Radial velocity analysis

Finding variable A/B-type stars

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Step-by-step

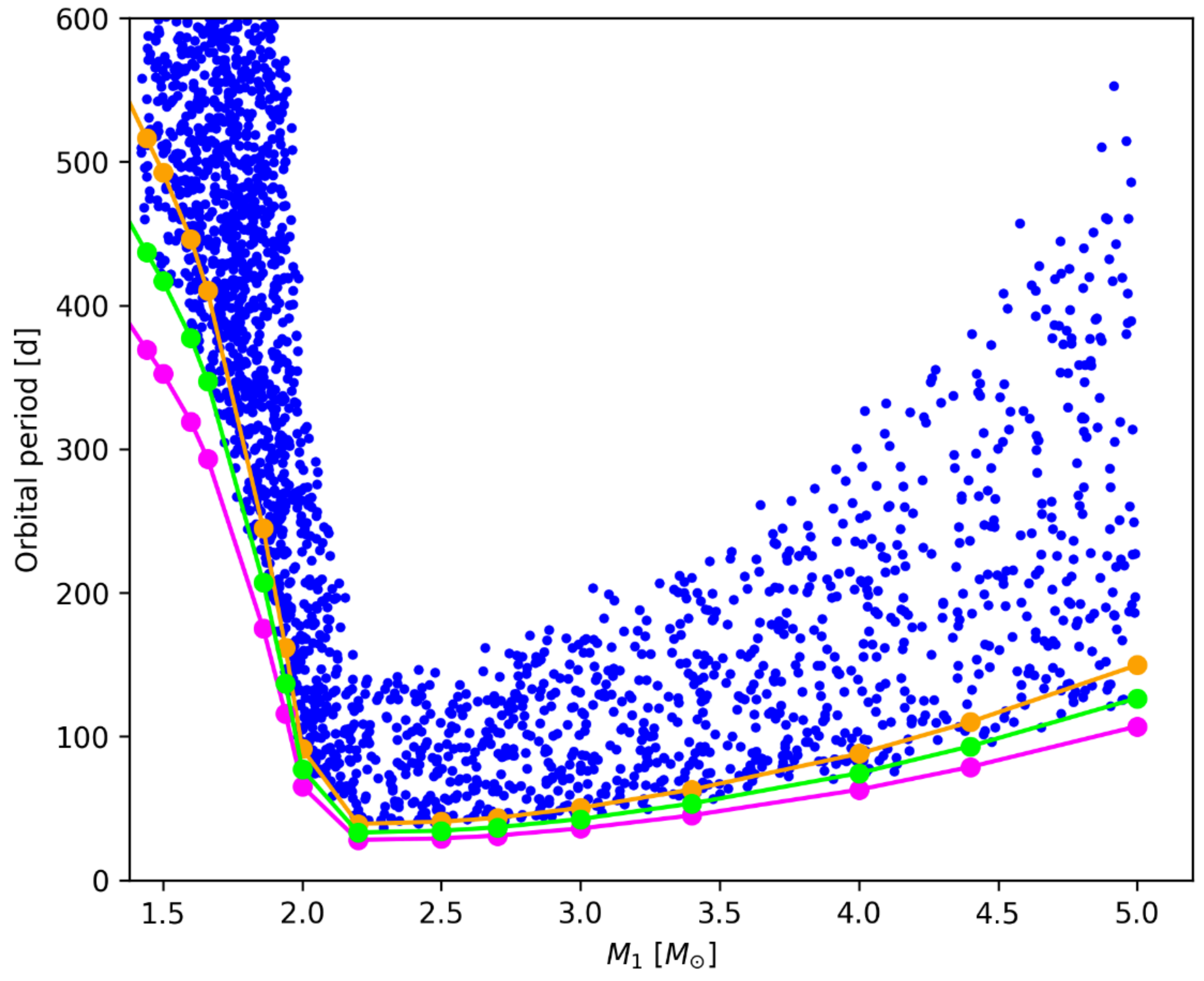
1
Conception & application
Have an idea and convince people to give you observation time
2
Observation
Obtain multiple spectra for each star, spaced by hours to days
3
Data reduction
Extract 1D spectra from raw 2D images
4
Extract information
Obtain radial velocity differences!
4.1
Gauss * Lorentz
model fit
4.2
„Auto-correlate“
Use best spectrum as model
5
Radial velocity variation!
Check if RV variation is consistent with random noise
6
Recommend targets for follow-up (in CMD)

1. Conception & application

Expected orbital periods for stars that will form hot subdwarfs by Marie Kurpas

- The first step is to have a scientific motivation to perform observations -> see Stephan's talk

- Convince someone with a big telescope to let you observe (or better: have them observe for you!)

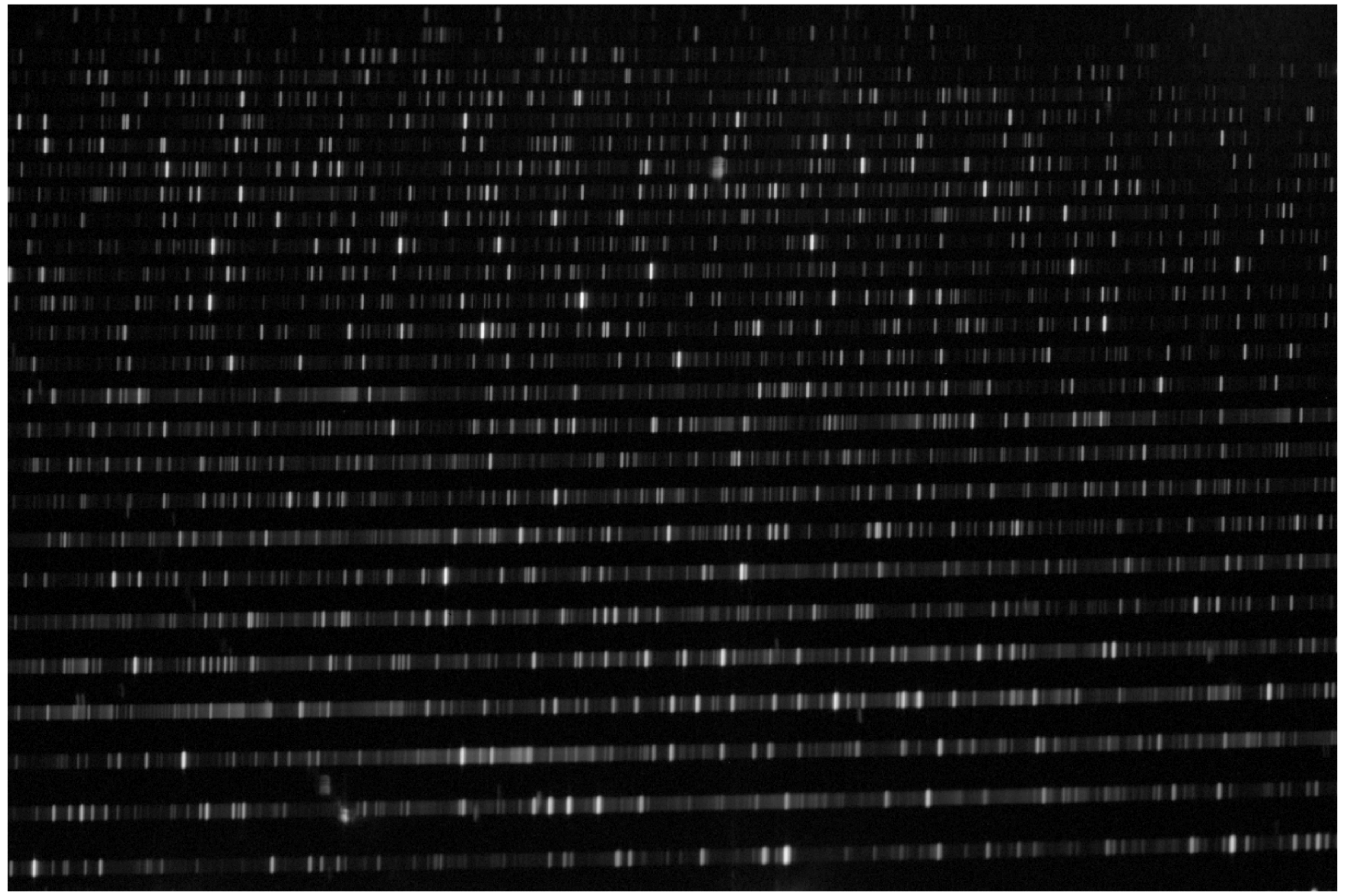


2. Observation & target selection

- Select which data you *need,* which *targets* you want to observe, which are *observable*
- Perform the observations, keeping constraints (airmass, clouds, brightness) in mind

