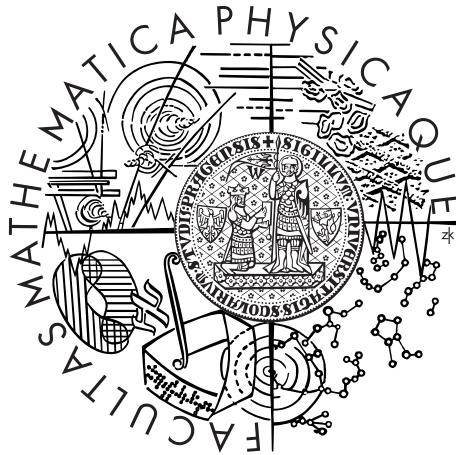


Charles University in Prague
Faculty of Mathematics and Physics

DOCTORAL THESIS



Brankica Šurlan

Radiation in stellar winds

Resonance line formation in inhomogeneous hot star winds

Astronomical Institute of the Academy of Sciences of the Czech Republic

Supervisor of the doctoral thesis: RNDr. Jiří Kubát, CSc.

Study programme: Physics, F1

Specialization: Theoretical Physics, Astronomy, and Astrophysics

Prague 2012

The thesis is dedicated to all those
from whom I have learned.

I declare that I carried out this doctoral thesis independently, and only with the cited sources, literature and other professional sources.

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In Prague, 20th of Jun, 2012

Brankica Šurlan

Název práce: Záření v hvězdných větrech. Formování rezonančních čar v nehomogenních větrech horkých hvězd

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Abstrakt: Zahrnutí třírozměrné (3-D) podstaty vytváření shluků do výpočtů přenosu záření a výzkum, jak různé parametry modelu ovlivňují formování rezonančních čar, je v této práci umožněn vlastním komplexním 3-D Monte Carlo kódem pro řešení rovnice přenosu záření v nehomogenních expandujících hvězdných větrech. Realistické 3-D modely, které popisují husté i zředěné složky větru, jsou použity pro modelování vzniku rezonančních čar v hvězdném větru se shluky hmoty. Je zahrnuto i nemonotónní rychlostní pole. Je ukázáno, že 3-D nehomogenity hustoty a rychlosti ve větru mají velmi silný vliv na formování rezonančních čar. Modely ukazují, že opacita v čarách je nižší pro vyšší separaci shluků a menší gradient rychlosti ve shlucích. Ukazují také, že pro získání empiricky správné rychlosti ztráty hmoty z profilů UV rezonančních čar je třeba vzít v úvahu vytváření shluků ve větru a i jejich 3-D podstatu.

Klíčová slova: přenos záření, horké hvězdy, hvězdný vítr, shlukování, ztráta hmoty.

Title: Radiation in stellar winds. Resonance line formation in inhomogeneous hot star winds

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Abstract: To incorporate the three-dimensional (3-D) nature of stellar wind clumping into radiative transfer calculations, in this thesis a newly developed full 3-D Monte Carlo radiative transfer code for inhomogeneous expanding stellar winds is presented and used to investigate how different model parameters influence resonance line formation. Realistic 3-D models that describe the dense as well as the rarefied wind components are used to model the formation of resonance lines in a clumped stellar wind. Non-monotonic velocity fields are accounted for as well. It is shown that the 3-D density and velocity wind inhomogeneities have very strong impact on the resonance line formation. The models show that the line opacity is lower for a larger clump separation and shallower velocity gradients within the clumps. They also demonstrate that to obtain empirically correct mass-loss rates from UV resonance lines, wind clumping and its 3-D nature must be taken into account.

Keywords: radiative transfer, hot stars, stellar wind, clumping, mass-loss.

Acknowledgements

First of all, I would like to thank my thesis supervisor Jiří Kubát who initiated my study of radiative transfer and provided me appropriate guidance and great support during my study. I am grateful for all inspirational discussions and advices which helped me to gain more into the subject.

I am equally grateful to Wolf-Rainer Hamann, whom I also consider as my supervisor, even though he was not an official one. I am thankful to him for many ideas, answered questions, programming tricks, and all what I learned from him. His approach to science has been an exceptional source of inspiration for my work. I feel a great honor to have a possibility to work with him and I believe that our collaboration will continue for a long time.

I would like to express my special thanks to Lidia Oskinova who has always generously shared her knowledge and has always been willing to discuss the scientific problems with me. Many her advices and encouragement helped me to make steps forward. I had a privilege to meet Achim Feldmeier, who introduced me with basics of hydrodynamics of hot star winds. I am deeply thankful to him for his help in writing the common project, which enabled my several visits to Potsdam University.

A lot of thanks go to my colleague Mary Oksala, who showed good will and invested her effort to read my thesis. Her comments and linguistic corrections contribute a lot to the improvement of my thesis.

I also would like to thank to all my colleagues from the Astronomical Institute in Ondřejov for a pleasant working atmosphere. Especially, I am grateful to Michaela Kraus with whom I shared my office, but also many my problems and dilemmas. As an experienced scientist, she helped me with many things. Her warm support meant a lot to me.

Finally, my grateful acknowledgements go to my family and all my friends, without whom all this work would not make sense. Their continuous support has been the largest source of energy for me to persist in the work.

This work has been funded by the scholarship P661/2008-10-I20-Z03 of the Ministry of Education, Youth and Sports of the Czech Republic and supported by the grant 205/08/H005 of the Grant Agency of the Czech Republic. My stay in Germany was supported by the PPP project D3-CZ2/2010-2011 of the Academy of Sciences of the Czech Republic and the German Academic Exchange Service.

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List of Abbreviations

HRD	Hertzsprung-Russell Diagram
ZAMS	Zero-Age-Main-Sequence
SN	Supernovae
IR	Infrared radiation
FIR	Far infrared radiation
UV	Ultraviolet radiation
XUV	Extreme ultraviolet radiation
IUE	International Ultraviolet Explorer
FUSE	Far Ultraviolet Spectroscopy Explorer
1-D	One-dimensional
2-D	Two-dimensional
3-D	Three-dimensional
CAK	Wind model named after Castor, Abbott, and Klein (1975)
LTE	Local thermodynamic equilibrium
non-LTE	Non local thermodynamic equilibrium
LBV	Luminous Blue Variable
S/N	Signal-to-noise
DAC	Discrete Absorption Component
CIR	Corotating Interaction Region
W-R	Wolf-Rayet star
LPV	The line-profile variation
DWEE	Discrete Wind Emission Elements
TVS	Temporal Variance Spectrum analysis
MC	Monte Carlo method
MCRT	Monte Carlo Radiative Transfer

PDF	The probability density function
CDF	The cumulative distribution function
SIE	Sobolev with Exact Integration
SMC	Small Magellanic Cloud
LMC	Large Magellanic Cloud

Chapter 1

Introduction

The main source of information about astrophysical objects is their radiation, which we are able to measure. Collected electromagnetic radiation, in the form of a spectrum, is the richest and most useful source of information about distant objects. To study these objects, it is necessary to understand how radiation is transferred through different media. Hence, the solution of the radiative transfer problem is one of the most important tasks in astrophysics.

This thesis is focus on very luminous, massive, hot stars that can be seen even at large distances. Due to their high luminosity and, consequently, very strong radiation field, almost all hot stars exhibit continuous outflow of material from their surface, called the *stellar wind*. Observed spectra of these objects contains “encoded” information about matter, which the stellar radiation passes through. To decode this information and deduce both stellar and wind properties, it is necessary to compute *synthetic spectra*, i.e. to solve the radiative transfer equation. In the context of hot stars and their winds, the radiative transfer equation should be solved first in static media (through the stellar atmosphere, lower part of the envelope), since the emergent radiation is formed in these regions, and then in moving media (through the stellar wind, upper part of the envelope), which changes the emergent radiation from the star¹. This is not an easy task, because to describe both static and expanding atmospheres accurately, numerous physical processes have to be taken into account. To obtain a consistent solution of the radiative transfer equation, efficient numerical tools are needed. The problem becomes even more complicated if the medium is not smooth, but rather inhomogeneous, as is the case of hot star winds.

¹ For hot stars it is not necessary to consider atmospheric convection.

1.1 Importance of hot star winds research

Hot stars (namely spectral types O and B) are spectacular objects in the Universe. They are very bright with surface temperatures $T \approx 10 - 50$ kK and high luminosities which reach $L \approx 10^5 - 10^6 L_{\odot}$. The most luminous among them are the most massive ones. They are located in the upper left corner of the Hertzsprung-Russell Diagram (HRD) with Zero-Age-Main-Sequence (ZAMS) masses of $M_{\text{ZAMS}} \approx 10 - 50 M_{\odot}$. Hot stars live short ($\sim 10^7$ years on the main sequence) but intensive lives, and usually end as core-collapse supernovae (SN), which are amongst the most powerful events in the Universe.

The most important property of massive, hot stars, which governs their evolution, is the mass-loss. Through stellar winds or during SN explosion, massive, hot stars expel tremendous amount of material and energy. Since hot stars are very luminous, their winds can be extremely strong, producing a very high mass-loss. During their main sequence lifetime, hot stars can lose a substantial fraction ($\sim 50\%$) of their initial mass, consequently, imposing a significant effect on the star's evolution, affecting the chemical composition, surface abundances, and luminosities of these stars. Thus, to correctly understand life and death of massive, hot star, mass-loss must be understood in all stages of their evolution.

Stellar winds enrich the interstellar medium with heavier elements, created by nuclear processes inside the stellar core, and significantly impacting the chemical composition of the interstellar medium, and evolution of the star's host galaxies (see e.g., Abbott, 1982; Leitherer et al., 1992).

The highly supersonic flow of hot star winds provides energy and momentum via interaction with the interstellar medium, and many shocks and mass compressions may develop. As cosmic engines, hot stars trigger and regulate star formation. It is believed that the first stars in the early Universe were massive hot stars and that they were responsible for the reionization of the early Universe (Bresolin et al., 2008).

As current detailed spectral modelling of the observed lines formed in these winds gives very good agreement between theory and observations, it is possible to infer important stellar properties (such as mass, luminosity, and temperature). It is thus possible to use massive, hot O and B stars, visible at large distances, as “standard candles” to infer distances to extragalactic objects (see, e.g., Kudritzki et al., 1992).

Although hot stars are rare and represent only a small fraction of the total stellar population, they are very important for studying the properties, physical conditions, and stage of both our own and other distant galaxies.

1.2 Motivation for the present work

The dominant spectroscopic signatures of hot stellar winds are the characteristic P-Cygni profiles, which are created in the differentially expanding medium. These profiles, but also other emission or absorption line profiles that may appear in spec-

trum of the massive, hot star winds, contain a lot of information about physical properties and kinematics of stellar winds. Therefore, it is important to understand how these line profiles are formed, and how wind matter with particular density and velocity distributions may affect line profile morphology.

Hot star winds are observed in different spectral regions, from the radio and infrared (IR) through the optical and ultraviolet (UV) to X-rays, and possibly γ -rays. All of these spectral regions can be used as diagnostic tools for estimating stellar wind properties. One primary diagnostic tool for studying hot star winds is the analysis of UV P-Cygni resonance line profiles, which are typically extremely broad and easily identified. Since they are formed over the entire wind range, they provide much information about wind conditions.

In observations from the Copernicus satellite observatory (Morton, 1976) and later with the International Ultraviolet Explorer (IUE) satellite (Prinja and Howarth, 1986), many time-dependent phenomena, namely large-scale structure, were observed. Far Ultraviolet Spectroscopy Explorer (FUSE) was the last launched UV satellite observatory with higher resolution and sensitivity in the spectral range between 900 to 1200 Å. Spectra obtained from this satellite provided a wealth of information, not only about large-scale structures, but also about the stochastic small-scale time-density inhomogeneities distributed throughout the wind, known as *clumping*.

The amount of matter expelled from the star by its wind is called the mass-loss. The stellar mass-loss rate can be predicted theoretically, usually assuming the stationary, homogeneous and spherically symmetric wind model (“standard wind model”). To check the validity of theoretical wind models, predicted mass-loss rates are compared with empirical ones inferred from observations. These two approaches give values of mass-loss rates which are not in agreement. The proper determination of mass-loss rates is further complicated by the fact that various mass-loss rate diagnostics provide different values of the mass-loss rate. The standard wind model assumptions provide mass-loss rates which are usually overestimated. It is believed that wind clumping is the main reason for this deviation. Since wind clumping influences almost all mass-loss diagnostics, such as analysis of UV P Cygni resonance profiles, H_α diagnostics, and thermal radio and far infrared (FIR) continuum emission, any mass-loss diagnostics which do not account for wind clumping may underestimate the actual mass-loss rates. Therefore, to obtain empirically correct mass-loss rates, this property must be taken into account.

For correct modelling of the resonance lines, which are a key diagnostic of stellar mass-loss, the stellar wind clumping turned out to be very important. To incorporate clumping into the radiative transfer calculation, three-dimensional (3-D) models are required. Various properties of the clumps may have strong impact on the resonance line formation, and therefore, on the determination of empirical mass-loss rates. Important steps have been made in the past five years to calculate emergent spectra from clumped winds. However, all previous studies have applied simplifications to the wind geometry. The structured stellar winds present essentially a 3-D problem,

and a full description requires a 3-D radiative transfer.

In this thesis, the full problem of resonance line formation with 3-D radiative transfer in the inhomogeneous (clumped) wind is treated for the first time. By synthesizing resonance line profiles using newly developed 3-D Monte Carlo Radiative Transfer (MCRT) code, it was investigated how different properties of clumps may influence the formation of resonance lines (mostly UV lines) and how it can affect mass-loss rate determination.

1.3 Overview

To easily follow and understand the problem to be solved in this thesis, Chapter 2 gives a brief overview of hot star winds, the basic theory of line-driven winds, and spectroscopic wind diagnostics that are used for the determination of wind parameters. In Chapter 3, wind clumping is introduced. Theoretical predictions and observational evidence of wind clumping and how it can influence the mass-loss determination are also discussed. Chapter 4 describes radiative transfer in a stellar wind, as well as methods used to solve radiative transfer equation, while in Chapter 5 a detailed description of the developed 3-D MCRT code for inhomogeneous medium is given. Chapter 6 presents results of the model calculations, namely how inclusion of wind clumping in density and velocity, as well as the effect of a non-void interclump medium, may affect the resonance line formation. Comparison with observation is shown in Chapter 7, with Chapter 8 providing a summary of this work and an outline of further work.

Chapter 2

Hot star winds

Hot stars emit their peak radiation in the UV wavelength region which is not accessible from ground. Only after the first rocket AERObEE was launched, it was possible to obtain stellar spectra in the UV region. In the first UV spectra of early-type supergiants, Morton (1967a) found that the resonance lines (such as N v $\lambda\lambda 1239, 1243$ Å, Si iv $\lambda\lambda 1394, 1403$ Å, and C iv $\lambda\lambda 1548, 1551$ Å) show P Cygni profiles, which are typical for outflows. He also found that the displacement in the profiles corresponds to outflow velocities in the range $1500 - 3000 \text{ km s}^{-1}$. These initial UV observation, combined with IUE observations, concluded that all hot stars with a mass larger than $15 M_{\odot}$ show a high velocity outflow, the so-called *stellar wind*.

Hot stars lose their mass via stellar winds. The *mass-loss rate*, $\dot{M}$, is the mass lost by star per unit of time and is usually measured in units of solar mass per year, $M_{\odot}/\text{yr}$. Morton (1967b) made the first estimate of the mass-loss rates of three OB supergiants (δ , ϵ , and ζ Orionis) from the observed strengths of shifted N v, and Si iv absorption lines. From the particular model adopted, the mass-loss rate was of the order of $10^{-6} M_{\odot}/\text{yr}$, a value that is still valid.

The velocity of the wind grows from the photosphere and reaches its maximum far from the stellar surface. Since no external force acts on the stellar wind there, the velocity of the wind remains constant at large distances from the star. Thus, it is called the *terminal velocity of the wind*, v_{∞} . Luminous hot stars have stellar winds with high terminal velocity of typically about $v_{\infty} \approx 1000 - 3000 \text{ km s}^{-1}$, i.e., of the order of 1 % of the speed of light, which is significantly higher than the local speed of sound (in the atmospheres of hot stars it is $\approx 10 - 30 \text{ km s}^{-1}$). Hence, the hot star wind is usually described as a supersonic continuous outflow of particles ejected from the upper atmosphere of a star into the interstellar medium.

Let us now make a comparison with the Sun. Since the Sun belongs to a different

temperature class of stars, it exhibits a different type of stellar wind. The solar wind consists of a hot ($T \sim 2 \times 10^6$ K) coronal wind driven by gas pressure and heat conduction. Via the weak solar wind with speeds between $400 - 700 \text{ km s}^{-1}$, our Sun loses only about $10^{-14} M_{\odot}/\text{yr}$. Comparatively, hot stars lose about five orders of magnitude more mass per year and their winds are much faster.

2.1 Line-driven wind theory

Theoretical study of hot star winds commenced when it was discovered that electromagnetic radiation carries momentum that can be transferred to matter in the process of light scattering. Milne (1924, 1926) and Johnson (1925, 1926) proposed that material can be ejected from the star by the absorption and scattering of the radiation. Milne (1926) showed that Doppler effect is important for the line radiative acceleration, and that the force acting on selected ions due to absorption of photons can exceed gravity and ions then can leave the surface of the star.

Observed resonance lines of Si IV, C IV, and N V put constrain on the temperature in the wind, their presence exclude temperature of the order of 10^7 K, which are necessary for the solar type coronal wind (Parker, 1958). However, such high temperature would completely shift the ionization balance to higher ionization degrees, and the lines of Si IV, C IV and N V would not be observed. Therefore, it was concluded that another mechanism other than gas pressure must play a role in hot star wind driving. To explain the observed mass-loss, Lucy and Solomon (1970) suggested that absorption in the UV resonance lines of abundant ions identified by Morton (1967b) can resist stellar gravity and set up continuous mass flow, i.e., to drive the wind. They suggested that momentum transferred from the photons to the particles of the wind (i.e. the radiation pressure) is responsible for the wind driving (see Lucy and Solomon, 1970). Mass-loss rates obtained were two orders of magnitude lower than the value obtained from observation.

Significant progress in hot star wind theory, Castor et al. (1975, hereafter CAK) made. Solving the hydrodynamic equations, which treated the wind as a single hydrodynamic fluid assuming that the wind is driven by the radiation pressure. Castor et al. calculated the force due to line absorption using the *Sobolev approximation* (Sobolev, 1947). With a parametrized line force, the authors constructed a wind model, the so-called *CAK model*, based on assumptions that the wind is spherically symmetric, stationary, homogeneous, and free of magnetic fields. This model is also known as the *standard wind model*.

