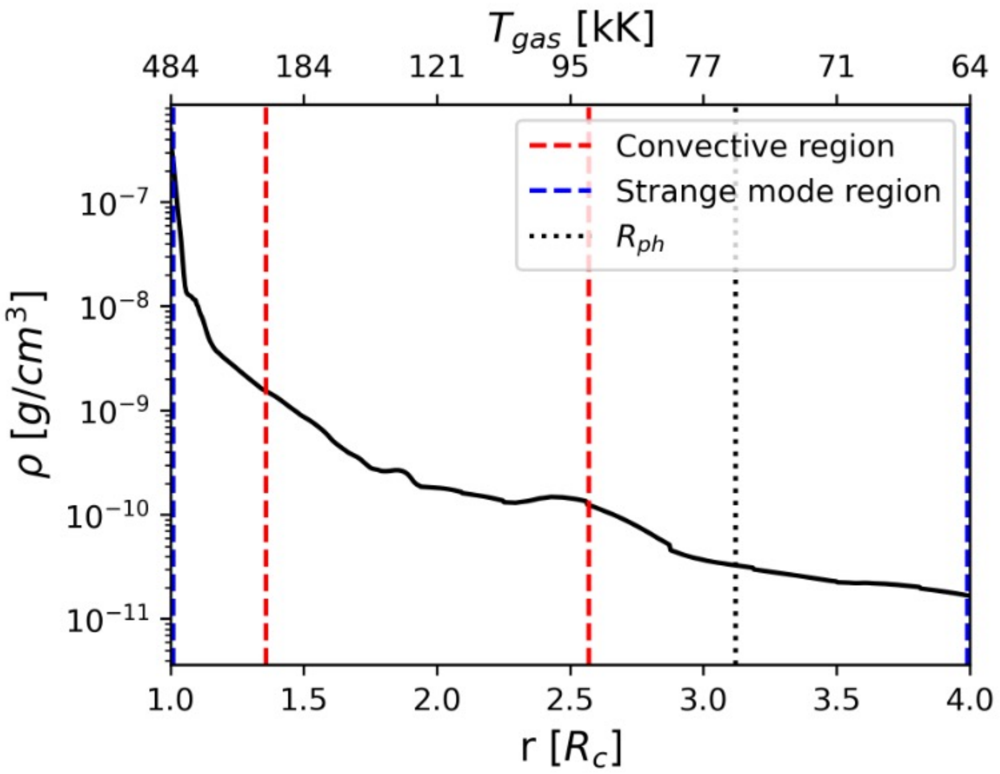


Initial conditions

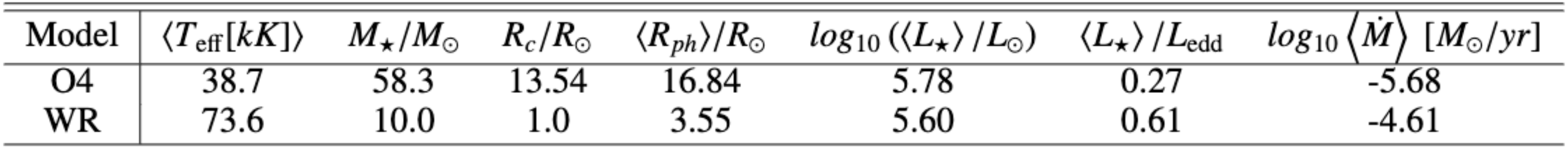
O-star



WR star



Model parameters



Model	$\langle T_{\rm eff}[kK]\rangle$	$M_\star/M_\odot$	$R_c/R_\odot$	$\langle R_{ph}\rangle/R_\odot$	$log_{10}\left(\langle L_\star\rangle/L_\odot\right)$	$\langle L_\star\rangle/L_{\rm edd}$	$log_{10}\left\langle \dot{M}\right\rangle$ $[M_\odot/yr]$
O4	38.7	58.3	13.54	16.84	5.78	0.27	-5.68
WR	73.6	10.0	1.0	3.55	5.60	0.61	-4.61

Effect of outflow

Characteristic advection time
$\boldsymbol{t \approx 1200\ s}$

Characteristic growth time
$\boldsymbol{t \approx 1500\ s}$

Perturbations don’t have time to grow before being advected out of Fe-bump region!