3. Data reduction

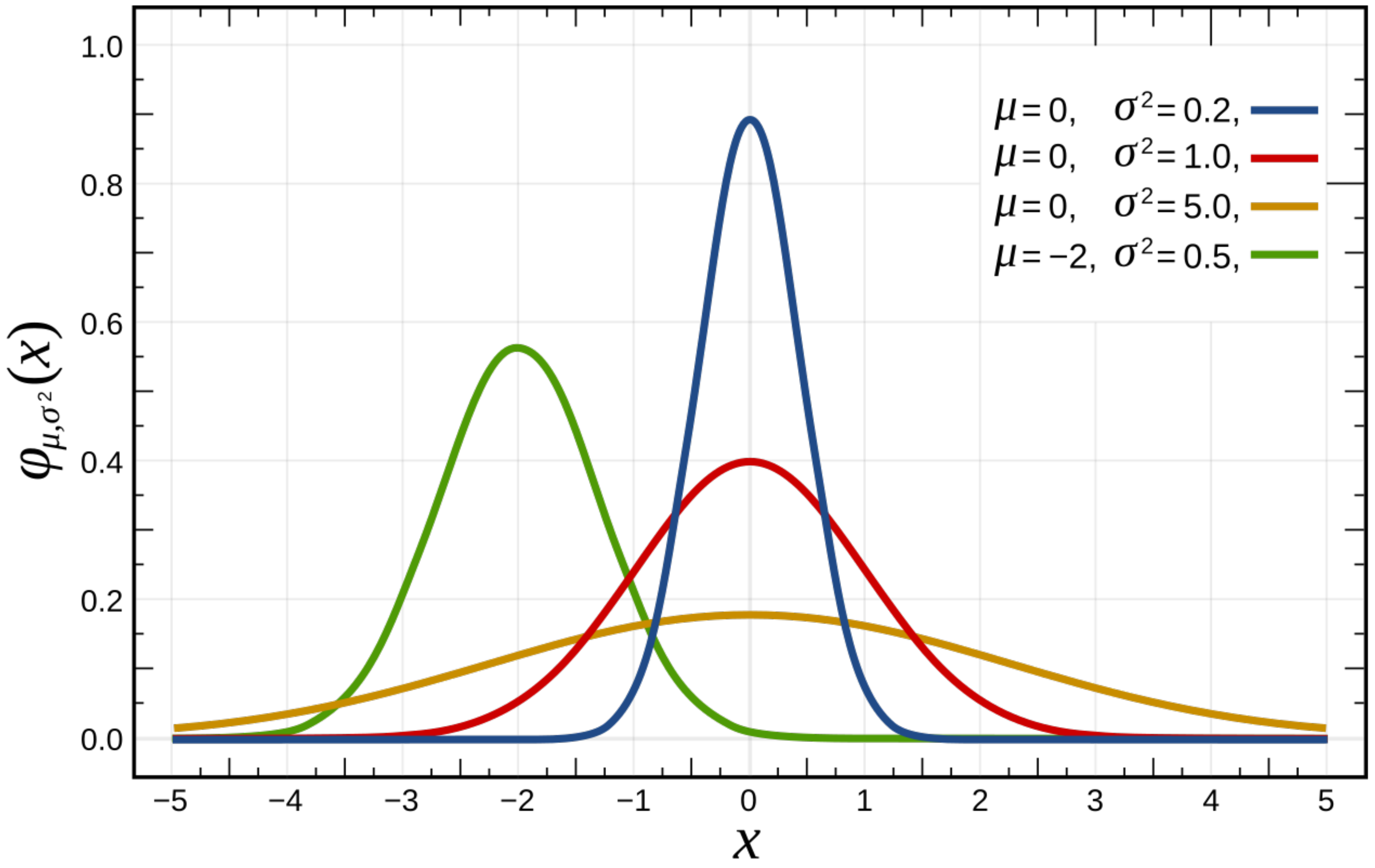
- Several step from 2D images of a spectrum to usable flux vs wavelength spectrum
- *Order extraction, apply bias, flat field correction, fit ThAr emission lines, solve pix-wavelength relations, normalise spectra, merge*



