## Photometric variability of binaries

Research workshop on evolved stars

## Veronika Schaffenroth

$$
10.09 .2021
$$

Institute for Physics and Astronomy
Email: schaffenroth@astro.physik.uni-potsdam.de
Room: 2.118


Introduction

Stars, whose brightness vary periodically, semi-periodically or irregularly as seen from earth

- extrinsic variables: variability is due to the eclipse of one star by another or the effect of stellar rotation
- intrinsic variables: variation is due to physical changes in the star or stellar system


## Extrinsic variables

Transiting planets/Eclipsing binaries


Rotating variables


Pulsating variables


Cataclysmic variables


Types of variable stars

## Binary Stars: Overview

## Binaries

$50 \%-80 \%$ of all stars in the solar neighbourhood belong to multiple systems.


Duchene \& Kraus 2013
$\rightarrow$ stellar evolution cannot be understood without understanding binary evolution

Rough classification:
apparent binaries: stars are not physically associated, just happen to lie along same line of sight ("optical doubles").
visual binaries: bound system that can be resolved into multiple stars
(e.g., Mizar); can image orbital motion, periods typically 1 year to several 1000 years.
spectroscopic binaries: bound systems, cannot resolve image into multiple stars, but see Doppler effect in stellar spectrum; often short periods (hours...months).

To determine stellar masses, use Kepler's 3rd law:

$$
\frac{a^{3}}{P^{2}}=\frac{G}{4 \pi^{2}}\left(m_{1}+m_{2}\right)
$$

where

- $M_{1,2}$ : masses
- $P$ : period
- a semimajor axis

Observational quantities:

- $P$ - directly measurable
- a - measurable from image if and only if distance to binary and the inclination are known


Problem when analysing orbits: orientation of orbit in space: "inclination"

In simplest case: real semimajor axis:

$$
a_{\text {observed }}=a_{\text {real }} \cos i
$$

## Spectroscopic Binaries





Spectroscopic binaries: Components close together: orbital motion via periodic Doppler shift of spectral lines.
SB2 = both spectra are visible
SB1 = only one spectrum visible
in eclipsing SB2 systems the inclination (close to $\mathrm{i}=90^{\circ}$ ) and masses for both components can be determined.


CD $-30^{\circ} 11223$ (Geier, ..., Schaffenroth et al. 2013, A\&A 554, 10)

Motion of star visible through
Doppler shift
in stellar spectrum:

$$
\frac{\Delta \lambda}{\lambda}=\frac{v_{r}}{c}=\frac{v \sin i}{c} \sin \frac{2 \pi}{P} t
$$

## Spectroscopic binaries

## Double-lined spectra, case SB2

Assume circular orbit ( $e=0$ )
$K_{1}, K_{2}$ velocity half amplitudes of components $1 \& 2$
$P \quad$ orbital period
$2 \pi a_{1 / 2}$ orbital radii of components $1 \& 2$

$$
\begin{aligned}
& K_{1 / 2}=\frac{2 \pi a_{1 / 2}}{P} \sin i \\
\Rightarrow & a_{1 / 2} \sin i=\frac{P}{2 \pi} K_{1 / 2}
\end{aligned}
$$

again $\sin i$ remains indetermined

## Spectroscopic binaries

centre of mass law:

$$
\frac{M_{1}}{M_{2}}=\frac{a_{2}}{a_{1}}=\frac{K_{2}}{K_{1}}
$$

Kepler's third law:

$$
\begin{gathered}
M_{1}+M_{2}=\frac{4 \pi^{2}}{G P^{2}} a^{3}, \\
a=a_{1}+a_{2}=\frac{P}{2 \pi}\left(K_{1}+\frac{P}{2 \pi} K_{2}\right) / \sin i \\
\Rightarrow M_{1}+M_{2}=\frac{4 \pi^{2}}{G P^{2}} \frac{P^{3}}{(2 \pi)^{3}} \frac{\left(K_{1}+K_{2}\right)^{3}}{(\sin i)^{3}}(\star) \\
\Rightarrow M_{1}+M_{2}=\frac{P}{2 \pi G} \frac{\left(K_{1}+K_{2}\right)^{3}}{(\sin i)^{3}}
\end{gathered}
$$

$$
\left(M_{1}+M_{2}\right)(\sin i)^{3}=\frac{P}{2 \pi G}\left(K_{1}+K_{2}\right)^{3}
$$