The CAK-theory parametrization was improved by Abbott (1982), and the theory continued to develop further (see, e.g., Friend and Abbott, 1986; Pauldrach et al., 1986; Owocki et al., 1988; Kudritzki et al., 1989, and a review by Kudritzki and Puls 2000). For more recent reviews of the physical mechanism of line driving and of line-driven stellar winds see, e.g., Krtićka and Kubát (2007), Puls et al. (2008), or Owocki (2010).

2.1.1 Radiation line-driven mechanism

After photons are emitted from the star's photosphere, they interact with wind material, mostly consisting of hydrogen and helium, heavier elements (e.g., ions of carbon, nitrogen, silicon, iron, nickel, ...), and free electrons. If energy of the photon is equal to the energy required to excite a particle in the wind, then the photon is absorbed. In this process, not only is the photon's energy transformed into excitation energy of the particle, but the momentum is also transferred. After a very short time (typically 10^{-8} s) the particle de-excites to its ground state and photon is re-emitted again. The resulting net-momentum of the ion due to absorption and emission of the photon follows from the vector-sum of the photon and ion momenta. Since photon frequencies remain almost unchanged, it seems that photons are simply scattered to another direction and thus this process is called the *line scattering*. If the lower level of the scattering transition is the ground level of the ion, then the process is called the *resonance line scattering*. Winds driven by the line scattering process, which is a bound-bound type transition, are known as *radiation line-driven winds*.

An additional process necessary to drive the wind are *Coulomb collisions*. Via Coulomb collisions, accelerated ions transfer their momenta to the bulk plasma of the wind (hydrogen and helium), and the total wind is accelerated outwards. Since hydrogen and helium have very few lines in the relevant spectral range in which early-type stars emit most of their radiation, metal lines are responsible for the line driving. Thus, hydrogen and helium are mostly passive components of the wind and inefficient for wind driving. If transfer of momentum between metallic and passive wind component is efficient, the wind is well-coupled and can be treated as one component (Castor et al., 1976). If the transfer of momentum is inefficient, the wind components may decouple (Springmann and Pauldrach, 1992, see also Krtićka and Kubát 2000).

2.1.2 Wind hydrodynamic equations

For the single-fluid treatment of a stellar wind, neglecting viscosity and forces due to electric and magnetic fields, the differential form of the equations of the continuity and momentum is as follows

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \mathbf{g}_{\text{ex}} \quad (2.2)$$

where $\mathbf{v}$ is the velocity field, ρ is the mass density, p is gas pressure. These are functions of position and time. The vector of external acceleration $\mathbf{g}_{\text{ex}}$ includes gravitation and radiation forces. To complete the hydrodynamic system of equations, an energy equation is also required. For hot star winds, an approximate solution of the energy equation is allowed (see Klein and Castor, 1978), assuming that the

electron temperature is approximately constant with radius and slightly less than the photospheric effective temperature, T_{eff} , i.e. isothermal wind.

Considering a spherical star of mass M_* , radius R_* , and bolometric luminosity L_* , with stationary and spherically symmetric wind, the hydrodynamic equations (Eq. 2.1 and Eq. 2.2) reduce to one-dimensional (radial) form as

$$\frac{1}{r^2} \frac{d}{dr} (\rho v_r r^2) = 0 \quad (2.3)$$

and

$$\rho v_r \frac{dv_r}{dr} = -\frac{dp}{dr} + g_{\text{grav}} + g_{\text{rad}} \quad (2.4)$$

where the radial component of the external acceleration is represented as a sum of the total radiative acceleration, g_{rad} (which will be discussed in detailed in Subsection 2.1.3), and the radial component of the gravitational acceleration given by $g_{\text{grav}} = GM_*/r^2$ (M_* is the stellar mass and G is the gravitational constant). The integral in Eq. 2.3 gives the total outward mass flux, i.e. mass-loss rate,

$$\dot{M} \equiv \frac{dM_*}{dt} = 4\pi \rho(r) v_r(r) r^2 = \text{const.} \quad (2.5)$$

The gas pressure p is given by an ideal gas equation of state,

$$p = \frac{\rho k_B T}{\mu m_H} = \rho a^2, \quad (2.6)$$

where k_B is Boltzmann's constant, m_H is the mass of a hydrogen atom, and μ is the mean molecular weight of gas particles.

2.1.3 The radiative force

The radiative acceleration (force per unit mass) due to a radiation field at a point r is given by

$$g_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} d\nu \oint_{\Omega=4\pi} (\kappa_{\nu} I_{\nu}(r, \mathbf{k}) - \eta_{\nu}) \mathbf{k} d\Omega \quad (2.7)$$

where κ_{ν} is a total (absorption plus scattering) mass extinction in cm^{-1} and η_{ν} is emission coefficient. These coefficients are assumed to be isotropic i.e. independent of direction $\mathbf{k}$. I_{ν} is the monochromatic radiative intensity, and the unit vector $\mathbf{k}$ is the direction of the radiation propagation. For isotropic emissivity, the integral over all angles vanishes as well as the second term in Eq. 2.7, and κ_{ν} can be factored out of angular integration. Then, the radiative acceleration is reduced to

$$g_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \kappa_{\nu} d\nu \oint_{\Omega=4\pi} I_{\nu}(r, \mathbf{k}) \mathbf{k} d\Omega = \frac{1}{c} \int_{\nu=0}^{\infty} \kappa_{\nu} \mathcal{F}_{\nu}(r) d\nu \quad (2.8)$$

where $\mathcal{F}_\nu$ is the monochromatic flux,

$$\mathcal{F}_\nu(r) = \oint_{\Omega=4\pi} I_\nu(r, \mathbf{k}) \mathbf{k} d\Omega. \quad (2.9)$$

Total radiative acceleration may be split to two components: the acceleration due to continuum opacity, and the acceleration due to an ensemble of spectral lines, i.e.,

$$g_{\text{rad}}(r) = g_{\text{cont}}(r) + g_{\text{line}}^{\text{tot}}(r). \quad (2.10)$$

Continuum opacity There are several types of continuum processes: atomic free-free and bound-free transitions and scattering on free electrons. Continuum opacity due to free-free and bound-free processes can be neglected in the winds of O and B type stars. Therefore, scattering of free electrons (Thomson scattering) is the main contributor to the continuum opacity. The continuum integral in Eq. 2.7 is proportional to the continuum bolometric flux $\mathcal{F}_* = L_*/4\pi r^2$, where L_* is the bolometric (total) stellar luminosity. Hence, the continuum acceleration can be written as

$$g_{\text{cont}}(r) = \frac{\kappa_{\text{th}}}{c} \int_{\nu=0}^{\infty} d\nu \oint I_\nu^c(r, \mathbf{k}) \mathbf{k} d\Omega = \frac{\kappa_{\text{th}} \mathcal{F}_*}{c} = \kappa_{\text{th}} \frac{L_*}{4\pi r^2 c}, \quad (2.11)$$

where I_ν^c is "direct" continuum intensity from the stellar surface, and c is speed of light. κ_{th} is the Thomson scattering opacity given by $\kappa_{\text{th}} = n_e \sigma_e$, where $\sigma_e = 6.65 \times 10^{-25} \text{ cm}^2$ is the cross-section for Thomson scattering and n_e is the number density of free electrons. The continuum acceleration has the same $1/r^2$ dependence on distance as the gravitational acceleration. Their ratio is conveniently expressed in terms of the so-called Eddington factor

$$\Gamma_e = \frac{g_{\text{cont}}}{g_{\text{grav}}} = \frac{\sigma_e L_*}{4\pi c G M_*}. \quad (2.12)$$

Γ_e is spatially constant, fixed by the luminosity-mass ratio, L_*/M_* , which is independent of radius and hence has a characteristic value for each star. For hot stars it is often within a factor of 2 below unity. The limit $\Gamma_e \rightarrow 1$ is known as the Eddington limit.

Wind driving requires that the radiative force become larger than the gravity force. The continuum radiative force is not sufficient, consequently some additional force is required.

Line opacity The general expression for the radiation acceleration due to an ensemble of lines in a medium moving with the velocity $\mathbf{v}(r)$ is given by

$$g_{\text{line}}^{\text{tot}} = \sum_{\text{lines}} \frac{\kappa_L}{c} \int_{\nu=0}^{\infty} d\nu \oint \phi(\nu - \nu') I_\nu(r, \mathbf{k}) \mathbf{k} d\Omega d\nu, \quad (2.13)$$

where sum is over all lines, and κ_L is line opacity (absorption coefficient) for one single line determined by atomic physics as

$$\kappa_L = \frac{\pi e^2}{m_e c} f_L n_i. \quad (2.14)$$

The first factor, $(\pi e^2)/m_e c = 0.02654 \text{ cm}^2 \text{ s}^{-1}$, e the elementary charge, m_e the electron rest mass, f_L is the oscillator strength which depends on the line, n_i is the number of atoms of ion i that can absorb in the line, and $\phi(\nu)$ is a normalized line profile function.

For gas in motion, the frequency of radiation ν' seen at position r will be Doppler shifted by

$$\nu' = \nu_0 \left[1 + \frac{1}{c} \mathbf{k} \cdot \mathbf{v}(r) \right], \quad (2.15)$$

where ν_0 is the frequency in the comoving frame and $\mathbf{v}(r)$ is flow velocity of the gas, assumed to be non-relativistic.

Let us now introduce the dimensionless frequency displacement (from line centre) in Doppler units as

$$x \equiv \frac{\nu - \nu_0}{\Delta\nu_D}. \quad (2.16)$$

If we assume that there is no turbulence in the wind, the Doppler width is given as

$$\Delta\nu_D = \frac{\nu_0}{c} v_{\text{th}}, \quad (2.17)$$

where

$$v_{\text{th}} = \sqrt{\frac{2k_B T}{m_i}} \quad (2.18)$$

is the ion thermal speed in the gas, and m_i is mass of driving ion i . The total line radiative acceleration can thus be written as

$$g_{\text{line}}^{\text{tot}} = \sum_{\text{line}} \frac{\kappa_L \Delta\nu_D}{c} \int_{x=-\infty}^{\infty} dx \oint_{\Omega=4\pi} \phi \left(x - \frac{\mathbf{k} \cdot \mathbf{v}(r)}{v_{\text{th}}} \right) I_\nu(r, \mathbf{k}) \mathbf{k} d\Omega \quad (2.19)$$

where $dx = d\nu/\Delta\nu_D$, and $\nu - \nu' = x - \mathbf{k} \cdot \mathbf{v}(r)/v_{\text{th}}$. The profile functions are normalized such that

$$\int_{\nu=0}^{\infty} \phi(\nu) d\nu = \int_{-\infty}^{\infty} \phi(x) dx = 1. \quad (2.20)$$

To determine the radiation intensity in Eq. 2.19, we need to solve the radiative transfer equation. The formal solution of radiative transfer equation along the ray, assuming that the source function is isotropic and independent of frequency, can be written as

$$I_\nu(\tau_\nu) = \int_0^{\tau_\nu} S_\nu(\tau'_\nu) e^{-(\tau_\nu - \tau'_\nu)} d\tau'_\nu + I_\nu^{\text{cont}} e^{-\tau_\nu}, \quad (2.21)$$

where I_ν^{cont} is the continuum intensity emerging from the core, and τ_ν the geometry-independent optical depth along the ray defined as

$$\tau_\nu(r) = \int_{R_*}^r \kappa_L \rho(r') \phi\left(x - \frac{\mathbf{k} \cdot \mathbf{v}(r')}{v_{\text{th}}}\right) dr', \quad (2.22)$$

where ρ is density distribution of the medium, and the path length ranges from the stellar surface at $R_* = 0$ to the same radius $r' = r$ in the wind.

For a monotonically increasing function of the velocity law, the optical depth integral in Eq. 2.22 can be simplified by invoking the Sobolev approximation (Sobolev, 1947, 1957). This approximation is valid only if the velocity gradient is sufficiently large. In this approximation the profile function can be approximated with a δ -function that is sharply peaked around the central line frequency. The Sobolev approximation is based on assumption that, due to the Doppler shift caused by the accelerating flow, the geometrical size in which a line can absorb photons with the fixed frequency is so small that flow variables κ_L and ρ change very little. This geometrical size, the ‘‘Sobolev length’’ L_S , is the distance over which the velocity of the flow increases by an amount equal to the thermal velocity of the scattering ions. The frequency of the line in the observer’s frame of reference is changed by a Doppler width over L_S . If the L_S is smaller than a typical length scale H of the flow, then the optical depth integral becomes a function of the local parameters only,

$$L_S \equiv \frac{v_{\text{th}}}{dv/dr} \ll H = \frac{\rho}{d\rho/dr} \sim \frac{v}{dv/dr}. \quad (2.23)$$

In the Sobolev approximation, the local variables κ_L and ρ are slowly varying and can be taken out of the optical depth integral in Eq. 2.22. Since the velocity of the flow is monotonic, it is possible to make a one-to-one change from the spatial into frequency space, such that

$$x' = x - \frac{\mathbf{k} \cdot \mathbf{v}(r')}{v_{\text{th}}} \quad (2.24)$$

and

$$dx' = -\frac{1}{v_{\text{th}}} d[\mathbf{k} \cdot \mathbf{v}(r')] = -\frac{1}{v_{\text{th}}} \mathbf{k} \cdot \nabla [\mathbf{k} \cdot \mathbf{v}(r')] dr'. \quad (2.25)$$

Then, the optical depth at radius r and frequency x given by Eq. 2.22

$$\tau(r) = \tau_L \int_{x'}^\infty \phi(x') dx', \quad (2.26)$$

where

$$\tau_L = \frac{\kappa_L \rho(r) v_{\text{th}}}{\left| \frac{dv_n}{dn} \right|} = \frac{\kappa_L \rho(r) v_{\text{th}}}{\mathbf{k} \cdot \nabla [\mathbf{k} \cdot \mathbf{v}(r)]} = \kappa_L \rho(r) L \quad (2.27)$$

is the *Sobolev optical depth*. The denominator in Eq. 2.27 represents the projected gradient of the projected velocity in a direction $\mathbf{k}$ of the ray. The integral in Eq. 2.26

is proportional to the error function for pure Doppler broadening, i.e., Gaussian profile, $\Phi(x')$.

A common assumption in hot star wind theory is coherent scattering and the complete frequency distribution (Hummer, 1969; Castor, 1970; Rybicki and Hummer, 1992). In this case, the source function, which consists of diffuse and direct “core” terms (see Eq. 2.21), can be written in a simplified form keeping only the direct continuum term from the formal solution for the intensity; the diffuse term is neglected. The continuum intensity I^{cont} can be assumed to be constant over the line and taken out of the integral in Eq. 2.21. The line radiative acceleration from a sum of spectral lines in the Sobolev approximation can be written in analytical form as

$$\begin{aligned} g_{\text{line}}^{\text{tot}}(r) &= \sum_{\text{lines}} \frac{\kappa_L \Delta\nu_D}{c} \oint I^{\text{cont}}(\Omega) \mathbf{k} \, d\Omega \int_{x=-\infty}^{\infty} e^{-\tau_L \Phi(x')} \, d\Phi(x') \\ &= \sum_{\text{lines}} \frac{\kappa_L \nu_0 v_{\text{th}}}{c^2} \oint I^{\text{cont}}(\Omega) \mathbf{k} \, d\Omega \left[\frac{1 - e^{-\tau_L}}{\tau_L} \right]. \end{aligned}$$

2.2 Spectral diagnostics of stellar winds

One of the most important diagnostics of stellar winds is spectral lines formed by different processes. Depending on process, which is responsible for line formation, variety of the line profiles can exhibit in the wind offering different wind diagnostics.

Typical wind lines can appear either as absorption or emission lines, or with a *P-Cygni line profile*². This type of profile consist of both a blue-shifted absorption and a red-shifted emission component. Wind lines are usually very broad and easily identifiable. They can be formed over the entire wind range, even in the deeper part of the wind with lower velocities. The mass density and velocity distribution of the wind, as well as the ionization balance of the wind are responsible for a variety of wind profile morphology. The presence and properties of stellar winds can be derived from spectroscopic observations of wind lines, and measuring the line quantities (e.g. line position, line strength, i.e., equivalent width, or line shape).

Two *global wind parameters* of hot stars, namely v_∞ and $\dot{M}$, are not directly observable, but can be determined from observations. Wind parameters obtained from observations are known as empirical or “observed” parameters, and are usually compared with theoretical (also known as predicted) wind parameters obtained from the radiation line driven wind theory and hydrodynamic models in order to check validity of theoretical wind models. However, derivation of global wind parameters from observation is possible only with the help of stellar photospheric models, which provide photospheric flux, and if the stratification of the *local wind parameters*, i.e., distribution of the mass density and velocity of the wind, is known.

² The P-Cygni line profile is named for the Luminous Blue Variable (LBV) P Cygni (HD 193237) that the line was first observed in.

For the stratification of the local wind parameters, common simplified assumptions about the stationarity, spherical symmetry, and homogeneity of the wind (i.e., “standard wind model”) are usually used. Under these assumptions, the density $\rho(r)$ of the wind follows from the continuity equation (see Eq. 2.5),

$$\rho(r) = \frac{\dot{M}}{4\pi r^2 v(r)}. \quad (2.28)$$

The velocity distribution of the hot star wind is usually approximated by the so-called β -law², given in the form,

$$v(r) = v_\infty \left(1 - \frac{b}{r}\right)^\beta, \quad (2.29)$$

where the constant b is given by

$$b = R_* \left\{ 1 - \left(\frac{v_{\min}}{v_\infty} \right)^{1/\beta} \right\}, \quad (2.30)$$

and $v_{\min} = v(R_*)$ is the velocity at the lower boundary of the wind of the order of the isothermal speed of sound. The parameter β describes the steepness of the velocity law, i.e., how fast the velocity of the wind is approaching v_∞ . In this simple wind model approach, which is based on analytical description of the wind, additional information is needed to calculate a synthetic spectrum, including the radiation field from the photosphere (lower boundary), which can be obtained from a photospheric model, and the electron temperature of the wind, which is usually assumed to be equal or somewhat smaller than the effective temperature T_{eff} of the star, assuming that the wind is in radiative equilibrium (Kudritzki and Puls, 2000). Additionally, the core-halo approximation is assumed, in which the radiation field of the photospheric radiation (underlying core) is influenced by the wind (halo) of lower density. The influence of the wind (halo) on the photosphere (core) is usually neglected. The stellar and wind parameters are determined assuming a smooth transition between the quasi-hydrostatic photosphere and the wind.