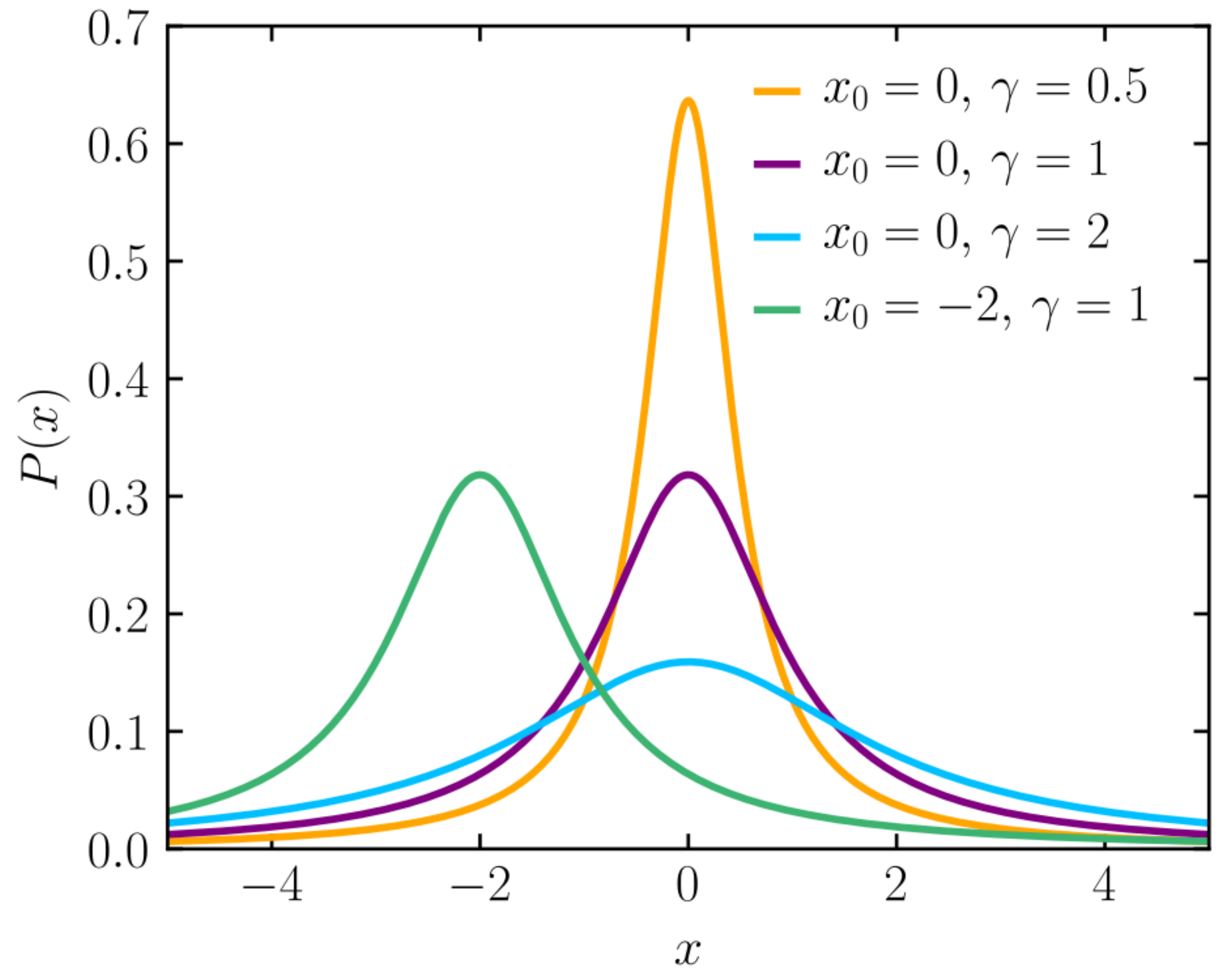
4. Extract information

Straight forward method
fit Gauss * Lorentz model

Obtain radial velocity shifts between spectra of the same star



$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$$

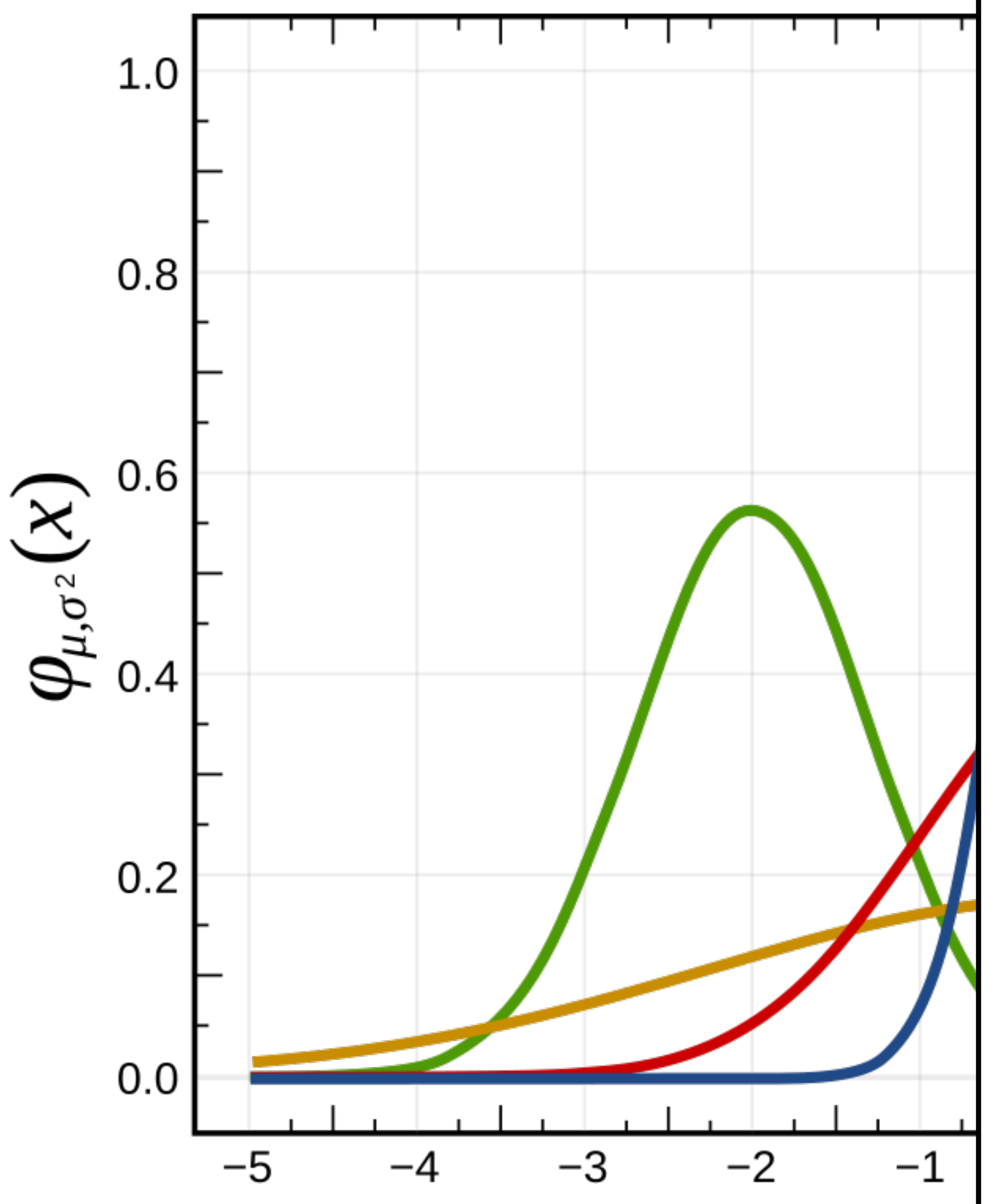


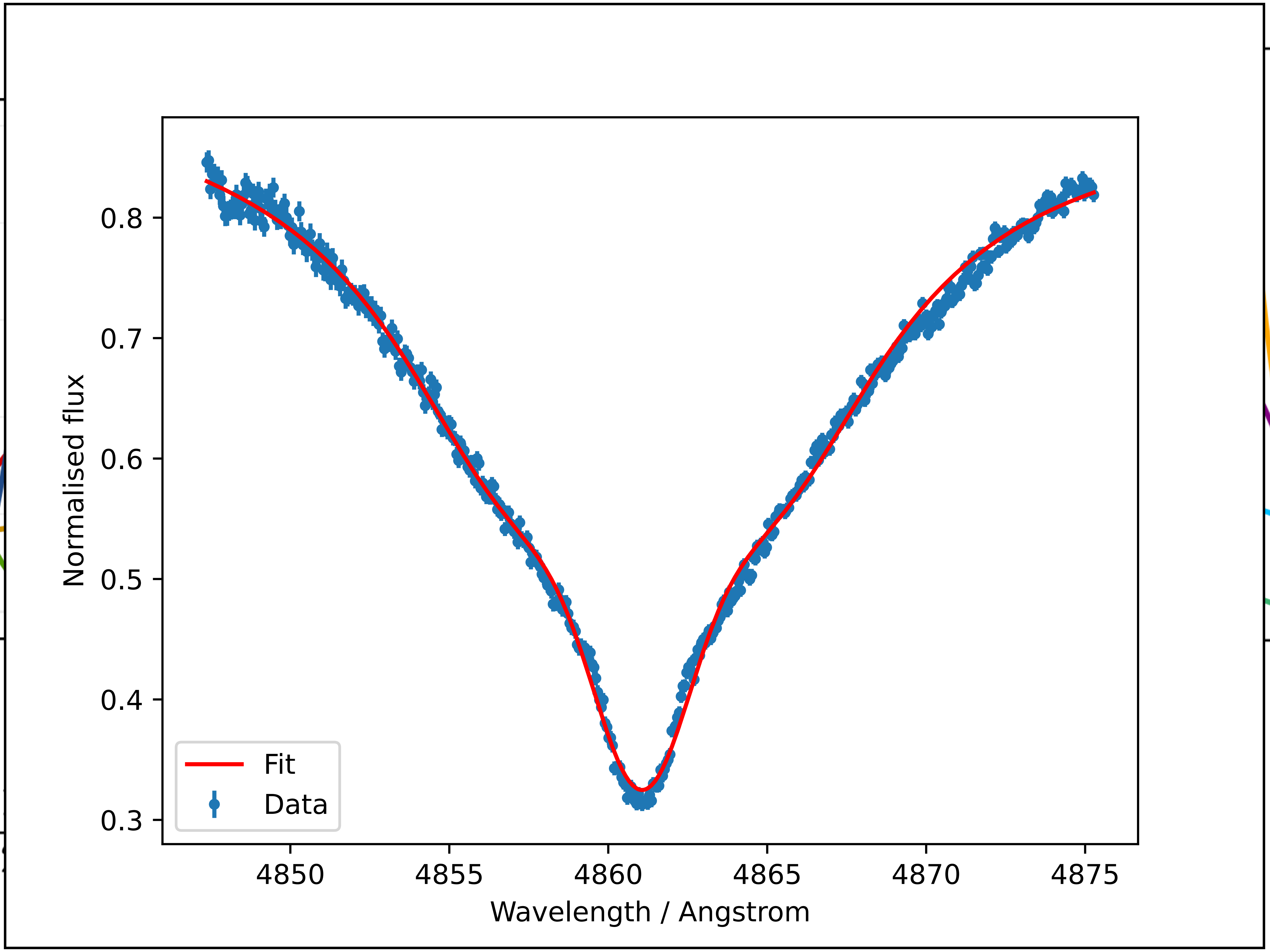
$$f(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + \left(\frac{x-x_0}{\gamma}\right)^2\right]} = \frac{1}{\pi}\left[\frac{\gamma}{(x-x_0)^2 + \gamma^2}\right]$$