$\Rightarrow$ two equations for three unknowns ( $\left.M_{1}+M_{2}, \sin i\right)$, $\sin i$ can only be determined for eclipsing binaries

## Spectroscopic binaries

## Single-lined spectra, case SB1

(only one spectrum visible):
$K_{2}$ unknown: $K_{2}=K_{1} \frac{M_{1}}{M_{2}}$
Insert in equation ( $\star$ ):

$$
\begin{gathered}
\left(M_{1}+M_{2}\right)(\sin i)^{3}=\frac{P}{2 \pi G}\left(K_{1}+K_{1} \frac{M_{1}}{M_{2}}\right)^{3} \\
\frac{M_{2}\left(1+\frac{M_{1}}{M_{2}}\right)(\sin i)^{3}}{\left(1+\frac{M_{1}}{M_{2}}\right)^{3}}=\frac{P K_{1}^{3}}{2 \pi G}
\end{gathered}
$$

Mass function $f(M)$ :

$$
f(M)=\frac{M_{2}(\sin i)^{3}}{\left(1+\frac{M_{1}}{M_{2}}\right)^{2}}=\frac{P K_{1}^{3}}{2 \pi G}
$$

## Spectroscopic binaries: Radial velocity curve


http://astro.unl.edu/naap/esp/animations/radialVelocitySimulator.html

## Light Curves of Eclipsing Binary Stars

## Eclipsing Binaries

Determination of diameters $d_{A}$ and $d_{B}$ from eclipse timing:
Duration of eclipse:

$$
\begin{equation*}
d_{A}+d_{B}=v\left(t_{5}-t_{2}\right) \tag{3.1}
\end{equation*}
$$

Duration of eclipse egress:

$$
\begin{equation*}
d_{A}-d_{B}=v\left(t_{4}-t_{3}\right) \tag{3.2}
\end{equation*}
$$

therefore:

$$
\begin{align*}
& d_{A}=\frac{1}{2} v\left(t_{5}-t_{2}+t_{4}-t_{3}\right)  \tag{3.3}\\
& d_{B}=\frac{1}{2} v\left(t_{5}-t_{2}-t_{4}+t_{3}\right) \tag{3.4}
\end{align*}
$$

Note: requires extremely accurate photometry

## Eclipsing Binaries



Stephan-Boltzmann-Law

$$
\begin{equation*}
L_{1 / 2}=4 \pi R_{1 / 2}^{2} T_{1 / 2}^{4} \tag{3.5}
\end{equation*}
$$

$$
\begin{array}{cc}
\frac{T_{1}}{T_{2}}=\left(\frac{F 1-F 2}{F_{1}-F_{3}}\right)^{1 / 4} & \text { (3.6) } \quad \frac{R_{1}}{R_{2}}=\left(\frac{F 1-F 3}{F_{2}}\right)^{1 / 2} \\
\frac{R_{1}}{a}=\frac{1}{2}\left(\sin 2 \pi \Phi_{a}-\sin 2 \pi \Phi_{b}\right) & \text { (3.7) }
\end{array} \frac{R_{2}}{a}=\frac{1}{2}\left(\sin 2 \pi \Phi_{a}+\sin 2 \pi \Phi_{b}\right) .
$$

## Eccentricity in eclipsing binaries










$$
\Delta t=\frac{2 P}{\pi} e \cos \omega
$$


R. Hynes

In a close binary system: Gravitational potential described by the Roche potential:
$\Phi_{R}(\mathbf{r})=-\frac{G M_{1}}{\left|\mathbf{r}-\mathbf{r}_{1}\right|}-\frac{G M_{2}}{\left|\mathbf{r}-\mathbf{r}_{2}\right|}-\frac{1}{2}(\vec{\omega} \times \mathbf{r})^{2}$
and where

$$
\vec{\omega}=\left(\frac{G M}{a^{3}}\right)^{1 / 2} \hat{e}
$$

Stellar surfaces are isosurfaces of this potential
$\Rightarrow$ stars are non-spherical
$\Longrightarrow$ Stellar magnitude changes with orbit. Roche radius:

$$
\begin{equation*}
\frac{R_{L}}{a}=\frac{0.49 q^{2 / 3}}{0.6 q^{2 / 3}+\ln \left(1+q^{1 / 3}\right)} \tag{3.11}
\end{equation*}
$$