To calculate a synthetic spectrum using the standard wind model, $\dot{M}$, v_∞ , and β are treated as fitting parameters determined from observations (i.e. spectrum). They are varied as long as satisfactory agreement with observation is obtained. Even if these “observed” wind parameters are derived from the observations, they are actually the result of theoretical modelling.

Spectroscopic diagnostics of hot stars and their winds were developed during the 1980s and 1990s (see reviews by Kudritzki and Hummer, 1990; Kudritzki and Puls, 2000). More sophisticated and precise diagnostic methods use the so-called “unified” non-LTE model atmosphere, in which stellar and wind parameters are derived simul-

² Note that this type of dependence was already derived by Milne (1926) and Chandrasekhar (1934) with $\beta = 1/2$. Today’s estimates of this value are a bit higher.

taneously and consistently. Current state-of-the-art wind models are based on the standard wind model assumption, non-LTE, and the radiative equilibrium approximation. Density and velocity stratification of the wind is derived from hydrodynamic calculations, or calculated from Eq. 2.28, and Eq. 2.29 assuming a smooth transition between photosphere and wind. For more about these unified models we refer to Gabler et al. (1989) and references therein.

To determine stellar wind parameters, the effective temperature, T_{eff} , and surface gravity, $\log g$, as well as the physical properties of the star such as mass, M_* , photospheric radius, R_* , luminosity, L_* , chemical composition, etc., must be known. T_{eff} and $\log g$ are prerequisites for abundance analysis, and are directly related to the physical properties of the star.

Depending on which of the global wind parameters we want to determine, a different part of the spectrum (i.e., different wind diagnostics) should to be used. Reliability of the estimated global wind parameters may vary depending on the underlying model used, but also depending on which part of the spectrum is used. The most commonly used diagnostics for wind parameters determination are: free-free continuum emission at radio wavelengths, UV resonance-lines, and H_α (or IR) lines. These diagnostics provide properties for different regions of the wind.

Physical properties of stars cannot be directly inferred from observation. Therefore, to deduce the atmospheric parameters (i.e. stellar and wind parameters), detailed quantitative modelling has to be performed.

2.2.1 Photospheric parameters determination

The photosphere is the region of the stellar atmosphere where the optical continuum is formed², and is below the sonic point (the point where the flow speed is equal to the sonic speed). In the 1930s, the first model photospheric calculations were performed based on very simple assumptions, including radiative and hydrodynamic equilibrium, plane-parallel stratification, LTE, parametrised opacity, and only hydrogen and helium as atmospheric constituents.

Determination of photospheric parameters begins with determination of effective temperature T_{eff} and surface gravitational acceleration g . One of the widely used methods to derived T_{eff} and $\log g$ is a comparison between photometric observations and models. Since changes of stellar flux with wavelength are characteristic of T_{eff} , $\log g$, and chemical composition of the star, a measurement of stellar flux at several wavelength intervals can be used to determine these parameters. A common procedure for T_{eff} determination is to use adopted calibrations of effective temperature and bolometric correction (for particular spectral type) to determine L_* and R_* , and then from de-reddened photometry determine T_{eff} , distance, and spectral type. Additionally, a comparison with evolutionary tracks in the HRD can provide M_* . A

² This is roughly valid in the case of O and B stars. However, in most Wolf-Rayet (WR) stars the entire continuum is formed in the wind.

more accurate T_{eff} determination is to use sophisticated model atmospheres.

In the late 1960s, Auer and Mihalas (1969) developed a new generation of atmospheric models using efficient computational techniques (complete linearisation) for non-LTE model atmosphere calculations (see e.g. Mihalas, 1972; Mihalas et al., 1975; Kudritzki, 1976; Hubeny, 1988). This generation of models were improved to include metal opacities and line blanketing (see e.g. Werner, 1988, 1989; Anderson, 1989; Hubeny and Lanz, 1995).

Additionally, an effect called metal line blanketing (effect caused by the presence of numerous metal lines in the UV region) represents one of the major difficulties in modelling the atmospheres of these stars. Reliable determination of stellar and wind parameters of hot stars can be achieved only with fully blanketed non-LTE models (also known as unified models) for an entire stellar atmosphere, including the photosphere (quasi-static) and the supersonic wind (see review by Hubeny et al., 2003).

2.2.2 Terminal velocity determination

The strongest saturated UV P Cygni lines can be used to determine the terminal velocity by measuring the frequency position of the blue edge of the profile (see, e.g., Howarth and Prinja, 1989; Prinja et al., 1990; Lamers et al., 1995; Howarth et al., 1997). Determination of v_{∞} , although in principle easy, is not straightforward. Ideally, the blue edge of the P Cygni profile should be sharp. However, often the blue edges of the absorption trough of strong lines are not well defined, but approach the continuum more gradually, making the measurement of the position of the blue edge quite inaccurate. This “softening” is interpreted as an indicator for existence of some extra velocity field, caused by additional small-scale or large-scale motions. The small-scale motions are usually referred to as “microturbulence” (see, e.g., Hamann, 1981a,b), and they act as an extra broadening. Thus, the value of v_{∞} determined from the “softened” blue edge of the line profile is overestimated.

An alternative explanation for the generation of “black troughs” (an extended region in the absorption part of saturated profiles with zero flux), suggested by Lucy (1982, 1983), is enhanced back-scattering in multiple non-monotonic velocity field. Detailed UV line formation calculations confirmed this scenario (see, e.g., Puls et al., 1993).

The main UV diagnostics lines for stars with strong winds and saturated P-Cygni profiles are N V, Si IV, C IV, N IV, O VI, and C III. These profiles can appear below 1000 Å, but they are contaminated with interstellar molecular and atomic hydrogen absorption and are not suitable for the determination of v_{∞} .

When UV spectra are not available, other diagnostics must be used, such as Balmer lines (H_{α} , H_{β} , H_{γ} , H_{δ}) and He I in the optical region. These lines can appear either as pure emission, pure absorption, or P-Cygni profiles. For pure emission lines, the line width is related to the wind velocities. To determine v_{∞} , these lines are fit with a synthetic spectrum computed from atmospheric models with different terminal

velocities. The best fit with corresponding v_∞ provides an indirect measurement of the terminal velocity.

When a spectroscopic diagnostic is not available (e.g., no UV observations exits), the terminal velocity can be approximately estimated from the relation between terminal velocity and escape velocity via the CAK force multiplier parameter α (Friend and Abbott, 1986; Kudritzki et al., 1989). Kudritzki and Puls (2000) provided a more general relation between v_∞ and v_{esc} with additional corrections. Typically, terminal velocity of stars hotter than $T_{\text{bs}} = 25 \text{ kK}$ (T_{bs} denotes the temperature of the bistability jump), is estimated to be $v_\infty/v_{\text{esc}} \sim 3$. For stars cooler than this temperature, $v_\infty/v_{\text{esc}} \sim 1.5$. Below this temperature, stars show systematically larger mass-loss rates and lower terminal wind velocities than objects above this temperature. The ratio v_∞/v_{esc} has a jump (usually refereed to as the “*bistability jump*”, see, e.g., Pauldrach and Puls, 1990; Lamers et al., 1995), and is due to a change in the ionization equilibrium of iron, the most important element for line-driving in the inner wind. At temperatures higher than T_{bs} , the radiation pressure is due to high-ionization lines which are mostly optically thick, and for temperatures lower, the radiation pressure is due to a large number of low-ionization lines which are more optically thin. Near T_{bs} , winds can exist in two states, either with high mass-loss rates and low v_∞ or low mass-loss rates and high v_∞ . Recent studies by Crowther et al. (2006) and Markova and Puls (2008) show that this discontinuity is not actually a real jump, but more a gradual decrease.

The wind velocity law (assuming the approximation of the β -law) is best determined from strong or saturated UV P Cygni resonance lines. By fitting the synthetic spectrum to observations, the line strength (and consequently the line opacity) and shape of the velocity law (v_∞ and β) can be determined.

In stars with denser winds, the parameter β can be determined with high precision from the shape of the emission peak, at the H_α line. For OB-type stars, β is found to be in the range $\beta = 0.7 - 1.5$, except for supergiants with somewhat higher values of β (see, e.g., Howarth and Prinja, 1989). For stars with thin winds (H_α in absorption), the determination of β from optical spectroscopy is impossible and a value of $\beta = 1$ is usually adopted. The parameter β of the velocity law can be also determined from other optical lines. Since a slower velocity law produces stronger and narrower central emission peaks, the height and width of the central part of the line profile can be used to constrain β , but with certain limitations as H_α observations are usually contaminated by telluric lines or nebular emission and efficient techniques are needed to properly remove these features.

2.2.3 Mass-loss rates determination

There are two main classes of mass-loss rates diagnostics, the density (ρ) and density squared (ρ^2) diagnostics. For instance, line absorption scales linearly with ρ . The corresponding opacity is proportional to the number density of absorbing ions. P Cygni resonance lines, mainly in the UV region, are also a ρ diagnostic. These

lines are sensitive to the wind density, which is directly related to the mass-loss rate, i.e. $\rho \propto \dot{M}/(r^2 v_\infty)$. Emission lines (such as H_α in the optical region) are typically formed in recombination cascades, and they are ρ^2 -dependent, with the opacity proportional to the product of the ion and electron number densities. Wings of strong emission lines, created from the frequency redistribution of line photons by electron-scattering in moving atmosphere, scale linearly with density and have opacity that is proportional to electron density. Free-free emission processes also scale with ρ^2 .

The most useful diagnostics for mass-loss rate determination ordered by increasing sensitivity are: free-free continuum radio emission; H_α line emission; and UV resonance-line absorption.

Free-free radio emission Excess flux due to free-free emission (bremsstrahlung) arises when free electrons move in the field of an atom but without being bound. During this interaction the electron is accelerated, and a photon is absorbed or emitted. From many such interactions, photons are emitted with a continuum spectrum. The excess flux becomes significant at longer wavelengths due to the λ^2 dependence of the cross-section.

Continuum flux measurements at long wavelength regions offer a tool for mass-loss determination depending only on observable stellar properties. Assuming a spherically symmetric wind and that the free-free emission may be treated using the LTE approximation (which is reasonable, since the free-free emission is a collisional process), measurement of the radio flux, wind terminal velocity, and the stellar distance is sufficient for this determination using expressions of Panagia and Felli (1975) or Wright and Barlow (1975). Therefore, measurements of the mass-loss rates in the radio region offer an excellent mass-loss rate determination tool, which does not depend on inferred quantities, as is case for other diagnostics. The velocity law can also be determined. However, to detect intrinsically weak flux from distant objects, high spatial resolution and instrumental sensitivity are required.

Longer wavelengths sample the outer part of the wind. The processes responsible for continuum radiation at long wavelengths are $\propto \rho^2$, are sensitive to the density contrast, and can thus be used as detection for the structure of the stellar winds i.e. to make an empirical scan for clumps, shocks or anisotropy throughout the entire wind.

H_α diagnostics An alternative mass-loss rate determination is to use the H_α line (transition between levels with quantum numbers 2 and 3 of the hydrogen atom) which is an indicator of stellar mass-loss. The ionization correction for H_α is much simpler, and the hydrogen abundance is usually much less uncertain. Hydrogen is completely ionized, and thus the influence of ionization and abundance becomes almost negligible.

The H_α diagnostics are based on emission recombination processes and have a ρ^2

dependence. High resolution of spectroscopic observations obtained from the ground-based observatories can determine the shape of these lines precisely. Even very weak, high velocity emission wings can be used for mass-loss rate determination.

Quantitative analysis of H_α lines is more complicated compared to UV P-Cygni lines as their theoretical prediction requires detailed non-LTE calculations of the statistical equilibrium to determine the relevant level populations. Klein and Castor (1978) were the first to compute H_α (and He II) line profiles, compare with observations, and derive mass-loss rates. This work showed that observed H_α emission strength (for stars with known mass and luminosity) can be used for mass-loss rate determination. Later mass-loss rates were updated using the same method (Leitherer, 1988; Drew, 1990; Lamers and Leitherer, 1993). Puls et al. (1996) presented a fast approximate method to determine mass-loss rates of O-stars from H_α line profiles.

UV P Cygni resonance lines Hot star wind lines in the UV spectral regions, typically show characteristic P Cygni profiles which can be saturated (e.g., C IV, N V in early O-type stars) or unsaturated (e.g., Si IV, P V). The shape of these lines can provide information about the wind velocity law, especially from saturated line profiles, as explained in Section 2.2.2. However, the density distribution of the wind cannot be spatially resolved at UV wavelengths. Fortunately, the strength of the absorption and emission components of the P Cygni profiles can provide information about the total number of ions present in the wind that are responsible for the line scattering. Therefore, mass-loss rates can be determined by calculating the integrated line strength.

Unsaturated UV P Cygni profiles from metal resonance lines are commonly used for mass-loss determination in hot stars. Estimates of the mass-loss rates by UV spectroscopy typically uses the Sobolev with Exact Integration (SIE) method (see, e.g., Lamers et al., 1987; Haser et al., 1998), first suggested by Hamann (1981a). In this method, for the mass-loss rates determination, additional information is needed about the ionization structure of the wind, abundance, and the form of the velocity law. The strength of UV P Cygni profiles is determined by the radial optical depth of the wind, $\tau_{\text{rad}} \propto \dot{M} q_i A_e$, where A_e is the abundance of an element e and q_i is its ionization fraction. To estimate the mass-loss rate directly, the abundance of the ion i is required. Usually q_i is not known and its estimate for any resonance line is possible only with detailed non-LTE modelling. Further, X-rays, blanketing or blocking effects may lead to incorrect predictions of q_i (see Pauldrach et al., 1994). Only for the dominant ion can q_i be approximated to be ~ 1 . Unfortunately, the lines of most abundant ions (such as C, N, O) are saturated and are not suitable for reliable determinations of mass-loss rates. Consequently, only UV wind lines of the ions of a trace elements with small ionization fraction ($q_i \lesssim 10^{-3}$) can be used to measure the product $\dot{M} \langle q_i \rangle$ (see, e.g., Lamers et al., 1999), where $\langle q_i \rangle$ is a spatial average of the ion fraction. Some attempts in mass-loss rates determination

have been done by e.g., Howarth and Prinja (1989), who inferred the product $\dot{M} \langle q_i \rangle$ from unsaturated profiles of large O-star samples. $\langle q_i \rangle$ was estimated by comparing the value $\dot{M} \langle q_i \rangle$ obtained from UV resonance lines with $\dot{M}$ determined from radio continuum measurements insensitive to ionization. UV P Cygni resonance lines, while the most sensitive indicators of mass-loss rates and the best qualitative tracers of the gas in stellar winds, are not the best estimators of the mass-loss rates. Thus mass-loss rate determination should better rely on measurements from free-free radio or H_α emission.

Chapter 3

Stellar Wind Clumping

The winds of hot stars are usually considered to be stationary, smooth, and spherically symmetric with a monotonic velocity field. These simplifications are generally suitable for describing overall wind properties. However, to explain some observational characteristics of hot star winds, the standard assumption about steady and homogeneous wind is inadequate and needs to be relaxed. For instance, some observational indications, including non-thermal radio emission (Abbott et al., 1981, 1984b), enhanced thermal infrared emission (Abbott et al., 1984a), UV lines with variable absorption features, known as "Discrete Absorption Components" (DACs) (Lamers et al., 1982), black absorption troughs (Lucy, 1982), and soft X-ray emission (Harnden et al., 1979; Seward et al., 1979), suggest that hot star winds are actually inhomogeneous and variable on a range of small and large spatial and temporal scales (Hamann et al., 2008). One promising theoretical explanation for appearance of many structures in hot star winds is a strong instability in the line-driving mechanism itself.

Existence of large-scale wind structure can be inferred directly from observed time variability even in low signal-to-noise (S/N) spectra. These structures can be caused by a variety of phenomena in the photosphere and wind of hot stars, e.g. convection (Cantiello et al., 2009), pulsation (e.g. Cranmer et al., 1996), rotation (e.g. Cranmer and Owocki, 1996), wind-wind collision (e.g. Pittard, 2009) or magnetic fields (e.g. ud-Doula and Owocki, 2002). Observational evidence of large-scale wind structure is variable absorption features, known as DACs, observed to propagate bluewards from the center to the blue edge of unsaturated P-Cygni line profiles of nearly all O-type stars (Abbott, 1985; Prinja and Howarth, 1986; Hamann et al., 2001). One possible explanation for DACs is corotating interaction regions (CIRs). A model for CIRs was first proposed by Mullan (1984), who suggested that DACs are formed in

rotating stars when high-density, low-speed wind streams collide with low-density, high-speed streams (see, e.g., Cranmer and Owocki, 1996).

In addition to observed signatures of large-scale structures in the wind, many observational phenomena suggest that the winds consist also of a widespread large number of stochastic structures on various (mainly small) scales, so-called "*clumps*". This section is dedicated to these small-scale wind inhomogeneities, i.e. *wind clumping*, which is theoretically predicted and has been indirectly observed in almost all hot star winds.

3.1 Theoretical predictions of wind clumping

As described in Chapter 2, hot star winds are understood to be driven by radiative force which arises due to continuous and line opacities. Lucy and Solomon (1970) have previously pointed out that the radiation driving force is intrinsically unstable. To better understand radiation driven instabilities, let us first start with a basic description of the *linear theory of radiation driven instabilities* following the review of Rybicki (1987).

In order to investigate the (linear) stability of physical quantities, such as density, velocity or the radiative acceleration, these quantities are expressed as a sum of an unperturbed part and an infinitesimal perturbed part. Substitution of these variables into the hydrodynamical equations yield the perturbation equations in which the second and higher order terms are neglected. To find a solution of the set of these equations, it is usually assumed that typical scale lengths of perturbations are smaller compared to a typical scale length H of the mean flow (unperturbed solution), i.e. $kH \gg 1$, where k is a wave-number of perturbation. The perturbation in the radiative acceleration is usually expressed in terms of perturbations in other hydrodynamical variables, namely opacity and radiative flux. The perturbation in opacity is caused by perturbation in thermodynamic variables (density or temperature perturbations), and in the case of spectral lines, those caused by velocity perturbations. Since the perturbation in the radiative flux depends non-locally on the state of the medium (heating, long distance interaction by radiation or non-LTE effects) this kind of perturbations is more complex. Radiation driven instabilities can be classified based on the nature of the functional dependence of the opacity and radiative flux perturbations (see Martens, 1985).

One of the strongest instabilities in hot star winds is due to Doppler shifting of spectral lines in supersonic winds. This *line-shape instability* has a higher growth rate than other instabilities, and therefore dominates the growth of perturbations. Milne (1926) has previously pointed out that radiation force due to absorption in spectral lines and transferred momentum causes some elements of the medium to move faster (since they feel a stronger radiation field) than neighbouring elements. These different velocities lead to strong instability. Lucy and Solomon (1970) first realised that the line-shape instability can also appear in hot star winds, and suggested

that this instability can grow into shocks and heat the gas.