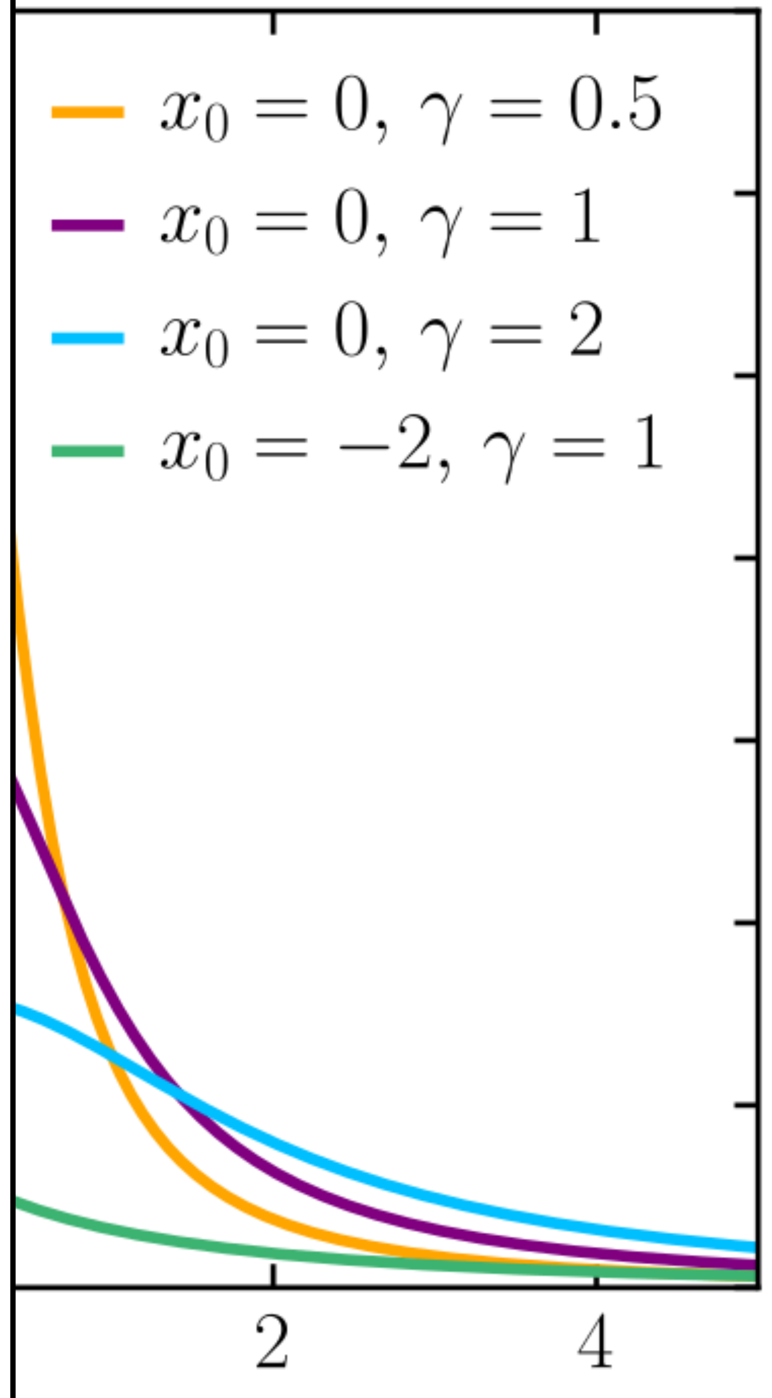
4. Extract information

Straight forward method
fit Gauss * Lorentz model

Obtain radial velocity shifts between spectra of the same star







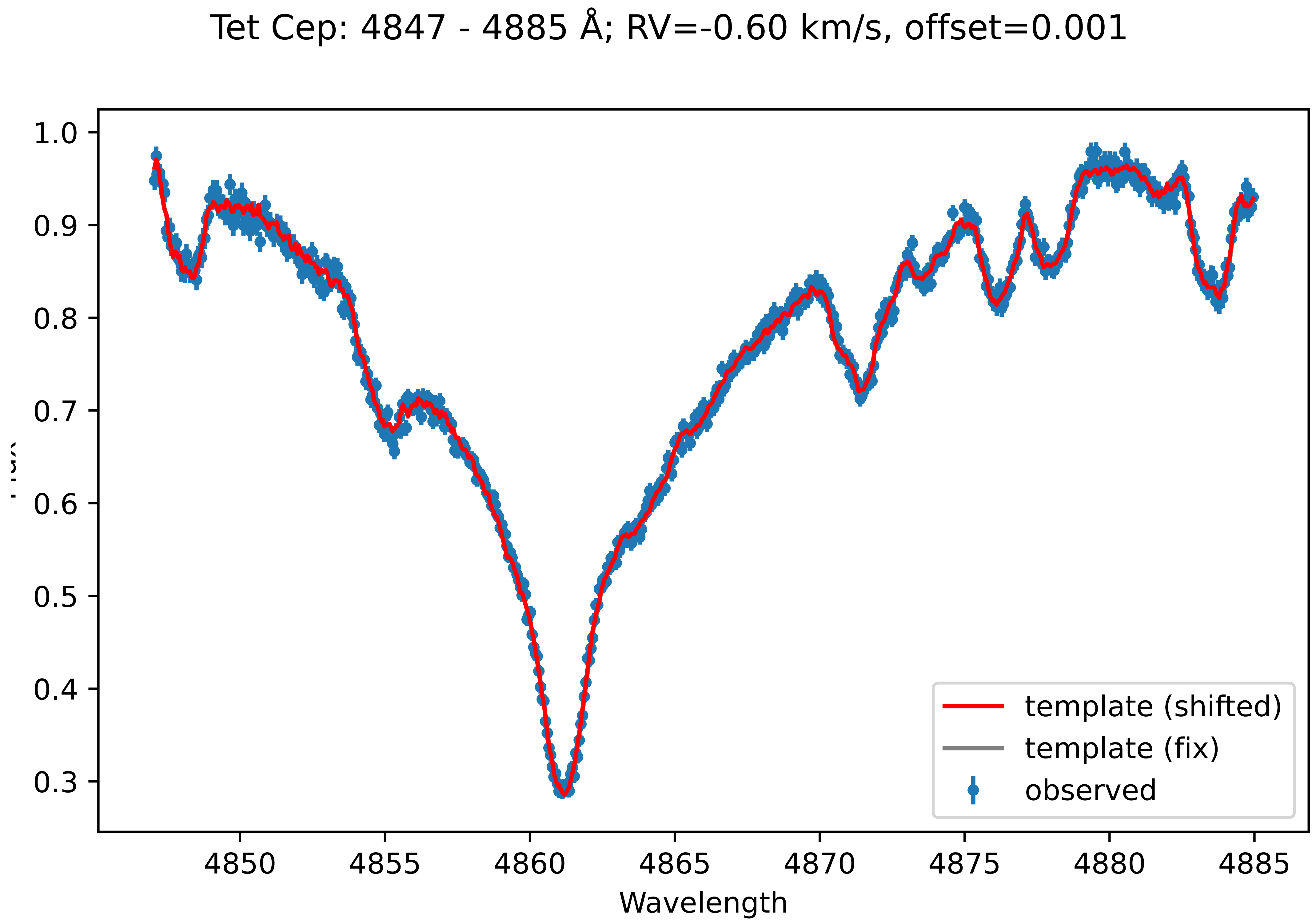
$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\Bigg(-$$

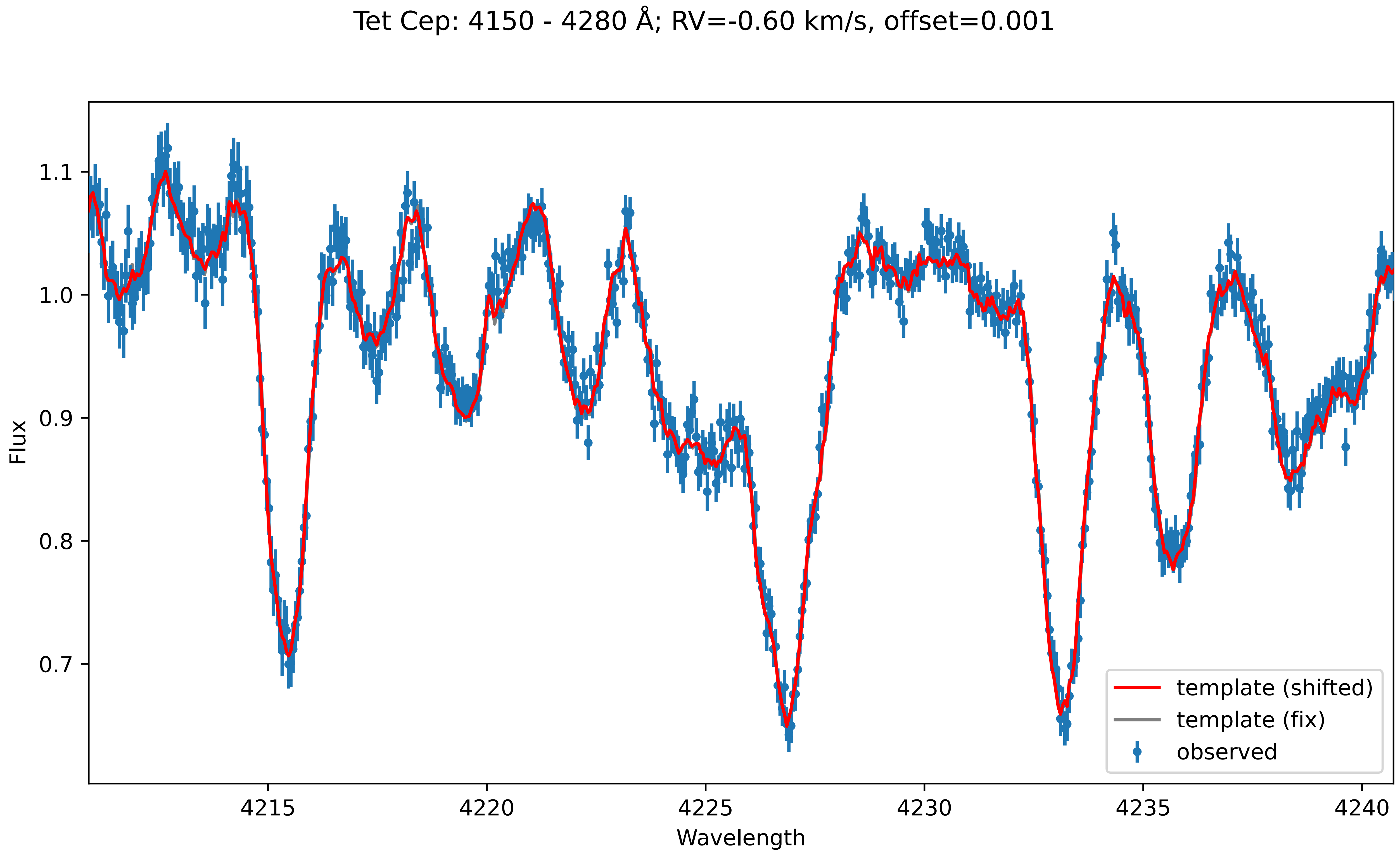
$$= \frac{1}{\pi}\left[\frac{\gamma}{(x-x_0)^2+\gamma^2}\right]$$