Carroll \& Ostlie

Approximations:

- stellar potentials are point-like (most of the stellar mass in concentrated in its core)
- Orbits are circularised (quickly established by tidal forces)
- rotation axes are perpendicular to the orbital plane
- stellar rotation is synchronous (tidally locked to the orbit)

The Roche Model


Detached Binaries

The Roche Model


Contact Binaries

The Roche Model


Overcontact Binaries

The Roche Model

light curves of eclipsing binaries: detached, contact, overcontact (top to bottom)


FIGURE 3.17. Center-to-limb variation. This figure shows the aspect angle (angle between normal vector $\mathbf{n}$ and radiation emission direction e) appearin in the mathematical formulation of the limb-darkening. The right part of th figure illustrates that the depth of the atmosphere region (and thus temperature accessible to an observer varies with the aspect angle $\gamma$.

Kallrath \& Milone (1999)

- intensity of the stellar disk decreases from the centre to the limb temperature is increaing with increasing photospheric depth
- can be measured for the sun
- can be measured by microlensing
- can be calculated from model atmospheres
- linear law: $I=I_{0}(1-\epsilon+\epsilon \cos \theta)$
$\epsilon=$ limb darkening factor, wavelength dependent sun in the UV ( $<1600 \AA$ ): limb brightening due to chromospheric temperature rise


## Limb darkening



Claret \& Bloemen (2011, A\&A 529, A75)

- limb darkening coefficient is temperature dependent
- other laws in use

Claret's law:

$$
\begin{align*}
& \quad I / I_{0}=1-a_{1}\left(1-\mu^{1 / 2}\right)-a_{2}(1-\mu)-a_{3}\left(1-\mu^{3 / 2}\right)-a_{3}\left(1-\mu^{2}\right)  \tag{3.12}\\
& \mu=\cos \gamma
\end{align*}
$$

HD 209458b: the first transiting exoplanet discovered, HST light curve:


- Transit is not central
- transit depth is not constant
$\bullet \longrightarrow$ caused by limb darkening


Brown et al. (2001, ApJ 552:699)

- non-spherical stars, surface gravity varies across the surface
- von Zeipel's Theorem: radiative atmospheres: black body: diffusion equation
- due to temperature gradient in star Flux $F_{R} \propto \nabla B \propto \frac{d B}{d \phi} \nabla \Phi$ $\propto g$
- in the convective case $\mathrm{F} \approx \mathrm{g}^{0.32}$ (Lucy's law, 1967)
- derive numerically from appropriate model atmospheres
- $F \propto g^{y}$ (tables by Claret \& Bloemen, 2011)

- non-spherical stars, surface gravity varies across the surface
- derive numerically from appropriate model atmospheres
- $F \propto g^{y}$ (tables by Claret \& Bloemen, 2011)

Tidally-distorted, limb-darkened, eclipsing, with and without grav-
ity darkening.


Heber et al. 2004, A\&A 420, 251

- light variation by irradiated hemisphere of the companion
- companion has phases like the moon or Venus
- e.g. HS2333+3927: Hot star (33000K) \& cool star (3000K)
- Albedo: percentage of light refelected from the irradiated surface.
- The refelction effect is not simply reflected light
- the irradiated hemisphere is strongly heated
- e.g. AA Dor: A hot subdwarf (40000K) \& brown dwarf (3000K)
- hemisphere is heated to more than 20000K
- redistribution of flux from one wavelengths range to the other $\longrightarrow$ albedo can be larger than 1 (100\%)
- synchronised rotation, no heat exchange expected

Vuckovic et al. 2016


- CoRoT 1b: Hot Jupiter: mass $\mathrm{M}=1.03 \mathrm{M}_{\mathrm{Jup}}$; radius: $\mathrm{R}=1.49 \mathrm{R}_{\text {Jup }}$
- CoRoT 1b: Reflection effect and eclipse of a transiting planet discovered for the first time (Snellen et al. 2009)
- Orbital period 1.509 d , light variation 0.01\%

Snellen et al., 2009, Nature 459,543

| i | Inclination |
| :--- | :--- |
| q | mass ratio $M_{2} / M_{1}$ for $M_{2} \leq M_{1}$ |
| $\Omega_{1}, \Omega_{2}$ | Surface potentials |
| $T_{1}, T_{2}$ | effecive temperatures |
| $A_{1}, A_{2}$ | albedos |
| $g_{1}, g_{2}$ | gravity darkening coefficients |
| $L_{1}(\lambda), L_{2}(\lambda)$ | monochromatische luminosities |
| $x_{1}(\lambda), x_{2}(\lambda)$ | linear limb darkening coefficients |
| $I_{3}(\lambda)$ | third light |

- parameters of the Roche model
- observe light curves, preferntially in several filters
- fit synthetic light curves, 17 free parameters!
- degenaracy of solutions, in particular for q
- RV curve $\longrightarrow$ limits q!