Hearn (1972, 1973) studied analytically the stability of the medium. He analysed the drift instability (Rayleigh-Taylor type instability, an instability of an interface between two fluids of different densities, which occurs when the lighter fluid is pushing the heavier fluid) of vertically propagating sound waves in atmospheres of early-type stars, and showed that isothermal sound waves can be non-linearly amplified by the continuum radiation force.

Nelson and Hearn (1978) have demonstrated the existence of a Rayleigh-Taylor type instability that results from the increase of the radiative force. They suggested that this instability would grow and break up in small horizontal scale lengths, damping sound waves. However, Martens (1979) showed that sound waves are not damped, but that they rapidly grow in the expanding atmospheres of hot stars and can be strongly amplified, causing density and velocity perturbations or a wave to grow. While the work of Nelson and Hearn (1978) concentrated only on a Rayleigh-Taylor instability in the upper photospheric region at the base of the wind, MacGregor et al. (1979) investigated instability of propagating line driven sound waves in the supersonic regions of O and B star winds. They used detailed calculations of the total radiative acceleration produced by an ensemble of lines to investigate the growth rates of small-amplitude sound waves in the wind of the O-type star ζ Pup.

Carlberg (1980) examined the stability of a radiation-driven stellar wind by applying a first order stability analysis to a physically realistic wind model. He found three types of instabilities. Both the line-shape and the radiation-driven sound wave instabilities were shown to propagate purely vertically. A new “gradient” instability was shown to propagate in both vertical and horizontal directions. He concluded that the radiation-driven sound wave and gradient instabilities suggest that the flow would tend to break up into clumps.

In all of the above mentioned treatments of stability analyses, perturbations in velocity are assumed to occur over a sufficiently small scale ($\lambda \ll L_S$, where L_S is the Sobolev length; see Eq. 2.23) that they remain optically thin and do not affect the radiation field. The perturbation in line optical depth can be ignored and the perturbed force is then computed only from the Doppler shift of the line profile. This treatment is known as the *Optically Thin Approximation*, which implies that short wavelength, optically thin perturbations are unstable.

Abbott (1980) investigated wave propagation in line driven winds assuming that the scale of the perturbations was sufficiently large that they could be treated by the Sobolev approximation. In Sobolev line-transfer, the force is proportional to the velocity gradient. In this case the velocity and perturbed force are out of phase by $\pi/2$, and so no net work is done on the perturbation. Consequently, long wavelength perturbations are stable. In contrast to works of MacGregor et al. (1979) and Carlberg (1980) that suggested the perturbations develop and grow, the main conclusion of Abbott’s work is that perturbations in driving force do not grow, cause presence of waves, and that the flow stays stable.

Owocki and Rybicki (1984) and Lucy (1984) demonstrated that these two contradictory results (stable or unstable flow) are due to different assumptions of the length scale of the perturbations. Therefore, Owocki and Rybicki developed a method that could treat the full range of possible perturbation scales, first assuming only pure absorption with no scattering (the effect of scattering later also was included; see Owocki and Rybicki, 1985). They derived the so-called “bridging law” in which they introduced the scale length λ_b , a “bridging length” found to be of the order of L_S . For $k\lambda_b \gg 1$, representing short wave-length, optically thin perturbations, they were able to recover the line-deshadowing instability. In the opposite limit, $k\lambda_b \ll 1$, representing long-wavelength, stable Abbott-waves are recovered. In this approach the wind is usually very unstable to disturbances on the scale less than L_S (the Sobolev length).

These findings suggest that perturbations have large growth rates, and any finite perturbation should quickly become nonlinear, leading to extensive small-scale structure and variability.

Since initially small perturbations grow into large amplitude velocity waves, nonlinear wind structures arise and develop shocks (Castor, 1987; Owocki et al., 1988). The nonlinear evolution of the instability by direct numerical simulations were performed by several authors, in one-dimensional (1-D) geometry (Castor, 1991; Feldmeier, 1995; Runacres and Owocki, 2002) and pseudo two-dimensional (2-D) geometry (Dessart and Owocki, 2003, 2005).

To conclude, the generally accepted origin of clumping in hot star winds is the radiative-acoustic instability, supported by several numerical simulations (see the review by Sundqvist et al., 2012b). An alternative explanation using adiabatic fluctuations (Chiueh, 1997) did not attract the attention of researchers. However, recent interesting idea of the influence of subphotospheric convection (Cantiello et al., 2009) offers a possibility of the existence of clumps in the part of the wind below the critical point, where the numerical wind simulations do not predict it (see e.g. Feldmeier, 1995; Runacres and Owocki, 2002). It seems that the most probable clumping scenario consists of multiple clump formation mechanisms. Although there is much unknown about clump formation, distribution, and clump properties, it is very important to develop a description of clumping.

3.2 Observational evidence of clumping

Before higher resolution observing technologies, hot star winds appeared smooth. With the new generation of high S/N detectors, echelle spectrographs with higher resolution, and large telescopes able to collect stronger signal, it is possible to resolve many wind structures (mainly on large scales). Therefore, the wind can no longer be considered smooth. With the new era of satellites (such as the *IUE* satellite, active between 1978 and 1995, and *FUSE* satellite with higher resolution, launched in 1999 and operated until 2007), observing the UV spectral region became possible, greatly

increasing our knowledge about the nature of hot star winds.

One of the first observational indications that hot star winds are structured was discovered by Cherepashchuk et al. (1984). Analyzing light curves eclipses of Wolf-Rayet (WR) star WN5 and O6 components of the binary star V444 Cygni in the optical, UV, and IR wavelength ranges, Cherepashchuk et al. determined a density structure of the expanding atmosphere and physical characteristics of the WN5 component. They found that the depth and width of a secondary minimum increase with wavelength, while the shape of primary minimum changes little from optical to near-IR wavelengths. This wavelength dependence of the light curve is possible to explain only if it is assumed that the most of the IR emission arises from free-free and free-bound processes (which are density squared dependent) in the wind of the WN5 component, which is supposed to be clumped. Otherwise, if the WN5 wind is supposed to be smooth, the light curve will not be wavelength dependent. This was one of the first indirect conclusions that hot star winds are not smooth.

Prinja and Fullerton (1994) investigated observed systematic variability in the He I (λ 5876 Å) P Cygni profile in the wind of the O-supergiant HD 152408. This variability on a hourly time scale was explained as a consequence of a possible wind structure that is moving outwards. He I line variability as a signature of a deeper, denser wind region with low velocities also suggested similarities to DACs, which are observed in UV P Cygni profiles of other O-type stars in the higher wind regions.

Small-amplitude Line Profile Variation (LPV) is a more direct evidence of wind clumping. These variabilities are often detected only with high S/N $\approx$ 100 – 1000 (unlike the large-amplitude LPVs due to large scale wind structure that can be even identified with low S/N $\sim$ 20). Eversberg et al. (1998) found stochastic variable structures in the He emission line of ζ Puppis, which drift with time from the line centre to the blue and red edges of the line, explained by accelerating wind inhomogeneities moving outwards. Lépine and Moffat (1999) performed high-resolution spectroscopic monitoring of LPVs in the emission lines of nine WR stars. They used phenomenological descriptions of wind structure such as Discrete Wind Emission Elements (DWEEs) to simulate LPVs. However, this description of wind structures was not based on real physical and hydrodynamical properties of the wind. To explain how wind structure is associated with observed emission LPVs, Dessart and Owocki (2002a,b) simulated the emission LPVs based on line-driven instability and used a “patch method” to mimic wind structure. They show that for the model with an angular patch size of about 3° it is possible to qualitatively reproduce the fundamental properties of observed LPVs.

Additionally, Markova et al. (2005) investigated the H_α LPVs for a large sample of O-type supergiants by employing the modified Temporal Variance Spectrum (TVS) analysis (firstly developed for the case of photospheric absorption lines by Fullerton et al., 1996) to account for the effects of wind emission. Markova et al. found that all stars from the sample show H_α LPVs with mean amplitudes between 1 and 4% of the continuum strength. The authors concluded that wind variability is the

dominant source of LPVs and that, even at lower wind densities, where variations generated by the photosphere are dominant, the wind variability has to be taken into account. Simulated line profiles using the simple models of coherent or broken shells uniformly distributed over the wind volume, when compared with the observations, succeeded to explain properties of the H_α variability for stars with intermediate wind densities. On the other hand, for stars at lower and higher wind densities Markova et al. found discrepancies between observed and simulated line profiles (mainly in the mean amplitude and in the symmetry properties of the TVS) suggesting that it can be caused by the presence of coherent large-scale structures.

Direct spectroscopic observation of five O-type massive stars of different evolutionary stages (Lépine and Moffat, 2008) suggest that clumpy structures are a common property and a universal phenomenon of all hot star winds. However, not all “rapid” line variability can be connected with clumping. For example, the spectral variabilities of three Of⁺ supergiants, investigated by De Becker et al. (2009) using TVS technique, suggest that these variabilities are not caused by clumping, but are a consequence of large-scale structures. These authors reported that wind clumping would more probably yield a uniform TVS, in contrast to the structured one they found.

Electron-scattering wings, seen in the strong emission lines of WR stars, represent additional evidence of clumping. To explain overestimation of the electron-scattering wing strengths of WR stars by standard spectra calculations assuming smooth wind, Hillier (1991) invoked a very simple model of clumping where size, spatial, and velocity distribution of clumps were defined as free parameters. He described the density structure of a clumped wind considering that the atmosphere is consisted of discrete zones (dense spherical shells) with a particular density described by a set of parameters, i.e. shell-like model. Later, using the blanketed non-LTE radiative transfer code CMFGEN, Hillier and Miller (1999) performed a detailed analysis of HD 165763 (a Galactic WR star) and found that overestimation of the strength of the electron-scattering wings by standard smooth wind models can be explained by the presence of wind clumping. These authors described clumping with the help of a volume filling factor, f (see the following Section) assuming f to be 0.1. Using this way of clump description, Hillier and Miller (1999) succeeded to obtain better agreement with observed line profiles, i.e. to fit the red wings better, and also suggested that clumping is the key to explain the high mass-loss rates derived for WR stars.

Observed X-ray radiation in hot stars represents one more piece of observational evidence of wind clumping. The first X-ray emission from hot stars was detected with the *EINSTEIN* satellite. High-resolution X-ray spectroscopy became possible with X-ray satellites *XMM-NEWTON* and *CHANDRA*. The most widely accepted explanation for the origin of X-rays is the wind-shock scenario caused by the line-driven instability (Feldmeier et al., 1997). Detailed investigation of X-ray transport in clumped winds showed that wind clumping may strongly affect the X-ray line formation (see e.g.,

Feldmeier et al., 2003; Oskinova et al., 2004; Owocki and Cohen, 2006). Analyzing the *CHANDRA* spectra of δ Ori A (Miller et al., 2002) and ζ Pup (Kramer et al., 2003), it was shown that attenuation of X-rays is smaller than expected in homogeneous winds, and is suggested to be explained by clumping, which causes lowering of the effective opacity. Consequently, this implies more symmetric line profiles which are consistent with observations.

Observed “black troughs” of UV P Cygni line profiles indicate that hot star winds are clumped. Lucy (1982, 1983) showed that deep absorption trough cannot be reproduced by a spherically symmetric wind with monotonic velocity. An extended absorption trough, with zero residual intensity, was proved to indicate a non-monotonic wind velocity. For more about this clump signature see also Owocki et al. (1988) and Puls et al. (1993).

Since interpretation of observed spectra is affected by clumping, detailed quantitative spectroscopy of stellar atmospheres provides much more indirect evidence of wind clumping. Determination of mass-loss rates from observation is also strongly influenced by clumping. Different mass-loss diagnostics (UV resonance lines, H_α and other recombination line emission, IR and radio continuum emission) of OB-star winds give different mass-loss rates. The possible way to reconcile these discordances is to assume that wind is clumped. Many studies have shown that including clumping in non-LTE line profile calculation gives a better agreement with observations (see Section 3.4).

3.3 Parametric description of clumped medium

1-D simulations indicate that clumps are gaseous regions in the wind with density higher than in their surroundings, possibly with velocities different from the smooth wind. Clumps are stochastic inhomogeneities on various spatial and temporal scales.

To study how wind clumping can affect line profile calculations, the key clump properties that have to be included into radiative transfer calculations are described below.

3.3.1 Void interclump medium

Because there is no consistent theory of clump formation or direct observation of wind clumps, the detailed treatment of clumping is usually restricted to using free parameters describing the properties of clumping. This approach was previously used for the description of condensations in nebulae by Osterbrock and Flather (1959) for the case of the Orion nebula, where they assumed void intercondensation (i.e. interclump) medium and used the size of condensations as the free parameter, since the observational data available to them could not resolve details. Contemporary advanced observational techniques able to resolve many details of the nebula made the usage of the free parameter unnecessary for this particular case. However, for

stellar winds, direct clump imaging is still not possible.

Void interclump medium is a common assumption for the description of a clumped wind, whereby all matter is concentrated into clumps, which are distributed in vacuum. Such medium can be described by one free parameter, which may be either the *(volume) filling factor*

$$f_V = \frac{V_{\text{cl}}}{V_{\text{w}}}, \quad (3.1)$$

a fractional volume which contains material at higher density (referred to as the filling fraction by Owocki and Cohen, 2006), or the *clumping (correction) factor*

$$D = C_c = f_{\text{cl}} = \frac{\langle \rho_{\text{cl}} \rangle}{\langle \rho_{\text{w}} \rangle} = \frac{1}{f_V}, \quad (3.2)$$

where $\langle \rho_{\text{w}} \rangle$ and $\langle \rho_{\text{cl}} \rangle$ are the mean densities of the wind and clumps, respectively. Just one of these two adjustable parameters is sufficient.

Hamann and Koesterke (1998) were the first to use the factor D . $D > 1$ represents the density enhancement. Within this description of clumping, the equations need to be solved only for clumped medium assuming that density of clumps is

$$\rho_{\text{cl}}(r) = D \rho_{\text{sw}}(r), \quad (3.3)$$

where $\rho_{\text{sw}}(r)$ is the density stratification of the smooth wind model with the same mass-loss rates (i.e. total mass of the wind) as the clumped one. In the radiative transfer equation, the smooth wind opacity, $\kappa(\rho_{\text{sw}}(r))$, and emissivity, $\eta(\rho_{\text{sw}}(r))$, must be replaced by $\kappa_c(\rho_{\text{cl}}(r))$ and $\eta_c(\rho_{\text{cl}}(r))$, respectively, and scaled by the factor f_V to give

$$\kappa_{\text{f}}(r) = f_V \kappa_c(\rho_{\text{cl}}(r)) \quad \text{and} \quad \eta_{\text{f}}(r) = f_V \eta_c(\rho_{\text{cl}}(r)). \quad (3.4)$$

For any given ray, f_V represents the fraction of ray path which crosses clumps (the rest fraction of the ray path crosses the interclump medium), thus scaling with the factor f_V needs to be included. In this formulation, for processes that are linear in density, f_V and D cancel out in the emissivities and opacities in Eq. 3.4. However, for processes which scale with the square of the density, the emissivities and opacities are effectively enhanced by the factor D (see Eq. 3.4). This description of the clumped medium is also known as microclumping (see Section 3.3.3).

As an example, Hamann and Koesterke (1998) showed that the electron-scattering wings of several WR stars can be reproduced using this model of clumping, obtaining better agreement with observations. By increasing the clumping factor D , reducing the mass-loss rates, but keeping the same value of product $\dot{M}\sqrt{D}$, the electron-scattering wings were reduced, while the strength of strong WR emission lines remained. Using this effect, WR star had a typical value of D between 4 and 10 (see also Hillier, 1991, the first to study this effect).

It is more realistic to assume that the parameter defined by (3.1) or (3.2) depends

on depth. This dependence was predicted by pioneering hydrosimulations of Owocki et al. (1988). To differentiate between structured and smooth wind emission in the IR continuum and in the blue wings of the H_α line, the structured wind was described with a radially dependent clumping factor. Hillier and Miller (1999), later introduced an explicit expression for the depth dependent filling factor (3.1) in the form

$$f(r) = f_\infty + (1 - f_\infty) \exp\left(-\frac{v(r)}{v_{cl}}\right), \quad (3.5)$$

where f_∞ corresponds to f_V in (3.1) and v_{cl} is the location in the wind where clumping becomes important. With this parameterised factor $f(r)$, the clumping reduce the electron-scattering optical depth, and the clumped profiles are generally in better agreement with observations. This confirmed Hillier's earlier suggestions that the electron-scattering wings in standard WR models are overestimated.

Many radiative transfer calculations have used this expression for $f(r)$. Recently, Bouret et al. (2003, 2005, 2012) used a simple, parametric treatment of wind clumping in the CMFGEN code (Hillier and Miller, 1998), assuming a void interclump medium and clumps which are small compared to the photon mean free path. The detailed implementation of clumping in CMFGEN is discussed by Hillier et al. (2003).

Puls et al. (2006) tried to determine the depth dependence of the filling factor from observations. Including clumping into their analysis of a sample of 19 Galactic O-type supergiants/giants, assuming the interclump matter void, they performed a combined H_α , IR, and radio analysis and derived clumping factors normalized to a certain minimum, usually located in the outermost wind (radio-emitting region). Their main conclusion states that the innermost region of denser winds is more strongly clumped than the outermost wind region, while thinner winds have similar clumping properties in the inner and outer regions.

Najarro et al. (2008) used observations from UV to radio regions of OB stars to simultaneously constrain the clumping structure throughout the stellar wind. To investigate radial dependence of clumping in the wind, they introduced a more complicated clumping law (see Eq. 1 in Najarro et al., 2008), whose terms describe some basic properties of wind clumping (i.e. the maximum degree of clumping reached in the stellar wind, the velocity of the onset of clumping, the clumping structure in the outer wind, etc.).

The assumption of void interclump medium simplifies the description of clumping, however, this is rather unrealistic. Physically, it is more correct to assume that space between clumps is also filled with some amount of matter.

3.3.2 Dense interclump medium

Abbott et al. (1981) considered a non-void interclump medium, however, it was abandoned by later studies. Recently, Zsargó et al. (2008) reintroduced the idea of a dense interclump medium. Their simulations showed that clumped wind models

that use the classical “volume filling factor” approach (clumps with void interclump medium) cannot reproduce the observed O VI profile of ζ Pup. Also, if the interclump medium is filled by a low density gas, observed features can be reproduced. This diluted wind component becomes easily ionized by hard radiation produced in wind shocks. Therefore, the interclump medium contributes to the spectral line profiles of ions.