4. Extract information

Also straight forward
use „autocorrelation“

Obtain radial velocity shifts between spectra of the same star





More precise, but only relative RVs

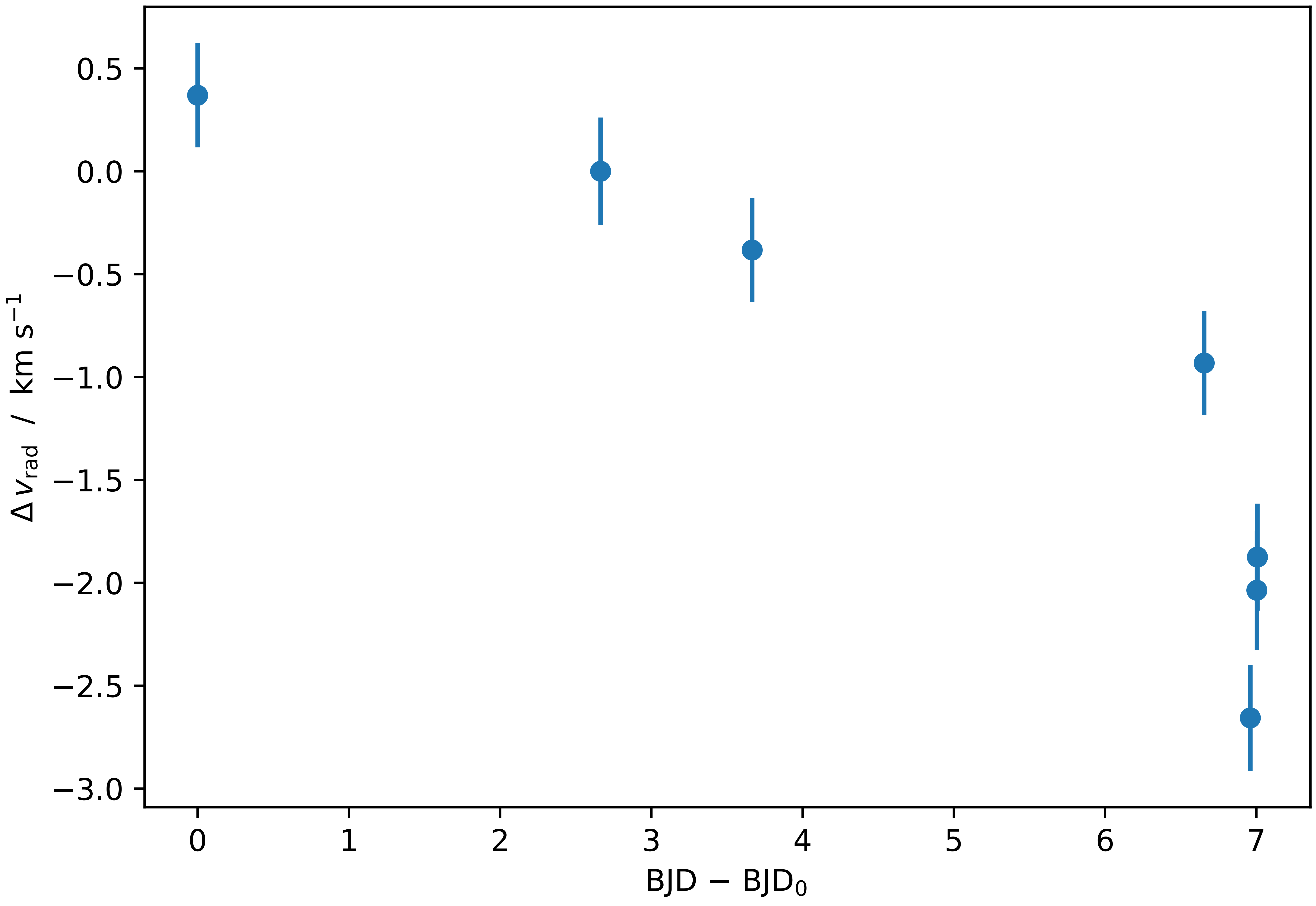
5. Radial velocity variation

$$\sigma_{v_{\rm rad}} = \sqrt{\sigma^2_{v_{\rm rad,stat}} + \sigma^2_{v_{\rm rad,sys}}}\ , \sigma_{v_{\rm rad,sys}} \approx 0.25\,{\rm km\,s^{-1}}$$

- After obtaining accurate relative radial velocities, plot them against BJD.
- How can we tell whether the star is variable or not?
- Assume that the star is constant and evaluate χ^2

$$\chi_i = (v_{\rm rad,i} - v_{\rm const})/\sigma_{v_{\rm rad,i}}$$

$$\chi^2 = \sum_i \chi_i^2$$



χ^2 distribution

distribution of the sum of the squares of k independent standard normal random variables

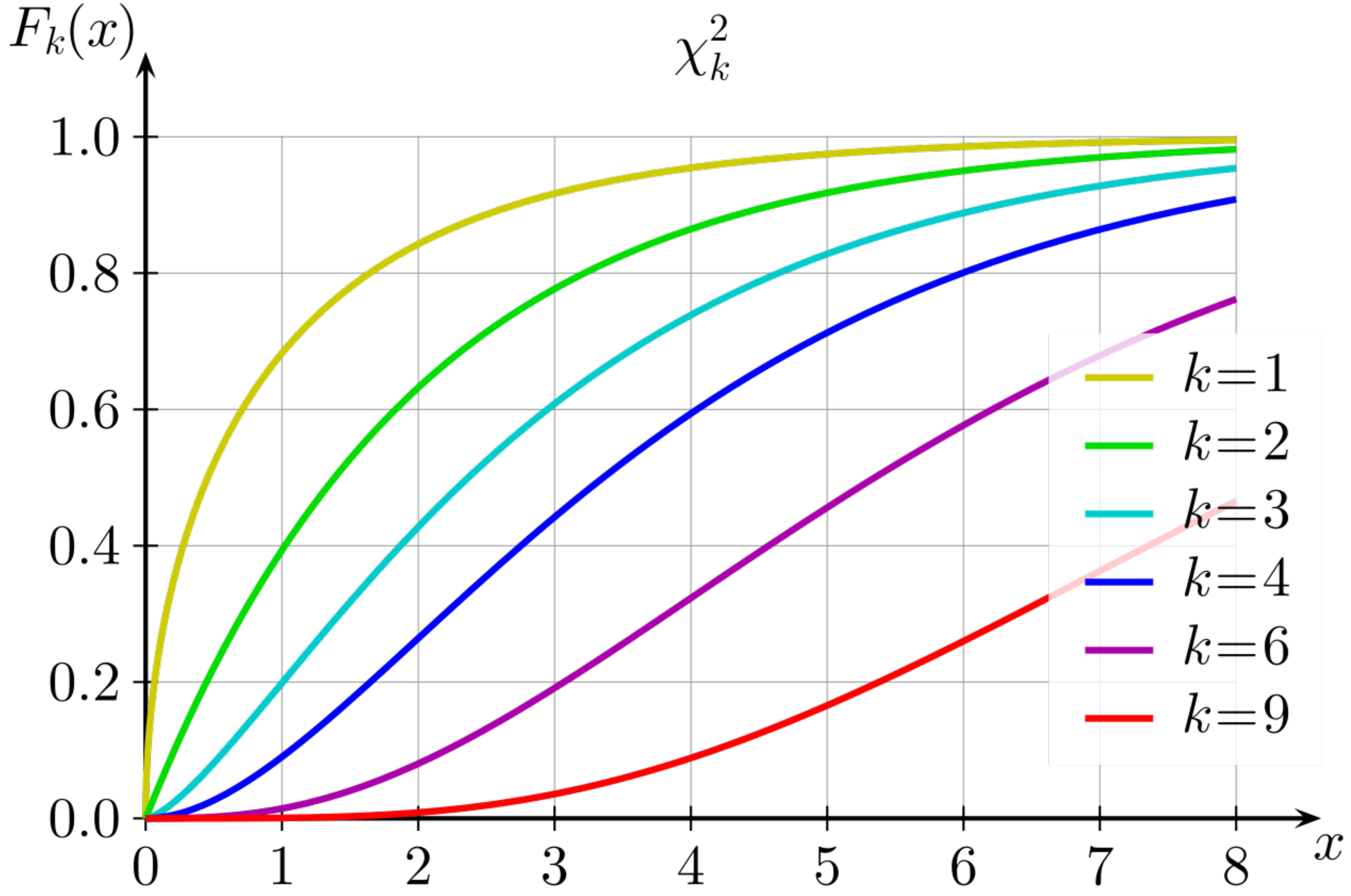
$$f_k(x) = \int_0^x \frac{t^{\frac{k}{2}-1} \cdot e^{-\frac{1}{2}t}}{2^{\frac{k}{2}} \cdot \Gamma(\frac{k}{2})} \, \mathrm{d}t$$

k is the „degree of freedom“

$$k = n_{\text{datapoints}} - n_{\text{free}}$$

Cumulative distribution function (CDF)

probability of observing χ^2 smaller than x



$$p = 1 - F_k(x)$$

```
pval = stats.chi2.sf(chisq_sum, dof)
```

RV-variable stars for follow-up

Mark your binaries in the initial catalogue of targets (~300 stars)