## Lightcurve analysis of eclipsing sdB+dM systems

Research workshop on evolved stars

## Veronika Schaffenroth

$$
10.09 .2021
$$

Institute for Physics and Astronomy
Email: schaffenroth@astro.physik.uni-potsdam.de
Room: 2.118


## Introduction

Hot subdwarfs in binaries with unseen companion discovered by RV method


CD-30¹122, $P=0.0498 \mathrm{~d}$ (Geier et al. 2013)


PHL 457, $P=0.3131$ d (Schaffenroth et al. 2014)

$$
f(m)=\frac{M_{2}^{3} \sin ^{3} i}{\left(M_{1}+M_{2}\right)^{2}}=\frac{K_{1}^{3} P}{2 \pi G}
$$

more than $50 \%$ of sdBs in close binaries ( $P<1 \mathrm{~d}$ )

## Formation of sdB binary



Han et al. $(2002,2003)$

Soker 1998 AJ

- Orbit of planet in envelope of evolved star
- fate of planet:
- evaporation
- merger with the core
- survival for $\geq 10 M_{\text {Jupiter }}$ depending on separation
$\rightarrow$ ejection of envelope
$\rightarrow$ studying the influence of planets on stellar evolution
- eclipsing binaries consisting of sdB and cool, low mass stellar or substellar companion
- 20 HW Vir systems published
- very short period $\sim 1.5-6 \mathrm{~h}$ (separation $\sim 1 R_{\odot}$ )
$\Rightarrow$ post common envelope system
- only sdB visible in spectrum
- unique lightcurve
$\Rightarrow$ huge reflection effect


Lightcurve of HW Virginis (Lee et al. 2009)

Observed mass distribution of sdBs


Fontaine et al. 2012


Schaffenroth et al. 2019 in press

$$
f(m)=\frac{M_{2}^{3} \sin ^{3} i}{\left(M_{1}+M_{2}\right)^{2}}=\frac{K_{1}^{3} P}{2 \pi G}
$$

## OGLE

Optical Gravitational Lensing Experiment

$\rightarrow$ observation of the lightcurve of many stars in different fields
$\rightarrow$ discovery of planetary transits, pulsators, eclipsing binaries CRTS, PTF, ZTF, BlackGEM, ....

## ATLAS

Asteroid Terrestrial-impact Last Alert System

$\rightarrow$ a robotic astronomical survey looking for near-earth objects
$\rightarrow$ located in Hawaii, planned in the southern hemisphere

150 HW Vir candidate systems: $P=0.05-1.26 \mathrm{~d}$


## EREBOS (Eclipsing Reflection Effect Binaries from Optical

 Surveys)- homogeneous data analysis of all newly discovered HW Vir systems
- photometric and spectroscopic follow-up of all targets to determine fundamental $(M, R)$, atmospheric ( $T_{\text {eff }}, \log g$ ) and system parameters ( $a, P$ )
- spectroscopic and photometric follow-up


## Key questions:

- minimum mass of the companion necessary to eject the common envelope?
- fraction of close substellar companions to sdB stars
- better understanding of the CE phase and the reflection
 effect


## Lightcurve analysis with Icurve



A light curve can be generated as follows:

- Generate grids covering all objects (stars, disc, ...)
- set their surface brightness including all effects, e.g. limb darkening, gravity darkening, reflection effect, Doppler beaming, ...
- At every phase compute what can and cannot be seen, add up the fluxes.