Sundqvist et al. (2010, 2011) also studied the effect of a dense interclump medium. Inclusion of the non-void interclump medium requires an additional free parameter (denoted by x_{ic} in Sundqvist et al., 2010) relating the interclump density to the clump density, $x_{\text{ic}} = \rho_{\text{ic}}/\rho_{\text{cl}}$. Line saturation is strongly affected by the dense interclump medium.

3.3.3 Clump properties

Several important properties of clumps must be defined to incorporate clumping into radiative transfer calculations, including the shape, size, density, and velocity of clumps.

Two general approaches are typically used. In the so-called “*microclumping*” approach, all clumps are assumed to be *optically thin*. However, this traditionally used assumption is far from realistic, and therefore must be relaxed. It is more accurate to assume that clumps can be of “*any optical thickness*”, i.e., that clumps can be optically thick in the cores of a resonance lines, while they may remain optically thin at all other frequencies. This approach is known as “*macroclumping*”.

Which assumption about optical thickness of clumps will be more suitable to apply for a particular case depends on the physical properties of medium (the abundance, ionization fraction or cross-section of the involved transition). For more about these approaches (microclumping vs. macroclumping) we refer to Oskinova et al. (2007) and Hamann et al. (2008). Below we give a summary of this work.

Microclumping In most radiative transfer calculations, clumps are assumed to be optically thin, which means that clumps are smaller than the mean free path of photons. Separation between clumps is assumed to be very small so that the mean opacity, $\bar{\kappa}$, can be used in the radiative transfer equation. This assumption is sometimes referred to as the “*microclumping*”. The treatment of clumping described in Section 3.3.1 is in fact microclumping.

In this simple approach, the stellar wind consists of clumps, while the interclump medium is void and the velocity field remains undisturbed (monotonic), like in a smooth wind. The density in clumps is enhanced by a factor D compared to a smooth wind with the same mass-loss rates and f_V is given as in Eq. 3.1 and Eq. 3.2. Microclumping can be easily implemented in non-LTE atmospheric codes.

As explained in Section 3.3.1, the main effect of microclumping is that, for processes which are linear in density, the mean opacity of a clumped medium is the

same as in the homogeneous case, while for processes which scale with the square of density, mean opacities and emissivities are enhanced by a factor D (Eq. 3.4). Additionally, radiative processes depend on the radial stratification of D , but they are not depend of clump size, geometry or distribution of clumps. The main consequence of microclumping is a reduction of empirical mass-loss rates. Diagnostics which scale with the local density squared in the microclumping approach give mass-loss rates which are lower by a factor of $\sqrt{D}$ than mass-loss rates assuming no clumping.

Macroclumping It is more natural to assume that clumps may be optically thick in some frequencies. The mean free path of photons is smaller than the size of the clumps, both in continua and lines. This situation referred to as “*macroclumping*” (Oskinova et al., 2007) or “*porosity*” (Owocki and Cohen, 2006). In this approximation, the escape of radiation increases through the porous space between clumps, leading to a reduction of the effective opacity compared to the opacity obtained in the microclumping approximation.

In an optically thick clump (macroclumping), photons interact with matter many times before eventually being destroyed or scattered in the clump, or escaping into the inter-clump matter. Porosity formalism was first introduced by Shaviv (2001) (see also Shaviv, 1998, 2000), who pointed out that the atmosphere of η Carinae is inhomogeneous, or “porous”, allowing more radiation to escape. Such an atmosphere yields a lower mass-loss rate for the same total luminosity. Feldmeier et al. (2003) described X-ray line emission from an opaque fragmented stellar wind. Owocki et al. (2004) then introduced a “*porosity length*”, h , defined as the ratio of the clump length scale l to the volume filling factor f_V , $h = l/f_V$. This porosity model has been applied by Oskinova et al. (2004, 2006) in synthesizing X-ray line profiles with a clumpy stellar wind described in the form of radially compressed shell fragments. Good agreement was obtained between synthetic and observed line profiles for non-magnetic O-type stars. Recently, Sundqvist et al. (2012a) used the porosity formalism to explain reduction in continuum opacity for the case of optically thick clumps. The formalism was also applied for the case of bound-free absorption (which is not directly affected by clumping) of X-rays in hot star winds. This case was also studied by Hervé et al. (2012), who made a basic comparison of the approximations by Oskinova et al. (2006) and Owocki and Cohen (2006).

Unlike microclumping, the distribution, size and geometry of the clump are important in macroclumping, so more parameters are needed to describe the medium. It is necessary to define clump size, $l(r)$, and an average separation between clumps, $L(r)$. Using these parameters, an expression for the effective opacity of a macroclumped (or porous) medium may be derived. Following the description of Oskinova et al. (2007), these parameters are related by the volume filling factor as

$$f_V = \frac{l^3}{L^3} = \frac{1}{D}. \quad (3.6)$$

The optical depth τ_c across a clump can be expressed as

$$\tau_c = \kappa_f l D = \kappa_f \frac{L^3}{l^2} = \kappa_f D^{2/3} L \quad (3.7)$$

where κ_f is given in Eq. 3.4, D is the density contrast (Eq. 3.2) and L is the average clump separation. For $L \rightarrow 0$ the clumps become optically thin and the smooth wind case is approached. The number density of stochastically distributed clumps can be expressed as $n_c \equiv L^{-3} = 1/(D l^3)$. The effective cross section of the clumps (i.e., their geometrical cross section) is multiplied by the fraction of photons that is absorbed when crossing the clump (first derived by Feldmeier et al., 2003), given by

$$\sigma_c = l^2 (1 - e^{-\tau_c}). \quad (3.8)$$

The effective opacity κ_{eff} for a clumped medium in the macroclumping approximation is

$$\kappa_{\text{eff}} = n_c \sigma_c = \frac{(1 - e^{-\tau_c})}{D l} = \frac{\kappa_f (1 - e^{-\tau_c})}{\tau_c} = \kappa_f C_{\text{macro}} \quad (3.9)$$

where κ_f is opacity of the clumped medium (Eq. 3.4) and $C_{\text{macro}} = (1 - e^{-\tau_c})/\tau_c$ is the factor which describes how opacity obtained is changed compared to microclumping. For the optically thin limit ($\tau_c \ll 1$), the microclumping approximation is recovered and $\kappa_{\text{eff}} \approx \kappa_f$, which depends on the volume filling factor. In the optically thick case ($\tau_c \gtrsim 1$), the effective opacity is strongly reduced by the factor C_{macro} .

Vorosity Most calculations assume an indentical smooth velocity field both inside and outside clumps. However, hydrodynamical simulations indicate that a non-monotonic field inside clumps is probably closer to reality. An inhomogeneous velocity field can affect only spectral lines. Owocki (2008) pointed out that for line transitions, the non-monotonic velocity field (“vorosity”) can be important, and may also reduce the effective opacity. He determined that the spatial porosity responsible for continuum absorption is not adequate for full description of the line absorption which is strongly affected by vorosity (porosity in velocity field). He introduced a “*velocity clumping factor*”, defined as a ratio between steepness of velocity inside the clumps, δv , to the velocity separation of the clumps, Δv .

This generalization was studied in more detail by Sundqvist et al. (2010, 2011), who described the velocity fields in stochastic wind models as a random perturbation of the velocity at the starting point of each clump. The jump in velocity in respect to the local smooth wind value given with β – law (see Eq. 4 in Sundqvist et al., 2010). Their results also showed that the detailed density structure, the non-void ICM, and a non-monotonic velocity field improve line fits.

Shape of the clumps Since no 3-D hydrodynamical simulations of the wind are available, there is no hint to the shape of clumps. In the parametric treatment

using the clumping factor, nothing is explicitly assumed about the clump shape and sizes. In more detailed calculations, different shapes were assumed, including shell fragments (Oskinova et al., 2004) and cubes (Muijres et al., 2011). Recently, Sundqvist et al. (2012a) studied how spherical and radially flattened clumps affect the X-ray line formation. It is not clear how important the detailed clump shapes is, but probably more critical is the distribution of clumps due to the stochastic nature of clumping.

3.4 Influence of clumping on empirical mass-loss rates

One large piece of indirect evidence that hot star winds are not smooth comes from the fact that there are large discrepancies between mass-loss rates derived from diagnostics with different density dependences. Here we list some of the important studies which show how wind clumping can affect different mass-loss diagnostics and empirical mass-loss rates determination.

Bouret et al. (2003) performed a detailed analysis of six O-type stars in the Small Magellanic Cloud (SMC), determining stellar and wind parameters and chemical abundances using non-LTE line-blanketed photospheric (TLUSTY) and wind (CMFGEN) codes. For three stars in their sample, mass-loss rates were lower by an order of magnitude or more than predicted from line-driven wind theory. Using a clumped model described with the volume filling factor and incorporated in the CMFGEN code, good match was achieved for the O v λ 1371 Å line. Bouret et al. (2005) later conducted a quantitative analysis of two Galactic O4 stars using far-UV spectra and derived stellar and wind parameters using the same codes as in their previous work. They could not fit observed lines (such as O v λ 1371 Å, P v $\lambda\lambda$ 1118 – 1128 Å, and N iv λ 1718 Å) with the smooth wind model, but matched these lines using clumped wind models also consistent with H_α lines. Bouret et al. derived mass-loss rates for several O stars which were a factor of 3 to 10 smaller than those obtained from smooth winds.

Markova et al. (2004) carried out a pure H_α profile analysis of 29 Galactic O-type stars with different luminosity classes in order to study their mass-loss and wind momenta. Their investigation relied on a method developed by Puls et al. (1996), which they modified to account for the effects of metal line-blocking/blanketing. Mass-loss rates derived from H_α were compared to those from radio free-free emission and for seven stars they found a very good agreement. However, for three stars with H_α in emission, the radio mass-loss rates are lower than those from H_α by a factor of about 2. Puls et al. (2006) combined H_α , IR, and radio data of a large number of Galactic O-stars and determined upper limit of mass-loss rates that are also factors of about 2 to 4 lower than mass-loss rates obtained earlier from H_α diagnostics using a smooth wind model.

To reconcile results obtained from different diagnostics, Oskinova et al. (2007), studied resonance and recombination lines assuming statistical distribution of clumps

and keeping a monotonic velocity field. These authors proposed that the discrepancies between the mass-loss rates derived from recombination lines and the P v resonance doublet can be solved by accounting for macroclumping.

Recently, Sundqvist et al. (2010, 2011) investigated the impact of clumping on diagnostics of ultraviolet resonance and optical recombination lines, by performing a multi-diagnostic study of α Cep (O-supergiant). They concluded that resonance lines are more affected by optically thick clumping than recombination lines. Using their stochastic wind models, Sundqvist et al. obtained consistent fits by increasing the clumping in the inner wind. A mass-loss rate they derived was about two times lower than what was predicted by the line-driven wind theory, but much higher than the corresponding rate derived when assuming optically thin clumps.

To investigate how the porous nature of the stellar wind might affect X-ray line attenuation, Owocki and Cohen (2006) assumed that wind material is compressed into localized, optically thick clumps. This study showed that a porous medium can reduce the effective absorption of X-rays emitted in an expanding stellar wind. Mass-loss rate reduction was also found by analyzing X-ray lines of ζ Pup (Cohen et al., 2010) and a very early O supergiant HD 93129A (Cohen et al., 2011). Consistent modelling of X-ray line profile shapes and broadband X-ray spectral energy distribution produced a reduction of mass-loss rates by a factor of 3 to 4 compared to the standard H_α mass-loss rate based on a smooth wind assumption. Porosity had only a small effect on X-ray line diagnostics in O stars, and therefore these diagnostics do indeed provide a good method for deriving O star mass-loss rates, insensitive to clumping.

Unsaturated resonance lines As explained in Section 2, reliable values of q_i (ion fraction of the species responsible for a particular wind line) are needed to determine mass-loss from UV resonance lines (density dependence diagnostics). However, it is very difficult to estimate q_i for most observed lines theoretically, or to measure it empirically. Therefore, the deduced mass-loss rates from UV resonance line diagnostics are poorly constrained. Hence, recent estimates of mass-loss rates for O-type stars are based on density squared diagnostics, such as H_α .

Saturated resonance ultraviolet lines fit theoretical spectra well, but their sensitivity to changes of wind parameters is relatively low. On the other hand, if lines are not saturated, they become a sensitive diagnostics tool. This happens for resonance lines of less abundant ions like Si IV, S V, Si IV, or P V. The Si IV resonance line ($\lambda\lambda$ 1393.76 Å, 1402.77 Å) was recently studied by Prinja and Massa (2010), who established spectroscopic evidence of optically thick clumps in the wind by measuring the ratios of the radial optical depths of the red and blue components of the Si IV doublets of B0 to B5 supergiants (see also Massa et al., 2008). These ratios have values between 1 and 2. Since they differ from the predicted value of two (a value that follows from the atomic constants for a smooth wind), this is a signature of optically thick clumping.

The possible importance of P v ($\lambda\lambda$ 1117.98 Å, 1128.01 Å) for mass-loss rates determination was pointed out by Crowther et al. (2002). In recent studies of O stars in the Large Magellanic Cloud (LMC) by Crowther et al. (2002) and SMC, Hillier et al. (2003) found that, to match P v in O supergiant, phosphorus needs to be underabundant. They have also concluded that a reduction of mass-loss rates relying on a smooth wind model might be necessary (similar results were obtained also by Massa et al., 2003).

Analyzing P v lines of 40 Galactic O-type stars, Fullerton et al. (2006) derived the product $\langle q_{\text{Pv}} \rangle \dot{M}$, where $\langle q_{\text{Pv}} \rangle$ is the mean ionization fraction of P v. Obtained values were a factor of 10 to 100 lower than values of mass-loss rates obtained from H_α or radio emission diagnostics. Since $\langle q_{\text{Pv}} \rangle$ for early and mid O-type stars is predicted to be ≈ 1 , estimated mass-loss rates seem to be extremely small that it would lead to questioning the line-driven wind theory. Therefore, it was assumed that optical properties of clumps can be different. Fullerton et al. concluded that either the mass-loss rates determined from P v resonance lines are wrong (which may be corrected by inclusion of wind clumping) or the abundance of P v is lower.

To test the abundance of P v, Krtićka and Kubát (2009) studied the non-LTE ionization balance of phosphorus in the wind. Using the code of Krtićka and Kubát (2004) they calculated the phosphorus ionization balance both with and without X-rays, and found that the changes were insignificant. Consequently, the abundance of P v is not lowered by presence of X-rays, which supports the clumping hypothesis. However, Waldron and Cassinelli (2010) suggested that XUV radiation may be important for phosphorus ionization balance. Detailed NLTE calculation (Krtićka and Kubát, 2012, submitted to MNRAS) agree, but the XUV radiation would at the same time destroy the driving ions of CNO, which would lower the radiation force and practically lower both the mass-loss rates and terminal velocity.

3.5 Influence of clumping on predicted mass-loss rates

While it is basically known how clumping (treated in the parametric way using the volume filling factor) influences the empirical determination of mass-loss rate for a *given* velocity and density structure, for mass-loss rate predictions the situation is much less clear.

Solution of hydrodynamic equations for given basic stellar parameters (e.g. the effective temperature T_{eff} , luminosity L_* , and radius R_*) and radiation at the lower boundary gives the density $\rho(r)$ and velocity $v(r)$ structure of the wind. The values of the mass loss rate $\dot{M}$ and terminal velocity v_∞ follow directly from the determined structure. They are typically called the predicted values.

Up to now, many comparison have been made between the mass-loss rates predicted by the radiation-driven wind theory and those estimated from observations. For example, Puls et al. (1996) concluded that there is a systematic discrepancy between mass-loss rates determined from observations (from radio data and from

analysis of H_α profiles) and those predicted by the radiation-driven wind theory. Mass loss rates for O-type stars derived from observations show higher values of about a factor of 2 from mass-loss rates predicted by the radiation-driven wind theory. One possible explanation for this discrepancy, which increases as a function of wind density, is an inadequate treatment of “multiple scattering” in radiation driven wind theory. Using Monte Carlo simulation and taking into account multiple scattering, Abbott and Lucy (1985) obtained a mass-loss of ζ Pup, which was a factor of about three higher than without multiple scattering (see also Puls, 1987).

Vink et al. (2000) developed a theoretical mass-loss recipe by Monte Carlo method in which multiple scatterings are taken into account. Mass-loss rates for OB stars were calculated for a wide range of stellar parameters. These theoretical mass-loss rates, for a large sample of O stars, showed disagreement with observations. For B stars, one group of stars’ obtained mass-loss rates were in good agreement with observations, while another group were not.

Krtićka et al. (2008b,a) studied the influence of clumping on predicted values of $\dot{M}$ and v_∞ using the stationary hydrodynamic code for non-LTE stellar winds (Krtićka and Kubát, 2004), which consistently calculates the radiative force without using the parameterization by the CAK parameters k , α , and δ . Clumping was treated in an approximate way ($\rho_{\text{clump}} = C_c \langle \rho \rangle$ – cf. Eq. 3.2); void interclump medium was assumed. This modified the opacity and emissivity in a different way for lines and continua, and consequently the radiative force was modified. Both optically thin and optically thick clumps were tested.

The results indicated that if clumping starts below the critical point, then the mass-loss rate increases. If clumping starts above the critical point, then the mass-loss rate does not change, but the terminal velocity increases. Clumps larger than the Sobolev length result in a decrease of the mass-loss rate. Clumping also influences the ionization balance, which has a strong impact on the line force, and consequently wind acceleration. Muijres et al. (2011) studied the effect of different portions of clumped and unclumped parts of the wind, as well as different clump sizes (assuming that clumps are cubes). Using the Monte Carlo wind code of Vink et al. (2000, 2001), optically thin clumps increase the mass-loss rate, whereas optically thick clumps decrease it.

Chapter 4

Radiative Transfer in stellar winds

To understand how radiation emerging from the stellar photosphere (static medium) changes passing through their expanding atmosphere, i.e. wind (moving medium), radiative transfer in both media has to be solved. The solution of the equation of radiative transfer in both media serves as a tool to analyse observed spectra to derive physical properties and kinematics of hot star winds.

Many methods and computer codes for static media are available, but they are not adequate to treat expanding atmospheres of hot stars. The problem of the transfer of radiation in moving media is more complicated, since the wind is expanding, and therefore the Doppler shift of the photon frequencies must be taken into account. Absorption and emission coefficients become angular dependent, and consequently, these coefficients become anisotropic and the aberration of light may become important (the latter effect is usually neglected for stellar winds where $v/c \lesssim 0.01$, see Mihalas et al., 1976). An additional complication is the coupling between angle and frequency, which makes the angular-frequency mesh very large. This makes a solution of the scattering problem in moving media computationally expensive.