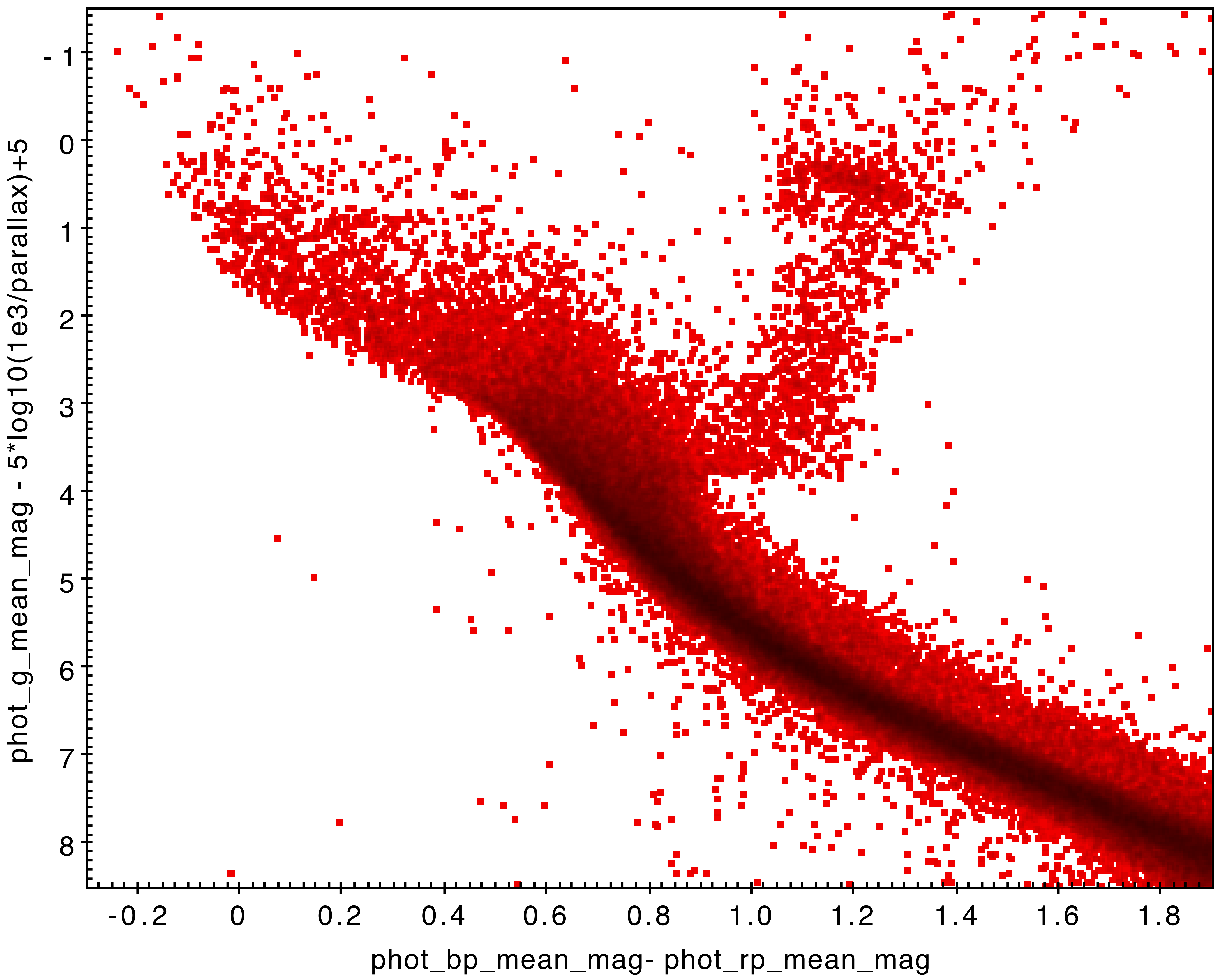
1

$$\log p < -4$$

means

probability that variation is caused by noise is < 0.01%

This is typically used as a threshold to detect variability



2

Plot / look at your spectra for each star and check if any have more than one set of lines (SB2+)

3

Provide a list of good candidates for binaries

Follow-up recommendation

Brankica will perform follow-up for your stars — you can help!

- Which stars are the most interesting (very negative log *p*, good predicted *P*)?
- Which stars are likely to be single (log *p* > ~-2)?

Practical information

- What is the optimal „frequency of pulses“ to achieve around SNR = 100 (in Mcounts)?
- What would be good exposure times for stars of magnitude ranges (4-5, 5-6, 6-7)?

Bonus: spectral energy distribution (SED)

Colours: differences between magnitudes

Use a model to fit the SED

Angular diameter

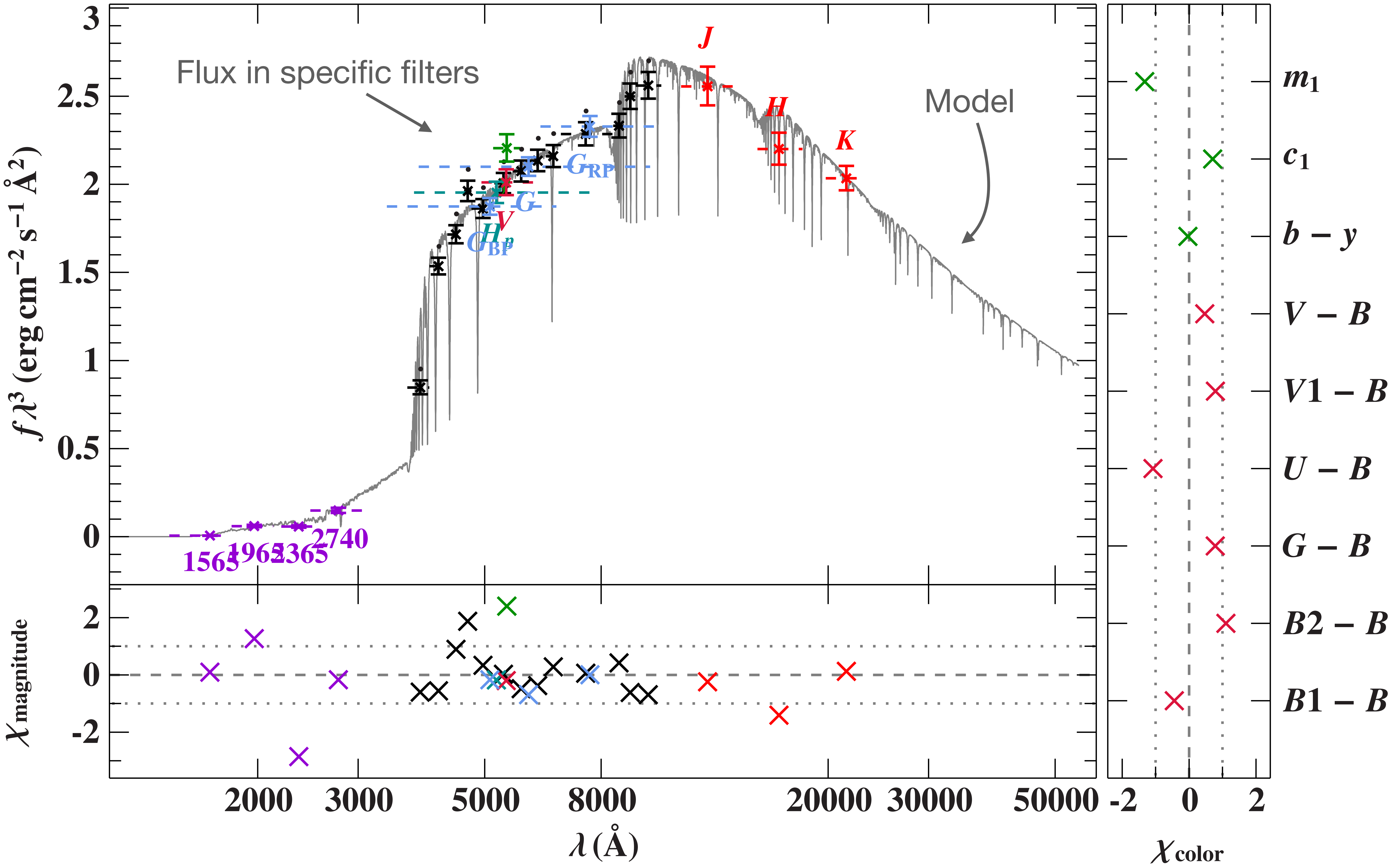
Θ

Effective temperature

$T_{\rm eff}$

Colour excess

$E(44-55)$



Residuals to be minimised

Apparent magnitude

What even is the „flux"?

In stellar astronomy, flux is often specified as the „astrophysical flux" F_λ

F_λ is a spectral flux density with units: $\mathrm{erg\,s^{-1}\,cm^{-2}\,\mathring{A}^{-1}}$

F_ν is a spectral flux density with units: $\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$

= flux density emanating at the surface of a spherical symmetric object with radius R

Also sometimes used: „Eddington flux" $H_\lambda = F_\lambda / 4\pi$

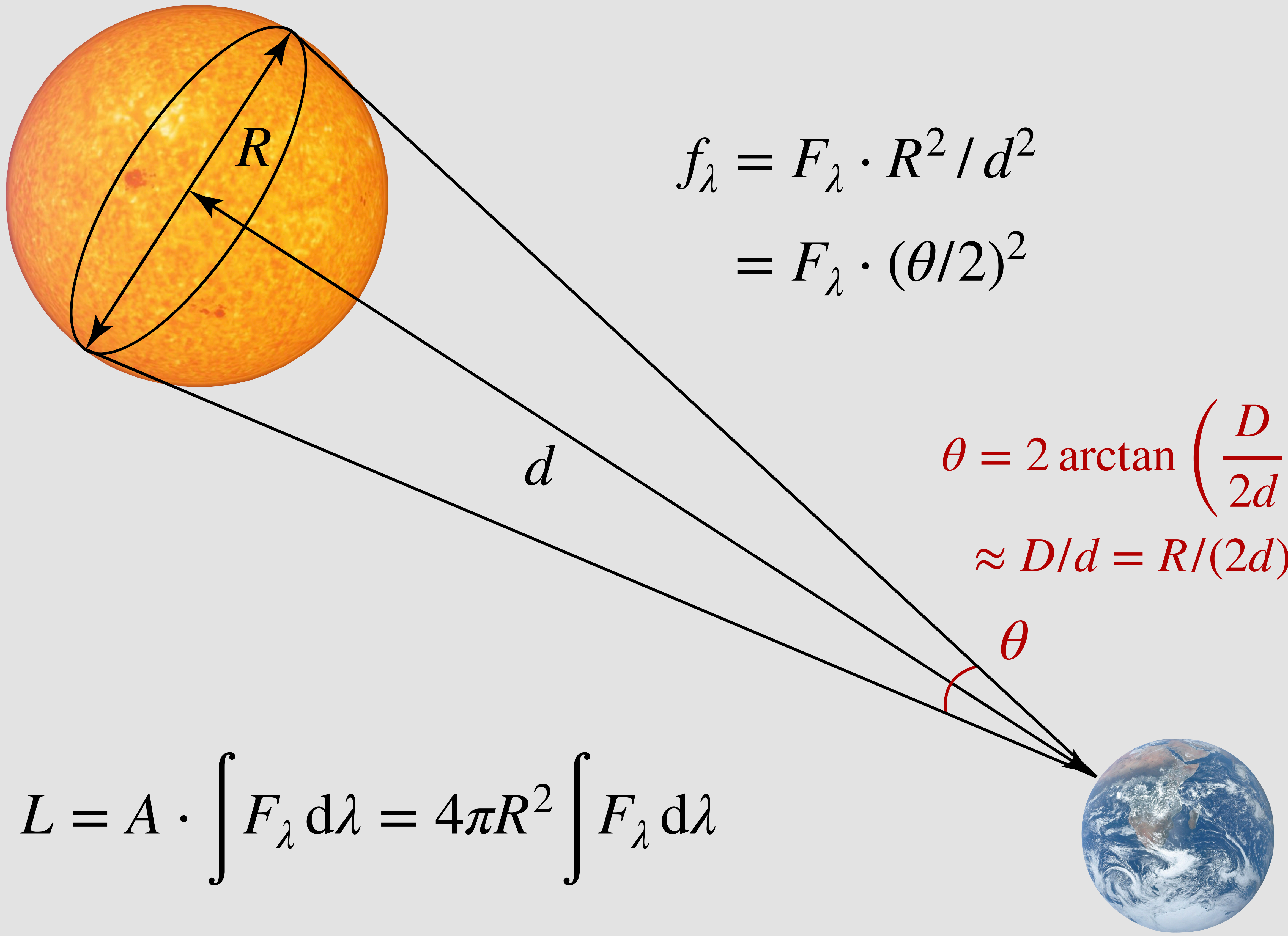
The apparent magnitude is defined relative to a reference