Iroche computes the light curve equivalent to a model of a sphere and a Rochedistorted star to model a white dwarf or subdwarf/main-sequence binary and can optionally include a disc and bright-spot as well.
Other physics included: Doppler beaming, gravitational lensing, Roemer time delays, asynchronous rotation of the stellar components

## Invocation

Iroche model data noise seed nfile [output] (device)]]
noise multiplier of the real error bars
seed Seed integer
nfile Number of files to store
output File to save the results in the form of rows each with time, exposure time, flux and uncertainty
device Plot device to use

## Data file

## Data file

- can be in any time units or phase
- must be in normalized flux not magnitudes
- combining data from different nights by phasing the data
- for deriving the period use Lomb-Scargle algorithm
- binning improves the $\mathrm{S} / \mathrm{N}$

Careful with combining data from different nights

- check normalization
- check for trends due to atmospheric dispersion
\#phase delta_phase flux flux_error weight factor
0.0000000 .0050000 .9986870 .00003911
0.0050000 .0050000 .9984290 .00003911
0.0100000 .0050000 .9986270 .00004011
0.0150000 .0050000 .9984450 .00003911 0.0200000 .0050000 .9982520 .00003911 0.0250000 .0050000 .9981460 .00003911 0.0300000 .0050000 .9979680 .00003911 0.0350000 .0050000 .9979220 .00003911 0.0400000 .0050000 .9977630 .00003911 0.0450000 .0050000 .9975870 .00004011 0.0500000 .0050000 .9975780 .00003911 0.0550000 .0050000 .9975950 .00003911 0.0600000 .0050000 .9974970 .00003911


## Parameter file - Physical parameters - Binary and stars

$x=$ initial_value param_space steps fitting(True/False) ignore_param(True/False)
$q \quad$ Mass ratio, $q=\mathrm{M} 2 / \mathrm{M} 1$
iangle Inclination angle, degrees
$r 1$ Radius of star 1, scaled by the binary separation
r2
$t 1$
t2 Temperature of star 2, Kelvin.
Idc1_1, etc Limb darkening for stars is quite hard to specify precisely.
Extrapolate from Claret et al.
velocity_scale sum of unprojected orbital speeds, used for accounting for Doppler beaming and gravitational lensing.
beam_factor 3-alpha factor that multiplies -v_r/c in the standard beaming formula where alpha is related to the spectral shape. Use of this parameter requires the velocity_scale to be set.

## Parameter file - Physical parameters - General

t0 Zero point of ephemeris, marking time of mid-eclipse
period Orbital period, same units as times.
pdot Quadratic coefficient of ephemeris, same units as times
deltat Time shift between the primary and secondary eclipses to allow for small eccentricities and Roemer delays in the orbit. Delay of -deltat/P by the secondary eclipse.
gravity_dark Gravity darkening coefficient. Only matters for the Roche distorted case. set gdark_bolom (see below) to 0 . Use Claret et al.
absorb The fraction of the irradiating flux from star 1 absorbed by star 2
slope, quad, factors to help cope with any trends in the data as a result of
cube e.g. airmass effects. The fit is multiplied by ( $1+x^{*}$ (slope $+x^{*}\left(\right.$ quad $+x^{*}$ cube) ))
third Third light contribution. Simply adds to whatever flux is calculated and will be subject to auto-scaling like other flux. It only applies if global scaling rather than individual component scaling is used. Third light is assumed strictly constant

## Computational parameters

delta_phase Accuracy in phase of eclipse computations nlat1/2f number of latitudes for star $1 / 2$ 's fine grid.

This is used around the phase of primary eclipse
nlat1/2c number of latitudes for star 1's coarse grid.
This is used away from primary eclipse.
phase1 This defines when star 1's fine grid is used abs(phase) < phase1. phase $1=0.05$ will restrict the fine grid use to phase 0.95 to 0.05 .
phase2
wavelength Wavelength (nm)
tperiod this defines when star 2's fine grid is used phase2 until 1-phase2. phase2 $=0.45$ will restrict the fine grid use to phase 0.45 to 0.55 .

The true orbital period in days. This is required, with velocity_scale, if gravitational lensing is applied to calculate proper dimensions.
gdark_bolom True, if gravity darkening coefficient represents the bolometric value limb1/2
'Poly' or 'Claret' determining the type of limb darkening law.
See comments on Idc1_1 above.


- find which models are consistent with the data, statistical and computational task
- different methods: LevenbergMarquardt method, simplex method, Markov Chain Monte Carlo (MCMC)
- much harder to find uncertainties in the parameters, than the best-fitting model itself.