This chapter is dedicated to the radiative transfer in a moving (expanding) medium beginning with basic theory and methods typically used for solving this problem. A description of the radiative transfer in inhomogeneous medium is given as well as an overview of a method to solve this problem. Finally, the Monte Carlo method used in the study of the wind clumping is described.

4.1 Basic theory

The main consequence of the atmosphere moving with some velocity is opacities and emissivities become affected by the velocity field through the Doppler shift. For

continuum radiation in a moving medium, the static equation of radiative transfer applies as the change in opacity and emissivity coefficients is negligible within the region of the Doppler frequency shift. For lines this does not hold because the changes of opacity due to Doppler shift may be large and become anisotropic.

There are two basic classes of methods (see, e.g., Mihalas and Kunasz, 1986) for solving the radiative transfer equation in moving medium:

1. *The solution in the observer's frame.* The frame of reference in which the observer is at rest with respect to the centre of the star (also called the inertial or laboratory frame). In this frame, the description of material properties is complicated. Due to the velocity field, the opacities and emissivities are anisotropic and angle and frequency are coupled making the scattering problems computationally very expensive. Thus, the solution of the transfer equation in this frame requires an enormous amount of frequency points to correctly describe the line profile. This restricts the velocity difference between neighbouring depth points to be smaller than several times the thermal velocity. At present, the computational power is high enough that this restriction is not longer valid (see Mihalas, 1980). The advantage of using this frame is that the partial differential equations for the radiation intensity are simple and any arbitrary velocity field (even non-monotonic) can be easily adopted and treated.
2. *The solution in the comoving-frame.* The comoving-frame moves with the outflowing material (also called the local reference frame). The advantage of using this frame for solving the radiative transfer equation is that the material properties (opacities and emissivities) are not affected by the velocity field, and can be treated as in the static case, i.e. no additionally anisotropy is introduced. Cases of both small and large velocity gradients can be solved in the comoving-frame, but including non-monotonic velocity field is quite complicated. For large velocity gradients, the Sobolev approximation (Sobolev, 1947) is often used. Algorithms have been developed for both planar and spherical geometry. However, the transfer equation in the comoving-frame is much more complicated.

A photon with frequency ν_{obs} , moving in material with velocity $\mathbf{v}$ in a direction $\mathbf{k}$, has a comoving-frame frequency

$$\nu_{\text{cmf}} = \nu_{\text{obs}} \gamma \left(1 - \frac{1}{c} \mathbf{v} \cdot \mathbf{k} \right). \quad (4.1)$$

This represents the frequency at which the photon interacts with matter, i.e. is absorbed or emitted. From Eq. 4.1, the opacity and emissivity become angle dependent (anisotropic) even if they are isotropic in the comoving-frame, and light aberration

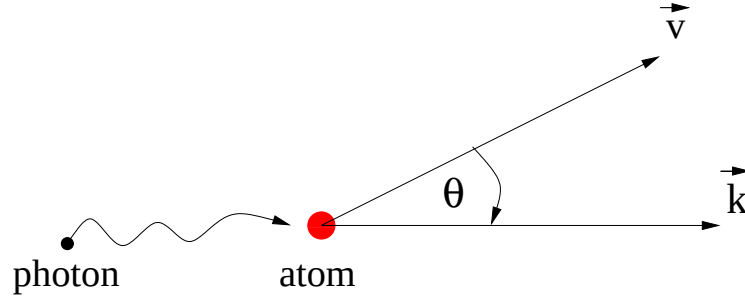


FIGURE 4.1: An atom of the medium which is moving with velocity vector $\mathbf{v}$ and a vector of the photon propagation $\mathbf{k}$ form an angle θ .

is generated. The direction of the photon $\mathbf{k}_{\text{cmf}}$ is given by following relation,

$$\mathbf{k}_{\text{cmf}} = \frac{\gamma \mathbf{v}/c + \mathbf{k} + (\gamma - 1)(\mathbf{k} \cdot \mathbf{v})\mathbf{v}/v^2}{\gamma(1 + \mathbf{k} \cdot \mathbf{v}/c)}. \quad (4.2)$$

For the non-relativistic case, when $v/c \ll 1$ (e.g. winds of hot stars), the relativistic Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ and all v^2/c^2 terms can be neglected, and Eq. 4.1 and Eq. 4.2 are simplified to

$$\nu_{\text{cmf}} = \nu_{\text{obs}} \left(1 - \frac{1}{c} \mathbf{v} \cdot \mathbf{k} \right) \quad (4.3)$$

and

$$\mathbf{k}_{\text{cmf}} = \frac{\mathbf{k} + \mathbf{v}/c}{1 + \mathbf{k} \cdot \mathbf{v}/c}. \quad (4.4)$$

Although aberration and advection effects can be important, they are negligible compared to the Doppler effect. Consequently, we may set $\mathbf{k}_{\text{cmf}} = \mathbf{k}$.

4.1.1 Observer's frame

The general time-dependent transfer equation in 3-D in the observer's frame can be written as

$$\left(\frac{1}{c} \frac{\partial I}{\partial t} + \mathbf{k} \cdot \nabla \right) I(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t) = \eta(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t) - \chi(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t) I(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t), \quad (4.5)$$

where $I(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t)$ is the specific intensity of radiation, $\mathbf{k}$ is the direction of propagation of radiation, $\mathbf{r}$ is the position vector, ν_{obs} is the radiation frequency, $\eta(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t)$ is the opacity, and $\chi(\mathbf{r}, \mathbf{k}, \nu_{\text{obs}}, t)$ is the emissivity of the medium. The radiative transfer equations in the observer's frame for static and moving media are the same.

4.1.2 Comoving-frame

The radiative transfer equation in a moving (expanding) medium is often solved in the comoving-frame. The main reason for this is that in this frame the only anisotropy is on an atomic level, which may be often neglected, and opacity and emissivity may be safely treated as isotropic. There is no need to account for the Doppler shift, and calculation of photon redistribution within the line becomes easy. However, for direct comparison with observations, the lines calculated in this frame must be translated to the rest frame of the star, i.e. to the line of sight of the observer at infinity. Many complexities of solving the radiative transfer equation in comoving-frame may arise due to the spatial and temporal variations of the velocity.

To derive the comoving-frame the radiative transfer equation, the Lorentz transformations (i.e. transformation between the inertial- and the comoving-frame) are used to convert for the specific intensity, opacity and emissivity into the inertial frame transport equation (see, e.g., Eq. 2.7 in Mihalas, 1980). Transformations are usually carried out only to order v/c , while higher orders are not needed for the fluid equations in classical mechanics.

The comoving-frame transfer equation (see Eq. 6.83 in Castor, 2004), neglecting relativistic effects and aberration ($\mathbf{k}_{\text{cmf}} = \mathbf{k}$), can be written in the following form:

$$\begin{aligned} \frac{1}{c} \frac{dI(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t)}{dt} + \mathbf{k} \cdot \nabla I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t) - \frac{1}{c} \mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k} \nu_{\text{cmf}} \frac{dI(\mathbf{k}, \nu_{\text{cmf}})}{d\nu_{\text{cmf}}} \\ - \frac{1}{c} \mathbf{k} \cdot \nabla \mathbf{v} (1 - \mathbf{k}\mathbf{k}) \cdot \nabla_{\mathbf{k}} I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t) + \frac{3}{c} \mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k} I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t) \\ = \eta(\mathbf{r}, \nu_{\text{cmf}}, t) - \chi(\mathbf{r}, \nu_{\text{cmf}}, t) I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t) \end{aligned} \quad (4.6)$$

where $\nabla \mathbf{v}$ is a rate-of-strain tensor of the velocity field, and $\nabla_{\mathbf{k}}$ is a derivative according to the coordinates of the vector $\mathbf{k}$. The frequency derivative of the intensity makes the transfer equation Eq. 4.6 more complicated than Eq. 4.5. If we suppose that v is not so large, the fourth and fifth term of the left hand side may be neglected. Then, Eq. 4.6 becomes

$$\begin{aligned} \frac{1}{c} \frac{dI(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t)}{dt} + \mathbf{k} \cdot \nabla I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t) - \frac{1}{c} \mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k} \nu_{\text{cmf}} \frac{dI(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t)}{d\nu_{\text{cmf}}} \\ = \eta(\nu_{\text{cmf}}, t) - \chi(\nu_{\text{cmf}}, t) I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t). \end{aligned} \quad (4.7)$$

4.2 Sobolev's Approximation

Back in the early 1930s, scientists started to develop theory for the formation of spectral lines in moving media (see, e.g., Beals, 1929, 1930, 1931; Chandrasekhar, 1934; Gerasimovič, 1933; Wilson, 1934). They tried to solve the radiative transfer equation in a spherically expanding medium by analytical methods, assuming that the matter is optically thin, but they were unsuccessful. Subsequently, Sobolev (1947)

developed a method which gives an approximate solution of the problem. By introducing a velocity gradient, the line transfer problem is simplified. Photon escape and thermalization become dominant processes, and the source function becomes geometrically localized. Since the Sobolev method is applicable in a medium with a large velocity gradient, this method is also known as the “supersonic” approximation. Solution of the transfer problem in Sobolev’s theory is based on calculation of radiation escape probability. This theory was later refined and extended by Castor (1970) and Rybicki and Hummer (1978).

If the line-of-sight velocity gradient $\mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k}$ in Eq. 4.7 is large, the first term on the left side of Eq. 4.7 can be neglected, leading to the *Sobolev equation* (see Eq. 6.85 in Castor, 2004),

$$-\frac{1}{c} \mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k} \nu_{\text{cmf}} \frac{dI(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t)}{d\nu_{\text{cmf}}} = \eta(\nu_{\text{cmf}}, t) - \chi(\nu_{\text{cmf}}, t) I(\mathbf{r}, \mathbf{k}, \nu_{\text{cmf}}, t). \quad (4.8)$$

This equation can be integrated over a frequency range of a particular line. Depending on $\nabla \mathbf{v}$, the comoving photon frequency may decrease in the case of expansion, or increase in the case of contraction. The total opacity and emissivity at the frequencies within the line can be assumed to be due to the line alone. The continuum opacity is ignored here, i.e. it is assumed that background continuum is not significant within the line and only line opacity κ_L is taken into account (this was generalized by Hummer and Rybicki (1985) to include the effect of continuous opacity on the line formation processes). Then, the total opacity in the Sobolev approximation can be written as

$$\chi(\nu_{\text{cmf}}) = \kappa_L \phi(\nu_{\text{cmf}}) \quad (4.9)$$

or

$$\eta(\nu_{\text{cmf}}) = \kappa_L S_L \phi(\nu_{\text{cmf}}) \quad (4.10)$$

where S_L is the line source function which can be either the Planck function for the LTE case or it can be calculated by non-LTE models. The line opacity is given by

$$\kappa_L = \frac{\pi e^2}{m_e c} g_l f_{lu} \left(\frac{n_l}{g_l} - \frac{n_l}{g_l} \right) \quad (4.11)$$

where n_l and n_u are level populations of the lower and upper level, and g_l and g_u are statistical weights of the lower and upper level, respectively. f_{lu} is the oscillator strength. $\phi(\nu_{\text{cmf}})$ is the line profile function which is normalized over frequency $\int \phi(\nu_{\text{cmf}}) d\nu_{\text{cmf}} = 1$. To find a solution to Eq. 4.8, we introduce a new variable y defined by

$$y = \int_{\nu_{\text{cmf}}}^{\infty} \phi(\nu'_{\text{cmf}}) d\nu'_{\text{cmf}} \quad \text{for} \quad \mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k} > 0 \quad (4.12)$$

and

$$y = \int_{-\infty}^{\nu_{\text{cmf}}} \phi(\nu'_{\text{cmf}}) d\nu'_{\text{cmf}} \quad \text{for} \quad \mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k} < 0. \quad (4.13)$$

Replacing ν_{cmf} with y in Eq. 4.8 we obtain following equation

$$\frac{\nu_0}{c} |\mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k}| \frac{dI(y)}{dy} = \kappa_L [S_L - I(y)], \quad (4.14)$$

which can be integrated using the boundary condition $I = I_{\text{cont}}$ at $y = 0$. This gives

$$I(y) = I_{\text{cont}} e^{-\tau(\mathbf{k})y} + S_L (1 - e^{-\tau(\mathbf{k})y}), \quad (4.15)$$

where $\tau(\mathbf{k})$ is the so-called *Sobolev optical depth*,

$$\tau(\mathbf{k}) = \frac{\kappa_L c}{\nu_0 |\mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k}|}. \quad (4.16)$$

$\tau(\mathbf{k})$ is a direction-dependent quantity and a measure of the optical thickness of the resonance zone, where particular photons may be absorbed by the line. If the velocity gradient is large this zone is very narrow.

If we integrate Eq. 4.8 for $0 < y < 1$, we obtain an average intensity over the line,

$$\bar{I}(\mathbf{k}) = I_{\text{cont}}(\mathbf{k})\beta(\mathbf{k}) + S_L(1 - \beta(\mathbf{k})), \quad (4.17)$$

where $\beta(\mathbf{k})$ is the *angle-dependent escape probability* given as

$$\beta(\mathbf{k}) = \frac{1 - e^{-\tau(\mathbf{k})}}{\tau(\mathbf{k})}. \quad (4.18)$$

Integration of Eq. 4.17 over all directions gives

$$\bar{J}(\mathbf{k}) = \frac{1}{4\pi} \oint I_{\text{cont}}(\mathbf{k})\beta(\mathbf{k}) + S_L(1 - \beta), \quad (4.19)$$

where β is the angular mean of $\beta(\mathbf{k})$ or the *Sobolev escape probability*,

$$\beta = \frac{1}{4\pi} \oint \beta \mathbf{k} d\omega = \frac{1}{4\pi} \oint \frac{1 - e^{-\tau(\mathbf{k})}}{\tau(\mathbf{k})}. \quad (4.20)$$

The Sobolev approximation is valid only for large velocity gradients. The steeper velocity gradient of the medium, the more accurate the method. With the large velocity gradient, the frequency displacement of the spectral line due to the Doppler shift is such that the radiative transfer in lines can be solved only locally. Radiation along a given ray changes only at certain point called *the resonance point*, where the Doppler shift is such that material can absorb or emit at frequency ν . The Doppler shifted frequency of the photon ν is equal to the frequency of the interaction. The

resonance condition is defined as

$$\frac{\nu - \nu_0}{\nu_0} = \frac{\Delta\nu}{\nu_0} = \frac{\mathbf{k} \cdot \mathbf{v}}{c} = \frac{v_l}{c} \quad (4.21)$$

where ν_0 is rest frame frequency and $v_l = \mathbf{v} \cdot \mathbf{k}$ is the line-of-sight velocity.

The Sobolev approximation is applicable when

$$\frac{\Delta l}{l_0} \approx \frac{v_{\text{th}}}{v_0}, \quad (4.22)$$

where l_0 is the typical scale length of the variation of medium, Δl is the width of the medium where absorption can happen (the so-called Sobolev length, L_S), v_{th} is the typical thermal velocity of the medium, and v_0 is the typical velocity of the medium.

4.3 Radiative transfer in inhomogeneous medium

Spectral analysis in many astrophysical branches, including hot star winds, is based on the assumption that radiation is formed in a homogeneous medium. This assumption is far from realistic, but it is a natural approach as it requires a minimum number of model parameters to fit observational data. The assumption of homogeneity enables direct and simple incorporation into well-established radiative transfer theory, however, as the scope and accuracy of observational data have increased, many structures and observed features (probably caused by inhomogeneities) became impossible to explain, and the assumption about homogeneity had to be relaxed.

Until recently, the computation of the radiation field was usually performed in 1-D assuming spherical symmetry and a homogeneous distribution of opacity sources. In such circumstances, the equation of radiative transfer can be solved analytically. However, since real astrophysical systems are inhomogeneous and not symmetric, the equations of radiative transfer can only be solved numerically.

To solve the radiative transfer problem in an inhomogeneous medium, multi-dimensional radiative transfer is required. In contrast to the 1-D formal radiation transport equation, which is affected only by conditions along the propagation path, multi-dimensional radiative transfer requires the state of material be known everywhere in the medium. Thus, all quantities must be specified as functions of (x, y, z) .

One key problem of radiative transfer in inhomogeneous medium is optically resolving multi-dimensional structures and the radiation in boundary layers (described by the optical depth, see, e.g., Auer, 2003). The radiation boundary layers are transition regions between optically thick and thin regions where computation of the radiation field is the most difficult. To resolve multi-dimensional structures it is also important to define relevant scale lengths, including the real physical size of an object, scale lengths of the structures within the object, the optical mean free path (gives the frequency dependence of opacity), and the thermalization length (distance passed by radiation between successive re-emission before converting to

thermal or kinetic energy). Scattering processes are also crucial for resolving the multi-dimensional structures.

To resolve the boundary layers optically, as well as small sources of radiation, to obtain a good resolution of the emergent radiation, the spatial and angular grid steps must be fine enough. For an irregular distribution of the structures, it is more convenient to use an adaptive grid which follows the underlying structures of the medium.

Solution of the radiative transfer problem in clumped hot star winds requires resolution of inhomogeneities not only geometrically, but also optically. This requires accounting for all inhomogeneities in velocity and density fields. There are two main approaches to solve the radiative transfer equation in such medium. One method solves the radiative transfer problem in a stochastic medium (see, e.g., Gu et al., 1995), originally used for solution of radiative transfer problem in turbulent medium (see, e.g., Thomas, 1960; Traving, 1964, 1980; Auvergne et al., 1973; Gail et al., 1974; Frisch and Frisch, 1976; Levermore and Pomraning, 1981; Pomraning, 1988). Another approach is the MCRT method which has been used by many authors and applied to different astrophysical problems, i.e., the formation of resonance lines in locally non-monotonic winds (Lucy, 1983), radiation pressure in hot star winds (Abbott and Lucy, 1985), supernovae (Lucy, 1987), multi-line transfer of Wolf-Rayet winds (Lucy and Abbott, 1993), line formation in cataclysmic variable winds (Knigge et al., 1995), the bi-stability jump in the winds of early-type supergiants (Vink et al., 1999), X-ray line calculations (Oskinova et al., 2004, 2006), Be star disks (Wood et al., 2004; Carciofi and Bjorkman, 2008), and resonance and recombination line calculation in hot star winds (Sundqvist et al., 2010, 2011).