$$m_\lambda = -2.5 \cdot \log_{10}\left(\frac{f_\lambda}{f_\lambda^{\mathrm{ref}}}\right)$$

f_λ → observed flux

f_λ^{ref} → reference flux

at Earth!



$$f_\lambda = F_\lambda \cdot R^2 / d^2$$
$$= F_\lambda \cdot (\theta/2)^2$$

$$\theta = 2\arctan\left(\frac{D}{2d}\right)$$
$$\approx D/d = R/(2d)$$

$$L = A \cdot \int F_\lambda \, \mathrm{d}\lambda = 4\pi R^2 \int F_\lambda \, \mathrm{d}\lambda$$

Bonus: SEDs

Radius, mass, and luminosity

©ESA

Radius R, mass M, and luminosity L from

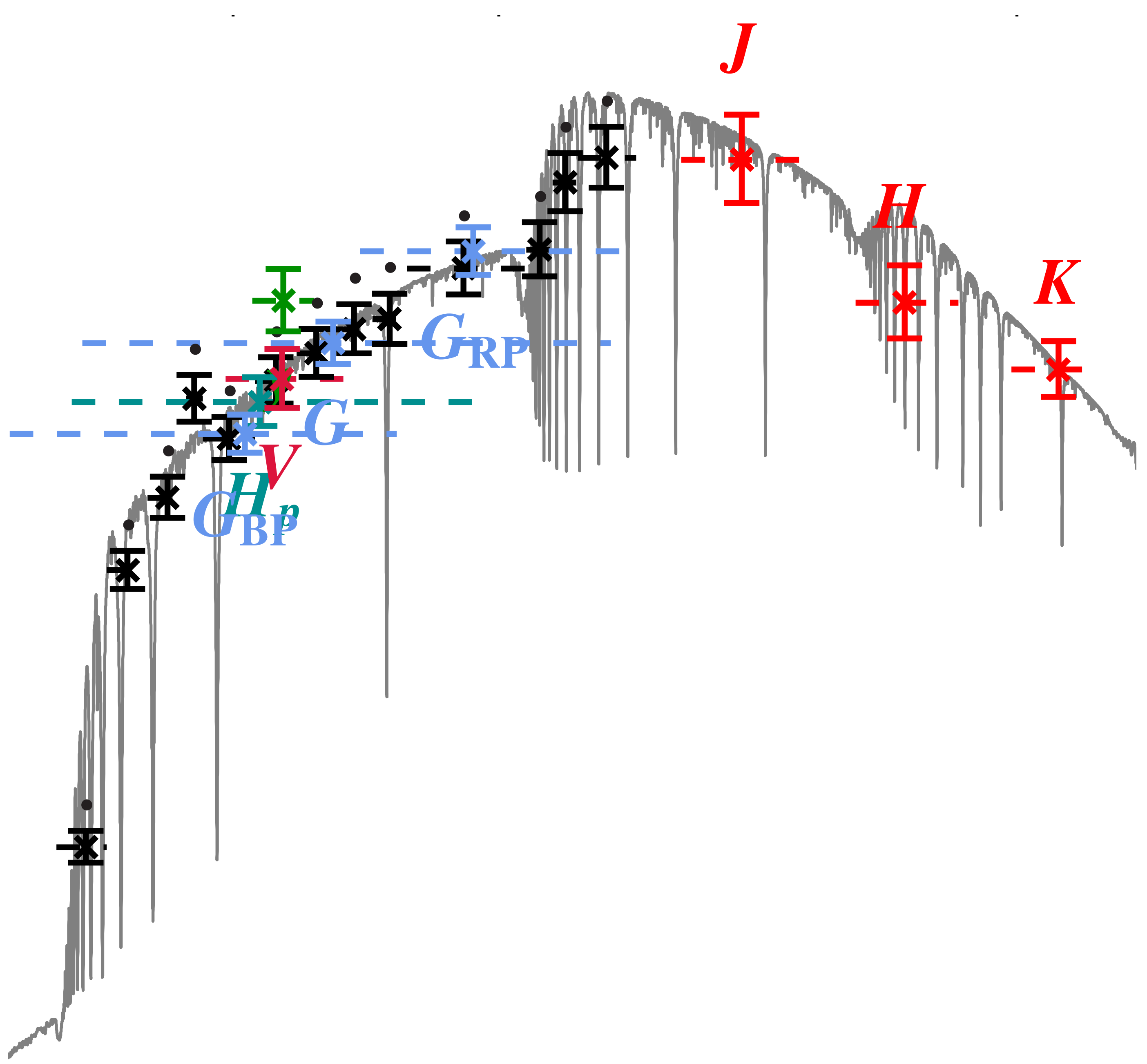
- Spectroscopy
  $\rightarrow$ surface gravity $g = GM/R^2$, T_{eff}
- SED fit using spec. atm. parameters
  $\rightarrow$ angular diameter Θ
- Parallax measurements by *Gaia* EDR3 $\rightarrow$ distance $d = 1/\varpi$

Then, with the gravitational constant G:

$$R = \frac{\Theta}{2\varpi} \qquad M = R^2 \cdot \frac{g}{G} \qquad L = 4\pi\sigma R^2 {T_{\text{eff}}}^4$$

Bonus: SEDs

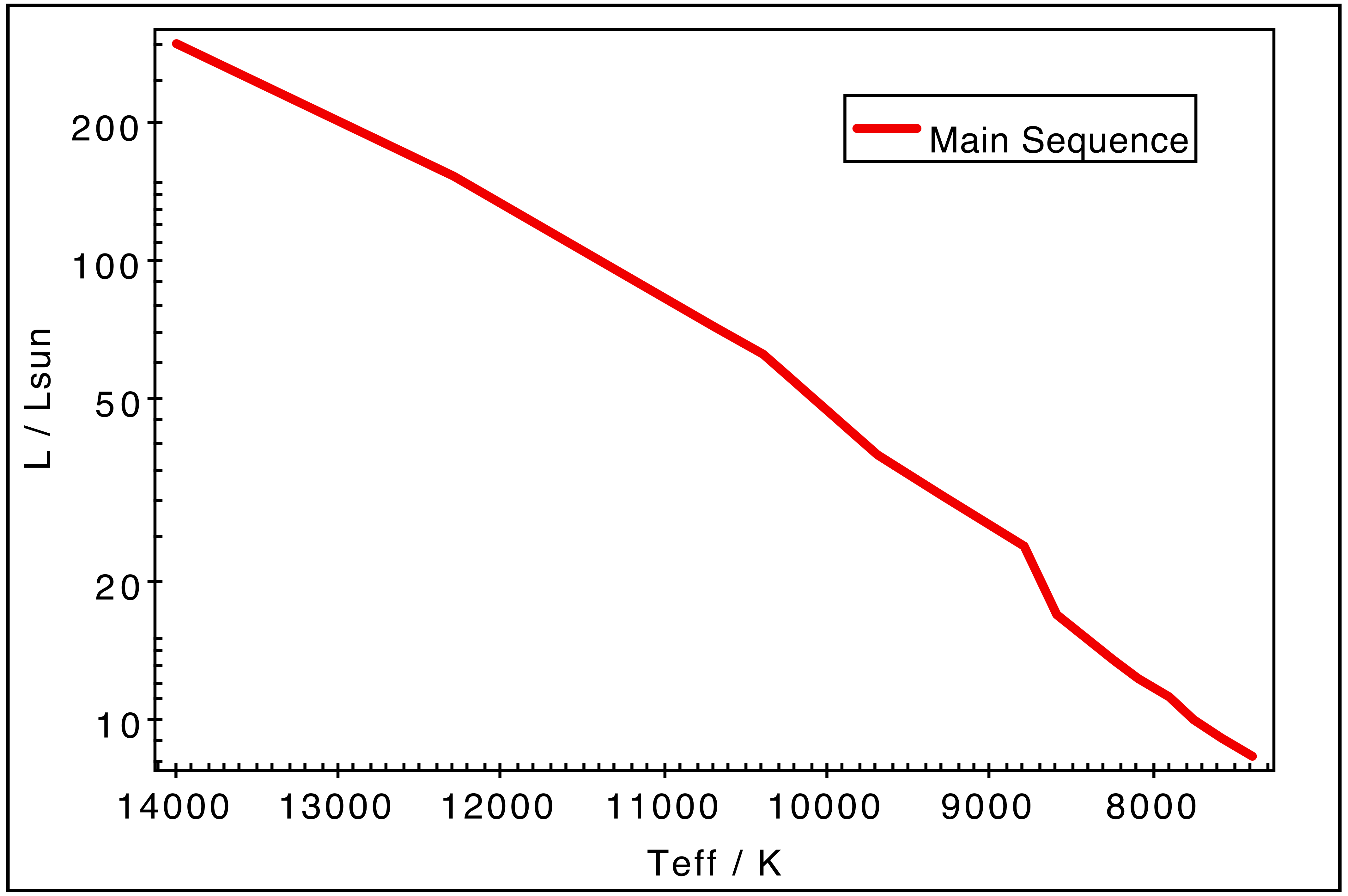
Example output parameters obtained from an SED fit