## Degeneracy in the light curve analysis

If a change in one parameter causes a change in the predicted light curve that can be matched by a change in another or several others, then the fit will be degenerate.

For a parameter to be well-defined, its effect on the light curve must be unique.

Degeneracy can

- make it impossible to uniquely constrain parameters,
- lead to strong correlations between multiple parameters,
- cause minimisation algorithms (e.g. Levenberg-Marquardt) to fail.

Bayesian methodology allows one to include prior information!
Use as many known parameters as possible from theory or spectroscopic observation ( $T_{1}, \log g_{1}, y$, limb darkening coefficients, $\ldots$ )

Degeneracy in the light curve analysis
iangle $=81.276055_{-0.904744}^{+0.50039}$


Spectrum

- Radial velocity curve $K_{1}$ and ideally $K_{2} \Rightarrow q=K 1 / K 2$
- effective temperature $T_{1}$
- $\log g_{1}$

Lightcurve

- orbital period $P$
- mass ratio $q$
- inclination $i$
- effective temperature $T_{2}$
- relative radius $r_{1} / a$
- relative radius $r_{2} / a$
- albedo
orbital separation

$$
\begin{equation*}
a=\frac{P}{2 \pi} \frac{K_{1}}{\sin (i)}(1 / q+1) \tag{5.1}
\end{equation*}
$$

radii

$$
\begin{equation*}
R_{1} / 2=\frac{r 1 / 2}{a} \cdot a \tag{5.2}
\end{equation*}
$$

masses

$$
\begin{gather*}
M_{1}=\frac{P}{2 \pi G} \frac{K_{1}^{3}(q+1)^{2}}{(q \sin i)^{3}}  \tag{5.3}\\
M_{2}=q \cdot M_{1} \tag{5.4}
\end{gather*}
$$

$$
\begin{equation*}
\log g=\log _{10}\left(\frac{G M_{1}}{\left(r_{1} / a\right)^{2} \cdot a^{2}}\right) \tag{5.5}
\end{equation*}
$$



Mass-radius relation for the companion Baraffe et al. 2003


Schaffenroth et al. 2017

- you can work remotely: ssh -X blockcourse@carina.astro.physik.uni.potsdam.de; password: late_stellar_evolution
- First play around with Iroche to get a feeling which parameters change what
- to invoke simplex algorithm: simplex model data
- when you found a good model use the Levenberg-Marquardt algorithm to estimate the error
- levmarq model data
- calculate the best model with Iroche to plot results
- with visualise model you get a nice visualization of both stars and their orbit


## Logical Order of Topics

- There is a generally accepted form for scientific papers called the IMRaD approach.
- I = Introduction
- $\mathrm{M}=$ Methods
- $R=$ Results, and
- $\mathrm{D}=$ Discussion

Example: https://iopscience.iop.org/article/10.1088/ 2041-8205/731/2/L22/pdf

## 

## Title and Abstract

- The title and abstract of your article help researchers quickly understand the topics covered in your article.
- They act as a short summary of your article. Most researchers do not have time to read complete articles, but rather rely on titles and abstracts when searching scientific literature.

A\&A style: https://www.aanda.org/for-authors

## Introduction of the article

- The introduction is the first part of your article that contains substantial amounts of text.
- Make the main goals of your study clear in the introduction.
- The introduction gives a statement of the problems that you are studying in the article. Provide the reasons for conducting this investigation.


## Observational or Experimental Work

- In articles about observational or experimental work, the corresponding "Methods" section discusses details of their observations or experiment.
- This section describes aspects of observational or experimental equipment.
- The methods section also highlights how the researchers analyzed their data.


## Results section

- The results section details the findings and outcomes of your study.
- This is especially useful in observational, experimental or data analysis studies.
This section often has tables with numerical data sets.


## Discussion for Observational Work

- Do the results agree with the current model of the phenomenon you are studying?
- If not, how do your results change the current understanding?
- Are you surprised by the outcome of your work?
- How does this advance the current state of knowledge of your field?


## Conclusion of your article

- The conclusion summarizes the information in your article and restates the major points.
- It attempts to tie all the different parts of the article together into a satisfying end.
- Try to answer all the questions you initially posed in the introduction.
- Conclusions are usually relatively short, around one page of text.
all slides: http://www.raa-journal.org/docs/RAA_Lectures/RAA
How to write a pdpaper