Since the MC method has already proved to be a powerful and simple method for studying radiative transfer in multi-dimensional geometrical problems with irregular distribution of inhomogeneities, in this thesis we implement this method for studying the resonance line formations in clumped hot star winds.

4.4 Monte Carlo Method

The Monte Carlo (MC) method is a general class of numerical methods for solving a variety of mathematical and physical problems using various distributions of random numbers and probability statistics. For a more detailed description of the MC method, we refer to the Metropolis and Ulam (1949).

The method uses random numbers generator for a statistical sampling of a quantity from a probability density function to calculate the solution of a problem. Since the method provides only approximate solutions (like all numerical methods), the approximation error must be analysed. Accuracy of the MC method depends on the number of samples taken. The more samples are taken, the more accurate estimate of a solution. Acquiring an accurate MC method is computationally very expensive. But since computers are now more efficient and faster, the computation time is not a

serious limitation for this method any more. This has made the method more widely used in many branches of science.

One of the advantages of the MC method is that even a complex problem which cannot be solved analytically, can be easily implemented in a MC algorithm. The MC techniques can provide a fast approximate solution with a high level of accuracy. Further, since all the samples in the MC procedure are independent, it makes it an ideal algorithm for parallel programming. Additionally, if a physical process can be represented by equations or by probability distributions, then the MC method can be used to simulate the transport problem, regardless of geometry or physical complexity.

Thanks to the many advantages of the MC method, solving multi-dimensional problems (such as radiative transfer through a clumped medium) are more suitable with MC than with some other methods which are traditionally used. Compared to “classical” solution methods, like the Feautrier scheme or short characteristics methods that are extremely time and memory consuming when more than one spatial dimension is required, the MC method allows relatively simple inclusion of wind inhomogeneities.

Before we introduce the MC method for solving radiative transfer problems and its application to inhomogeneous hot star winds, let’s start first with some general definitions of probability theory should be defined.

4.4.1 Basic definitions of probability

An *random event* has more than one possible outcome with associated probability. The probability describes the degree of occurrence of a particular event and can be between 0 and 1. A probability of unity means that an event has only one certain outcome, while the probability of zero means that an event will not occur.

A random event may be associated with a *random variable* X , which can take different possible numerical values $(x_1, x_2, \dots)$ corresponding to different outcomes with a certain probability. If a random variable can take only certain discrete values, then the variable is called a discrete random variable. The probability that the value x_i is chosen is given by $\tilde{p}_i = P(X = x_i)$, where $i = 1, 2, \dots$. On the other hand, if a variable can take any value from a given interval $\langle a, b \rangle$, then the variable is said to be a continuous random variable. The set of these probabilities $\tilde{p}_i$ forms a *probability distribution* of the random variable.

Continuous variables, are sampled from the *probability density function* (PDF), $p(x)$. The *sampling procedure* is an algorithm that produces a sequence of random variables $x \in \langle a, b \rangle$ such that for any subinterval $\langle \alpha, \beta \rangle \subseteq \langle a, b \rangle$, the probability that the variable is in $\langle \alpha, \beta \rangle$

$$P\{\alpha \leq x \leq \beta\} = \int_{\alpha}^{\beta} p(x) dx \leq 1 \quad (4.23)$$

where $p(x)$ is normalized over the interval $x \in \langle a, b \rangle$ such that

$$\int_a^b p(x) dx = 1. \quad (4.24)$$

Usually random variables are independent and uniformly distributed on $\langle 0, 1 \rangle$, and can be generated by computer procedures called *pseudo-random numbers generators*.

The parameter is sampled from the so-called *cumulative distribution function* (CDF) which describes the probability $F(x_0)$ that a random variable X with a given PDF, $p(x)$ will be found at a value less than or equal to x_0 . This represents the *fundamental principle* given in the following form

$$F(x_0) = P(x \leq x_0) = \int_a^{x_0} p(x) dx. \quad (4.25)$$

x_0 is obtained by generating a random number ξ , such that $F(x_0) = \xi$ and then applying the inverse integral transform method, i.e., $F^{-1}(\xi) = x_0$. In cases where the PDF is complicated, an explicit formula for $F^{-1}(\xi)$ is sometimes impossible to find. In such situations, alternative methods, which are more efficient than the inverse transform method, can be used. One such method is the *acceptance-rejection method* (or the von Neumann rejection method, see Press et al., 1992).

Assuming that the random variable $x \in \langle a, b \rangle$ has the PDF $p(x)$ and CDF $F(x)$. This method aims to find an alternative PDF $g(x)$ (known as *majorizing function*), such that the function $g(x)$ is greater than $p(x)$ for all x in a given interval, i.e. $p(x) \leq m g(x)$, where $m > 1$ and the ratio $p(x)/g(x)$ is bound by constant m which corresponds to the maximum of the $p(x)$. Hence, $m g(x)$ represents an envelope distribution from where sampling is easier from. The basic of acceptance-rejection algorithm for continuous random variables is to sample two random numbers ξ_1 and ξ_2 uniformly between 0 and 1. Then, we sample $x = (b - a)\xi_1 + a$ from $g(x)$ and calculate $y = m\xi_2$, where x and y are uniformly distributed on $\langle a, b \rangle$ and $\langle 0, m \rangle$, respectively. Then, $p(x)$ is calculated, and if $p(x) > y$, then y is accepted as a value of x . Otherwise, it is rejected and a new sampling for ξ_1 and ξ_2 is started again.

This method is used in this thesis to distribute clumps within the wind by determining their positions (see Section 5). For generating random numbers the `ran2` function (Press et al., 1992) represents a long period uniformly-distributed, pseudo-random number generator. This generator provides sufficiently fast calculation with a long sequence of independent random numbers.

4.4.2 Monte Carlo Radiative Transfer

The MC method has been proven ideally suited to solve radiative transfer problems in complicated situations such as a multi-dimensional system, in moving medium, for polarized radiation, etc. Currently, this method is widespread in astronomy.

The MC approach is the most promising way for treatment of radiation transfer in

a clumped stellar wind. “Classical” solution methods for the radiative transfer equation, like the Feautrier scheme or short characteristics methods, become extremely time and memory consuming and need very sophisticated solution schemes when we go to more spatial dimensions than one (e.g., Korčáková and Kubát, 2003, 2005). This is not the case of MC methods, where the extension to 2-D or 3-D is relatively simple. These methods also allow simple inclusion of wind inhomogeneities.

Basic definitions A beam of radiation can be characterised by its specific intensity I_ν defined as (see Mihalas, 1978)

$$I_\nu = \frac{dE_\nu}{\cos \theta dA dt d\nu d\Omega}, \quad (4.26)$$

where dE_ν represents the energy which passes through a surface area dA at angle θ in respect to the surface normal, within a solid angle $d\Omega$, in time dt and frequency range $d\nu$. In MCRT method the energy dE_ν represents the photon packet which is followed during the MC simulation. Usually, the initial total energy is split into an equal amount of N photon packets that are followed in order to constrain their statistical properties. These packets are related to the specific intensity. Integrating the specific intensity I_ν over all angles gives the flux defined as

$$F_\nu = \int I_\nu \cos \theta d\Omega, \quad d\Omega = \sin \theta d\theta d\phi. \quad (4.27)$$

The equation of radiative transfer can be written in the following form,

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \quad (4.28)$$

where $S_\nu = \eta_\nu/\chi_\nu$ is the source function, and τ_ν is the optical depth, defined as

$$\tau_\nu = \int_L \chi_\nu dl. \quad (4.29)$$

Physically, the optical depth can be explained as the number of photon mean paths l_{ph} over a distance L . The mean free path of the photon is $l_{\text{ph}} = 1/\chi_\nu$ and represents the mean distance passed by photon between any two interactions. The probability that photon will not interact within an infinitesimal length dx is given as

$$p(x) = 1 - \frac{dx}{l_{\text{ph}}} = 1 - \chi_\nu dx \quad (4.30)$$

A distance x passed by photon can be divided to N number of segments dx with the

probability of no interaction within dx written as

$$p(x) = \left(1 - \frac{\chi_\nu x}{N}\right)^N. \quad (4.31)$$

For sufficiently large N , the probability becomes

$$p(\tau_\nu) = e^{-\tau_\nu}. \quad (4.32)$$

Basic concept The basic concept of MCRT method deals with the propagation of so-called "photon packets" and their interaction with matter within a medium. The followed photon in the MCRT procedure is not a single particle but a collection of n photons with the same frequency ν and total energy $E_\nu = n h \nu$. Henceforth, photon will mean a packet of photons.

The initial photon position, frequency, direction of the photon propagation, the point of interaction and new direction after interaction are all sampled from suitable PDFs. After a photon is created at some initial position $\mathbf{r}_0 = (x_0, y_0, z_0)$ at the surface of the star (the photosphere), it is sent in a randomly chosen initial direction defined by the angles θ_0 and ϕ_0 . The angular distribution function for photons emitted by the unit area of a photosphere follows from the definition of specific intensity. We release photons uniformly in ϕ and with a PDF $\propto \mu d\mu$ in μ (see e.g., Lucy, 1983). From Eq. 4.27 and Eq. 4.24 it follows that the PDFs for θ and ϕ are

$$p(\mu) = 2\mu \quad \text{and} \quad p(\phi) = \frac{1}{2\pi}, \quad (4.33)$$

where $\mu = \cos \theta$ is a direction cosine. Since photons are sent only upwards, then $0 \leq \mu \leq 1$. From Eq. 4.25, CDFs for μ and ϕ are

$$F(\mu) = 2 \int_0^\mu \mu' d\mu' = \xi_1 \quad \text{and} \quad F(\phi) = \frac{1}{2\pi} \int_0^{\phi_0} d\phi' = \xi_2. \quad (4.34)$$

Finally, after applying the inverse method, the angles are

$$\begin{aligned} \mu_0 &= \sqrt{\xi_1} \\ \phi_0 &= 2\pi\xi_2 \end{aligned} \quad (4.35)$$

where ξ_1 and ξ_2 are two different random numbers.

The probability that a photon will not interact between 0 and τ is $p(\tau) = e^{-\tau}$ (see Eq. 4.32), defining how far photon will travel. From Eq. 4.25, the CDF is

$$F(\tau) = P(\tau \leq \tau_0) = \int_0^{\tau_0} p(\tau) d\tau = \int_0^{\tau_0} e^{-\tau} d\tau = 1 - e^{-\tau_0}, \quad (4.36)$$

where $0 \leq F(\tau) \leq 1$. Finally, after selecting a random number and setting it equal

to CDF, i.e. $F(\tau)=\xi$, and inverting it,

$$\tau_0 = -\ln(1 - \xi), \quad (4.37)$$

where τ_0 is a randomly chosen optical depth which determines the location of the photon interaction. The term $1 - \xi$ can be replaced by ξ (since ξ is equally distributed on $0 \leq \xi \leq 1$ then $1 - \xi$ will be also, see Avery and House, 1968 and Caroff et al., 1972).

During the random walk the photon is moved from one position to another, and the new photon position is recalculated by

$$\begin{aligned} x &= x_0 + L_{\text{ph}} \sin \theta \cos \phi \\ y &= y_0 + L_{\text{ph}} \sin \theta \sin \phi, \\ z &= z_0 + L_{\text{ph}} \cos \theta \end{aligned} \quad (4.38)$$

where L_{ph} is the total distance between two photon interactions, and $\mathbf{r}_0 = (x_0, y_0, z_0)$ and $\mathbf{r} = (x, y, z)$ are the vectors of the current and the next photon position, respectively. During the walk, the optical depth is accumulated. After every random step, it is checked whether the current optical depth τ_{new} is larger than the one randomly chosen. If not, then the random walk is continued until this occurs or until the photon exits. If $\tau_{\text{new}} > \tau_0$, then the photon interaction happens between the current and previous photon position.

After the photon interacts, its direction changes. Assuming isotropic scattering, the new direction is sampled randomly from an isotropic distribution of solid angle $d\Omega = \sin \theta d\theta d\phi$. From Eq. 4.27 and Eq. 4.24, the PDFs for μ and ϕ are

$$p(\mu) = \frac{1}{2} \quad \text{and} \quad p(\phi) = \frac{1}{2\pi}. \quad (4.39)$$

where $-1 \leq \mu \leq 1$. From Eq. 4.25, the CDFs for the new μ and ϕ after scattering are

$$F(\mu) = \frac{1}{2} \int_{-1}^{\mu} d\mu' = \xi_1 \quad \text{and} \quad F(\phi) = \frac{1}{2\pi} \int_0^{\phi} d\phi' = \xi_2. \quad (4.40)$$

Finally, after applying the inverse method, the angles of the new photon direction after interaction are

$$\begin{aligned} \mu &= 2\xi_1 - 1 \\ \phi &= 2\pi\xi_2. \end{aligned} \quad (4.41)$$

For non-isotropic scattering $p(\mu)$ and $p(\phi)$ are different than Eq. 4.39.

This procedure is repeated until the photon leaves the medium. One photon trajectory represents one MC event and is independent of all other trajectories. The

photon's random walk satisfies the definition of a Markov chain². Once the photon exits the top of the atmosphere, its frequency and angle of exit are tabulated and placed into angle and/or frequency bins. The number of bins determines the resolution of the resulting distribution function obtained after photons finish travelling and all placed into the proper bins. Since MCRT is a stochastic process, the emission or absorption of the photon is a Poisson process and obtained errors must obey Poisson statistics. With an infinite number of realizations the MC methods will give an expectation which has a standard deviation. The accuracy of a MC simulation, is a function of the number of realizations N and scales with $1/\sqrt{N}$. In the standard MC simulation the weight of every sample is the same ($w = 1$). In some particular cases it is desirable to adjust the weights in order to reduce computational costs and to equalize errors i.e. to obtain the same level of the MC noise.

4.5 Monte Carlo test calculation for hot star winds

To study the performance of the MC method in stellar winds, some test calculations were made using the code developed for the formal solution of the radiative transfer equation for given velocity, temperature, and density stratification. This task has already been done by many authors for different astrophysical applications (e.g. Abbott and Lucy (1985); Whitney (1991); Lucy and Abbott (1993); de Koter et al. (1997); Vink et al. (1999); Wood et al. (2004) and Kromer and Sim (2009), to name at least some of them). Nevertheless, to understand the processes more deeply, to feel how the Monte Carlo procedure works, and to have better control on the code behavior, we repeated this step.

Toy model The velocity structure $v(r)$ of the wind, is calculated using a β -velocity law (see Eq. 2.29), the density structure $\rho(r)$ follows from the continuity equation (see Eq. 2.5), and temperature $T(r)$ was taken as independent of r . Stellar and wind parameters were taken from Crowther et al. (2006), see Table 4.1.

For depth discretization of the wind we used equidistant spacing in the logarithmic scale. Density $\rho(r)$ is taken to be constant within a zone between two depth points, and equal to the value at the lower radius of the zone. The radial velocity $v(r)$ is linearly interpolated inside the zone.

We further introduced several crude simplifying assumptions. We assumed that all electrons in the wind come from hydrogen ionization, and that hydrogen is fully ionized for the calculation of electron density. The mass of heavier atoms was neglected. The electron density, necessary for calculation of the electron scattering opacity, can be simply calculated from the total density dividing by a sum of electron and proton masses. The opacity of the medium consists of only two processes,

² A Markov chain is a mathematical system, that evolve over the time from one state to another, and represents a special kind of stochastic process in which the next state of the system depends only on the present state and not on preceding states.

line scattering under Sobolev approximation, and electron scattering. Ionization and excitation state of wind is calculated using LTE approximation with constant $T = 3/4 T_{\text{eff}}$.

Further, we calculated the profiles of H_α lines as they would be resonance lines following the Monte Carlo method, as in Lucy (1983) and Abbott and Lucy (1985).

Monte Carlo radiative transfer The frequency of the newly created photons for MC calculations is determined from the static hydrogen-helium photosphere, calculated by a non-LTE model atmosphere code using the emergent flux distribution (Kubát, 2003), with the help of the acceptance-rejection method (see Chapter 4.4.1). The initial direction of the photon is randomly chosen according Eq. 4.35, and photons are sent from the stellar surface ($R_\star = 1$) outwards.

After its creation, each photon obtains information how far it is allowed to travel before it undergoes interaction with some particle. This is done by randomly choosing an optical depth $\tau_\xi = -\ln \xi$ (see Eq. 4.37). The actual optical depth, which the photon travels, is calculated by summing the opacity contribution along its path. This path determines the place of interaction.

We take into account two events which may happen to photon, namely the scattering of free electrons, and the resonance line scattering. While the first process acts on all photons with any frequency in the same way, the second process requires the frequency of a photon to be equal to a Doppler shifted frequency of a scattering atom. If we denote the photon frequency in the observer frame as ν_{obs} and the frequency of the line transition as ν_{line} , then the condition that the line scattering may occur is

$$\nu_{\text{line}} = \nu_{\text{obs}} \left(1 - \frac{\mathbf{k} \cdot \mathbf{v}(r)}{c} \right), \quad (4.42)$$

where $\mathbf{k}$ is a direction of photon propagation, c is the light speed, and ν_{line} corresponds to the ν_{cmf} in Eq. 4.3.

For a radial flow the corresponding optical depth is then

$$\tau_{\text{line}} = \frac{\pi e^2}{m_e c} f_{\text{line}} n_a \left[\mu^2 \frac{dv}{dr} + (1 - \mu^2) \frac{v}{r} \right]^{-1} \frac{c}{\nu_0}, \quad (4.43)$$

where μ is the cosine of the angle between photon propagation direction and the radial direction, n_a is the lower level number density (in our test case arbitrarily taken as $9 \cdot 10^{-9}$ electron density), f_{line} is the line oscillator strength, and m_e is the electron mass. The value in the parentheses represents the strain rate projected on the ray direction, $\mathbf{k} \cdot \nabla \mathbf{v} \cdot \mathbf{k}$ (see Eq. 4.16) in spherical coordinates. The optical depth is evaluated at the radius where line scattering occurs.

The optical depth of the electron scattering τ_{elsc} is calculated as

$$\tau_{\text{elsc}} = n_e(r) \sigma_e \quad (4.44)$$

Name	T_{eff} [K]	$\log g$	R_* [$R_{\odot}$]	v_{∞} [km/s]	$\dot{M}$ [$10^{-6}M_{\odot}/yr$]	β
HD 14818	18500	2.40	46.1	565	0.55	2.0
HD 13854	21500	2.55	37.4	920	0.85	2.0
HD 115842	25500	2.85	34.2	1180	2.00	1.5
HD 30614	29000	3.00	26.0	1560	5.00	1.5

Table 4.1: Stellar parameters were taken from Crowther et al. (2006).

for all depths. If the total optical depth is

$$\tau_{\text{line}} + \tau_{\text{elsc}} > \tau_{\xi}, \quad (4.45)$$

then a photon terminates its travel in the layer where this condition was first fulfilled.