Object: HD 211211	68% confidence interval
Color excess $E(B-V)$ from SFD (1998)	0.259 ± 0.010 mag
Color excess $E(B-V)$ from S&F (2011)	0.223 ± 0.008 mag
Color excess $E(44-55)$	$0.014^{+0.016}_{-0.014}$ mag
Extinction parameter $R(55)$ (fixed)	3.02
Angular diameter $\log(\Theta\,(\mathrm{rad}))$	$-8.924^{+0.006}_{-0.007}$
Parallax ϖ (*Gaia*, RUWE = 1.21, ZPO = 0 mas)	12.16 ± 0.08 mas
Distance d (*Gaia*, mode)	82.2 ± 0.5 pc
Distance d (*Gaia*, median)	82.2 ± 0.5 pc
Effective temperature T_{eff}	9560^{+290}_{-260} K
Surface gravity $\log(g\,(\mathrm{cm\,s^{-2}}))$	3.88 ± 0.12
Microturbulence ξ (fixed)	$0\,\mathrm{km\,s^{-1}}$
Metallicity z (fixed)	0 dex
Helium abundance $\log(n(\mathrm{He}))$ (fixed)	-1.05
Radius $R = \Theta/(2\varpi)$ (mode)	$2.17 \pm 0.04\,R_\odot$
(median)	$2.17 \pm 0.04\,R_\odot$
Mass $M = gR^2/G$ (mode)	$1.21^{+0.41}_{-0.29}\,M_\odot$
(median)	$1.3^{+0.5}_{-0.4}\,M_\odot$
Luminosity $L = (R/R_\odot)^2(T_{\mathrm{eff}}/T_{\mathrm{eff},\odot})^4$ (mode)	$35^{+5}_{-4}\,L_\odot$
(median)	36^{+5}_{-4}
Gravitational redshift $v_{\mathrm{grav}} = GM/(Rc)$	$0.35^{+0.12}_{-0.09}\,\mathrm{km\,s^{-1}}$
Escape velocity $v_{\mathrm{esc}} = \sqrt{2gR}$	$470^{+80}_{-60}\,\mathrm{km\,s^{-1}}$

Bonus: SEDs

Example output parameters obtained from an SED fit



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Bonus: SEDs

In your terminal

```
isis photometry_auto.sl 6114877567905306496
```

↑

Gaia DR3 ID

This works for any star, independent of spectroscopy.

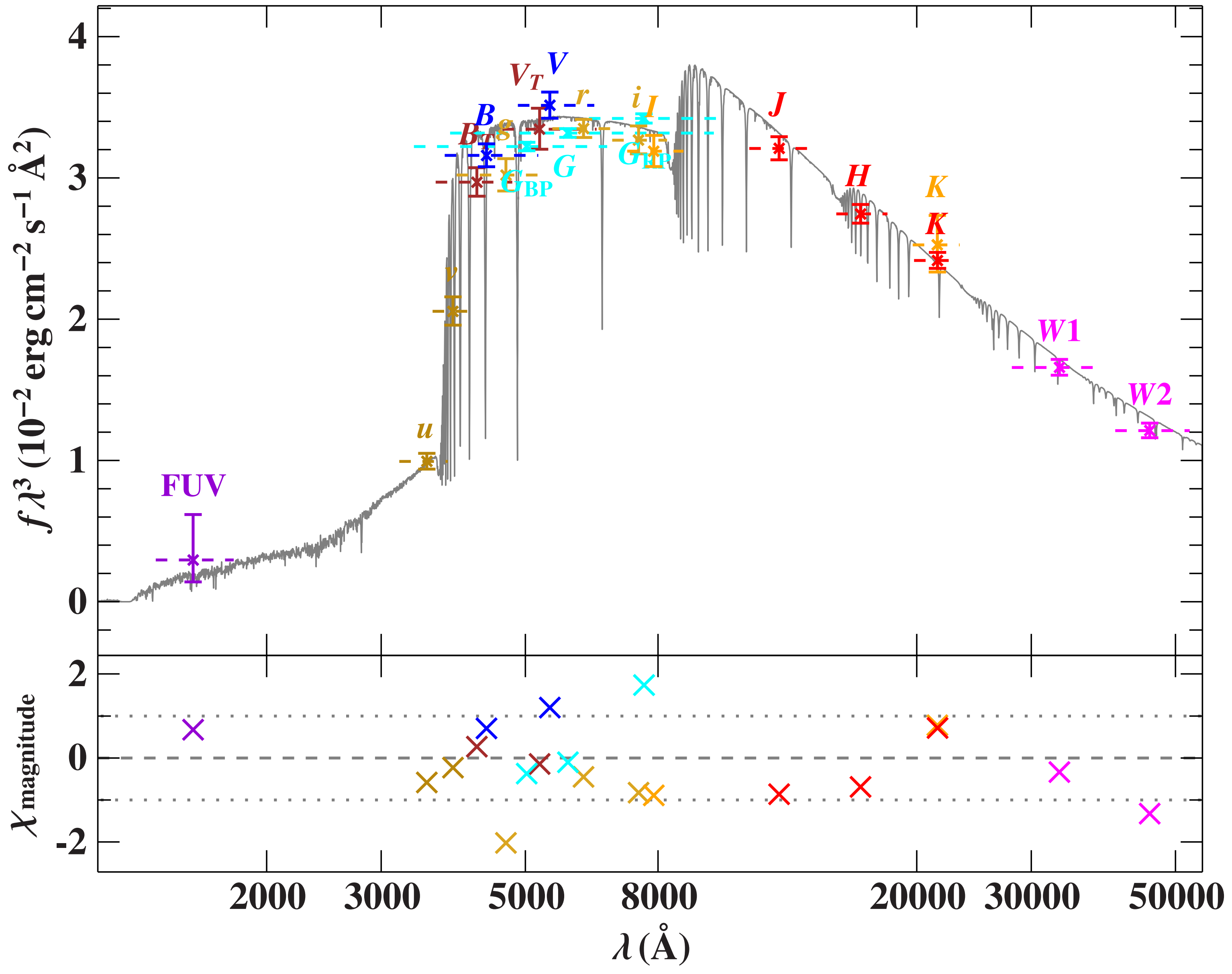
Limited only by model grids:

- MS grid: $2300 \leq T_{\mathrm{eff}} \leq 15000$ K, $2.0 \leq \log g \leq 5.2$
- BHB grid: $9000 \leq T_{\mathrm{eff}} \leq 20000$ K, $3.8 \leq \log g \leq 7.0$

Bonus: SEDs

In your terminal

```
isis photometry_auto.sl 6114877567905306496
```



Object: CD-38 8806	68% confidence interval
Color excess $E(B-V)$ from SFD (1998)	0.0745 ± 0.0016 mag
Color excess $E(B-V)$ from S&F (2011)	0.0641 ± 0.0014 mag
Color excess $E(B-V)$ from Stilism (Capitanio+ 2017)	0.043 ± 0.020 mag
Distance from Stilism and $E(44-55)$	510^{+280}_{-330} pc
Color excess $E(44-55)$	$0.040^{+0.017}_{-0.021}$ mag
Extinction parameter $R(55)$ (fixed)	3.02
Angular diameter $\log(\Theta\,(\mathrm{rad}))$	$-9.943^{+0.010}_{-0.009}$
Parallax ϖ (*Gaia*, RUWE = 1.19)	1.52 ± 0.05 mas
Distance d (*Gaia*)	658^{+22}_{-21} pc
Effective temperature T_{eff}	10600^{+400}_{-500} K
Surface gravity $\log(g\,(\mathrm{cm\,s^{-2}}))$	4.0 ± 0.4
Microturbulence ξ (fixed)	$0\,\mathrm{km\,s^{-1}}$
Metallicity z (fixed)	0 dex
Helium abundance $\log(n(\mathrm{He}))$ (fixed)	-1.05
Radius $R = \Theta/(2\varpi)$ (mode)	$1.66 \pm 0.07\,R_\odot$
(median)	$1.67 \pm 0.07\,R_\odot$
Mass $M = gR^2/G$ (mode)	$0.5^{+1.1}_{-0.4}\,M_\odot$
(median)	$1.0^{+1.2}_{-0.6}\,M_\odot$
Luminosity $L/L_\odot = (R/R_\odot)^2(T_{\mathrm{eff}}/T_{\mathrm{eff},\odot})^4$ (mode)	32 ± 6
(median)	32 ± 6
Gravitational redshift $\upsilon_{\mathrm{grav}} = GM/(Rc)$	$0.18^{+0.39}_{-0.12}\,\mathrm{km\,s^{-1}}$
Generic excess noise δ_{excess}	0.010 mag
Reduced χ^2 at the best fit	1.00