After a scattering (either line or electron), the photon obtains a new direction, chosen randomly for the case of isotropic scattering as in Eq. 4.41. For line scattering, a photon obtains a Doppler shifted frequency (in the observer frame)

$$\nu_{\text{obs,new}} = \nu_{\text{line}} \left(1 - \frac{\mathbf{k} \cdot \mathbf{v}(r)}{c} \right)^{-1}$$

and a new optical depth τ_{ξ} may be randomly chosen again.

If the randomly generated optical depth τ_{ξ} is lower than the total optical depth along the path travelled by the photon, the latter escapes from the wind region either back to the stellar surface or towards the observer. We define a frequency grid, which determines frequency intervals. Each escaping photon is then counted according to its frequency to the proper frequency interval.

We plot the profiles of the H_{α} line of four OB stars obtained as a result of our calculation. Stellar and wind parameters are taken from Crowther et al. (2006) (see Table 4.1). The line has wavelength and oscillator strength corresponding to the hydrogen H_{α} line. The flux is expressed as relative intensity with respect to local continuum. These results were previously presented in Šurlan and Kubát (2011).

This test calculations confirmed that the new wind method developed is applicable for the problem we want to study.

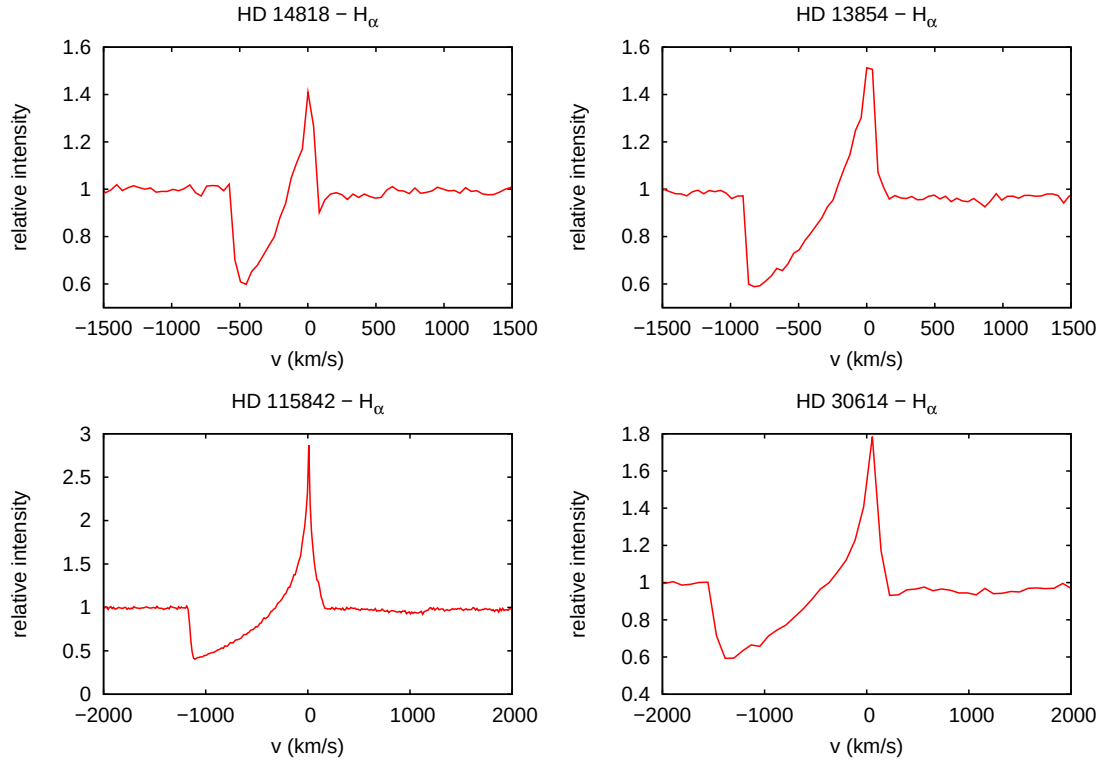


FIGURE 4.2: P Cygni line profiles (H_{α}) calculated for four different stars, whose names are donated above the panels, for the stellar and wind parameters given in Table 4.1.

Chapter 5

Description of the method

In this Chapter a method developed to solve the radiative transfer problem through the clumped massive, hot star wind, as well as calculation of resonance line profiles are described. First, the basic assumptions the method is based on, and a general description of the method and the wind model used are given. Further, the properties of clumps, interclump medium, and their velocities and opacities are introduced giving explanation how these properties are incorporated into the wind model. Last, the developed 3-D MCRT code is described, and some test calculations are shown.

5.1 Basic assumptions

To study the basic effects of clumping on the resonance line formation (both singlets and doublets), a core-halo model is adopted i.e. only the line opacity is taken into account, while the continuum opacity is neglected (line-free continuum). The intensity entering the wind from the photosphere is assumed to be constant in frequency and angle (no limb darkening). Only pure scattering is considered. Only pure Doppler-broadening is allowed and complete redistribution is assumed.

5.2 Radiative transfer solution

To solve the radiative transfer through a clumped wind, first a snapshot of the clump distribution is generated. Then, using the MC approach (see Sect. 4.4.2) the photons are followed along their paths. The density and velocity of the wind can be arbitrarily defined in a 3-D space. To overcome possible geometrical problems and to make tracing the photon inside the wind as simple as possible, the calculations are carried out in the comoving frame following the prescriptions by Hamann (1980).

The Cartesian coordinate system is used with position vector $\mathbf{r}$ of the photon expressed by a set of coordinates (x, y, z) .

The radiative transfer equation is solved in the dimensionless form. In the model, all quantities are dimensionless. Dimensionless frequency in the observer's frame x^{obs} , measured relative to the line centre in units of standard Doppler-widths $\Delta\nu_D$ is introduced (see Eq. 2.17) by

$$x^{\text{obs}} = \left(\frac{\nu}{\nu_0} - 1 \right) \frac{c}{v_l}, \quad (5.1)$$

where v_l is the line-of-sight velocity (see Eq. 4.21) and $v_l = \Delta\nu_D(c/\nu_0)$. During the calculations the local comoving frame frequency x^{cmf} of the photon (the rest-frame frequencies of spectral lines) is strictly kept, meaning that the frequency of the followed photon at a certain coordinate point can be expressed using the scalar product of the vector of the local macroscopic velocity ($\mathbf{v}$) and the unit vector in the direction of the photon's propagation ($\mathbf{k}$),

$$x^{\text{cmf}} = x^{\text{obs}} - \mathbf{k} \cdot \mathbf{v}. \quad (5.2)$$

When the photon exits the wind, x^{cmf} must be transformed into the observer's frame by

$$x^{\text{obs}} = x^{\text{cmf}} + \mathbf{k} \cdot \mathbf{v}. \quad (5.3)$$

Note that all velocities are dimensionless and measured in units of v_l .

The code is gridless and does not require any symmetry. Instead of using a predefined grid, an adaptive integration step Δr is introduced. After choosing Δr , the velocity and opacity of the wind are calculated along the photon path. This accounts for arbitrary density and velocity inhomogeneities (for a detail description see Sect. 5.2.2).

5.3 The wind model

A wind model is assumed to consist of both smooth and clumped regions. The clumped region is comprised of two density components: the tenuous or void inter-clump medium and the dense clumps. All distances in the wind are expressed in units of stellar radius. A parameter r_{cl} defines where wind clumping sets on. The region $r_{\text{min}} < r < r_{\text{cl}}$, where r_{min} is the lower boundary of the wind at the surface of the star ($r_{\text{min}} = 1$), represents the smooth region. The region $r_{\text{cl}} < r < r_{\text{max}}$, where r_{max} is the outer boundary of the wind, represents the clumped region (see Fig. 5.1). Since r_{cl} and r_{max} are free parameters of the model, wind regions can have arbitrary size. By properly setting these parameters, the studied wind can be fully clumped from r_{min} (for $r_{\text{cl}} = r_{\text{min}}$), totally smooth (in the case when $r_{\text{cl}} = r_{\text{max}}$), or a lower part of the wind may be smooth and the rest clumped.

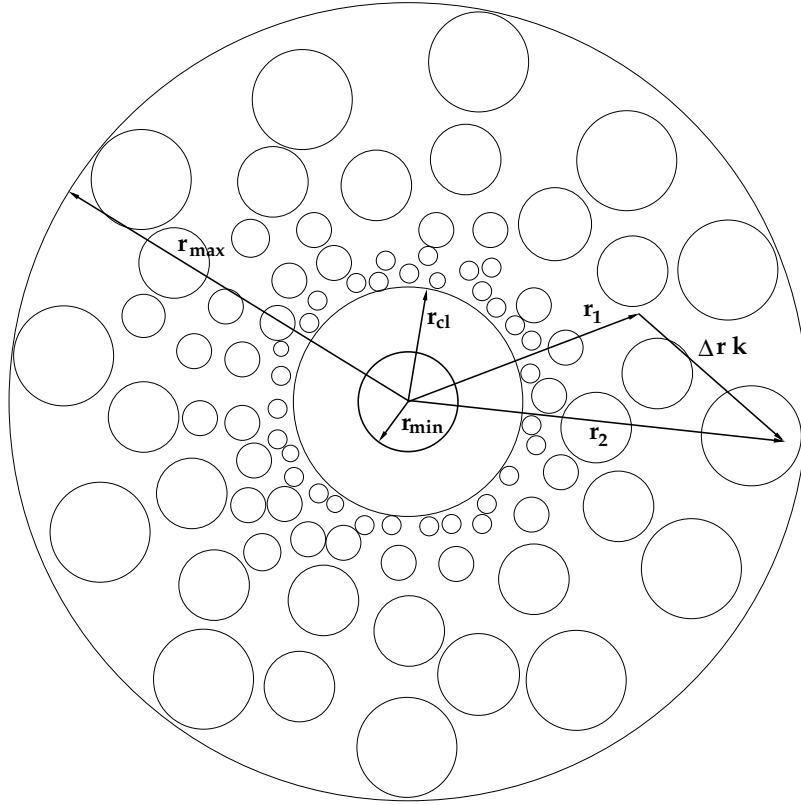


FIGURE 5.1: A schematic view of the wind model. $r_{\min}$ is the lower boundary of the wind (the surface of the star), r_{cl} is the onset radius of the wind clumping, $r_{\max}$ is the outer boundary of the calculation, and $\Delta \mathbf{r}$ is an integration step. Vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{k}$ are described in Sect. 5.2.2.

5.3.1 Wind velocity

The velocity field in the model calculations can be any arbitrary three-component velocity. For simplicity, the velocity field is assumed radial and for the underlying smooth wind we adopt the standard β -velocity law

$$v(r) = v_{\infty} \left(1 - \frac{b}{r} \right)^{\beta}, \quad (5.4)$$

where $v(r) = v_r$ is the radial component of the velocity vector, v_{∞} is the terminal wind velocity and b is chosen such that $v_{\min} \leq 10^{-3} v_{\infty}$ and is obtained using the radial velocity of the gas at the photosphere $v_{\min} = v(r = r_{\min})$,

$$b = r_{\min} \left(1 - \left(\frac{v_{\min}}{v_{\infty}} \right)^{\frac{1}{\beta}} \right). \quad (5.5)$$

The derivative of Eq. 5.4 gives the radial velocity gradient,

$$v' = \frac{dv_r}{dr} = v_r \beta \frac{b}{r(r-b)}, \quad (5.6)$$

which is important for choosing an adequate integration step (see Section 5.5.2).

5.3.2 Wind opacity

A line opacity calculation in a moving medium requires accounting for the Doppler shift, which leads to opacities that are anisotropic in the observer's frame. The line transfer problem in moving media is usually solved assuming the Sobolev approximation (see Section 4.2), which is valid only if the resonance zone (region where the interaction in a given spectral line can only happen) is very sharp. However, for clumped media where the velocity field may be non-monotonic, it is more convenient to assume a resonance zone within which physical quantities are allowed to change. Therefore the Sobolev approximation is not adopted here, but the radiative transfer equation in the comoving frame is solved. Since in this frame the fluid is at rest, the opacity is isotropic, and the anisotropy is taken into account by the frame transformation.

For the opacity, the same parametrization as Hamann (1980) is used,

$$\chi(r) = \frac{\kappa_0}{r^2 \frac{v(r)}{v_\infty}} q(r) \phi_x \quad (5.7)$$

where κ_0 is a free parameter of the model corresponding to the line strength and proportional to the mass-loss rate and abundance of the absorbing ion, $q(r)$ reflects the depth-dependent degree of the ionization of the absorbing ion and for simplicity is chosen to be $q(r) \equiv 1$ i.e. constant ionization condition. The absorption profile ϕ_x for singlet lines is assumed to be Gaussian

$$\phi_x = \frac{1}{\sqrt{\pi}} e^{-x^2}, \quad (5.8)$$

where x is the dimensionless frequency given by Eq. 5.1. The reference velocity v_l is implied to be the Doppler-broadening velocity v_D (i.e. $v_l = v_D$) including a contribution from both thermal broadening and microturbulence and is constant over the whole wind.

In this parametric formalism, the smooth wind opacity $\chi(r)$ is proportional to smooth wind density ρ_{sw} . In the calculations, wind density is not directly calculated, but the depth-dependent line opacity $\chi(r)$ which is enough for calculation of the emergent radiation from the wind.

5.4 Description of clumping

Based on previous attempt to parametrise properties of the clumped medium (see Section 3.3), in this section the properties of clump and inter-clump media of the inhomogeneous wind model are described.

5.4.1 Clump properties

Allowing for an arbitrary optical depth, clumps can be optically thick in the cores of resonance lines, while they remain optically thin at all other frequencies. Clumps are assumed to be statistically distributed within the stellar wind. The average clump separation $L(r)$, measured between their centers, is variable with the distance r from the star. For simplicity, it was assumed that each clump has the spherical shape with the volume $(4\pi/3)l^3$, where l is the radius of the particular clump. The clump radius varies with the distance from the star, $l = l(r)$.

The density ρ_{cl} inside the clump at a distance r from the center is assumed to be higher by a factor D than the smooth wind density ρ_{sw} at the same radius. The factor D (density contrast) is the first free parameter of the model and it is assumed to be either constant (depth independent) or depth dependent.

Depth independent clumping factor The density ρ_{cl} inside the clump at a distance r from the center for a constant clumping factor is

$$\rho_{\text{cl}}(r) = D \rho_{\text{sw}}(r) \quad (5.9)$$

and $D \geq 1$ (see Eq. 3.3). Using L and l , the factor D according to Eq. 3.6 can be expressed as

$$D = \frac{L^3(r)}{\frac{4\pi}{3}l^3(r)}. \quad (5.10)$$

In the model, clumps are assumed to be preserved entities. It is assumed that clumps are accelerated radially and neither split nor merge, but expand as they move outwards. Therefore, the number of clumps per unit volume n_{cl} has to satisfy the equation of the continuity

$$n_{\text{cl}} \propto \frac{1}{r^2 v_r}. \quad (5.11)$$

For the average clump separation it holds that $L = n_{\text{cl}}^{-1/3}$. With Eq. 5.11, this can be expressed as

$$L(r) = L_0 \sqrt[3]{r^2 w(r)}, \quad (5.12)$$

where L_0 is the second free parameter of the model, representing the typical clump separation in units of the stellar radius, and $w(r) = v_r/v_\infty$ is the velocity in units of

the terminal speed. As in Eq. 5.12, the clump radius can be written as

$$l(r) = l_0 \sqrt[3]{r^2 w(r)}, \quad (5.13)$$

where

$$l_0 = L_0 \sqrt[3]{\frac{3}{4\pi D}} \quad (5.14)$$

follows from Eqs. 5.10 and 5.12. With this procedure, clumps are created whose size varies with the radial distance (see Fig. 5.2).

The volume filling factor f_V is defined as the ratio of the total volume that clumps occupy (V_{cl}) to the whole volume V_w (see Eq. 3.1) in which the clumps are distributed (between r_{cl} and r_{max}),

$$f_V = \frac{V_{\text{cl}}}{V_w} = \frac{\sum_{i=1}^{N_{\text{cl}}} V_{\text{cli}}}{\frac{4\pi}{3}(r_{\text{max}}^3 - r_{\text{cl}}^3)}, \quad (5.15)$$

where $V_{\text{cli}} = (4\pi/3)l_i^3$ is the volume of i -th clump and N_{cl} is the total number of the clumps.

The total mass of the wind can be defined as

$$m_w = \int_{V_w} \rho_{\text{sw}}(r) dV = \langle \rho_{\text{sw}} \rangle V_w. \quad (5.16)$$

For the case of a void inter-clump medium, the entire mass of the wind in the clumps can be written as (using Eq. 5.9)

$$m_w = \sum_{i=1}^{N_{\text{cl}}} m_{\text{cli}} = \sum_{i=1}^{N_{\text{cl}}} \rho_{\text{cli}} V_{\text{cli}} = D \sum_{i=1}^{N_{\text{cl}}} \rho_{\text{sw}}^i V_{\text{cli}} = D \langle \rho_{\text{sw}} \rangle \sum_{i=1}^{N_{\text{cl}}} V_{\text{cli}}, \quad (5.17)$$

where m_{cli} and ρ_{cli} are the mass and density of the i -th clump, respectively, and ρ_{sw}^i is the value of the smooth wind density at the location of the i -th clump. If the inter-clump is void,

$$f_V = \frac{1}{D}, \quad (5.18)$$

consistent with Eq. 3.2.

Depth dependent clumping factor In the case of a depth dependent clumping factor, Eq. 5.9 can be generalized by allowing the clumping factor $D(r)$ to be depth dependent,

$$\rho_{\text{cl}}(r) = D(r)\rho_{\text{sw}}(r). \quad (5.19)$$

A simple picture for a varying clumping factor may be derived assuming that clumps, once created, do not change during their travel downstream in the wind. Conse-

quently, their volume and density ρ_{cl} do not depend on the radius r . Then, the clumping factor may be expressed as

$$D(r) = \frac{\rho_{\text{cl}}}{\rho_{\text{sw}}(r)}, \quad (5.20)$$

where ρ_{cl} gives the density inside clumps at the inner radius r_{min} (this is a reference density inside clumps),

$$\rho_{\text{cl}} = D(r_{\text{min}})\rho_{\text{sw}}(r_{\text{min}}). \quad (5.21)$$

Combining Eq. 5.20 and Eq. 5.21, the depth variable clumping factor is defined as

$$D(r) = D(r_{\text{min}}) \frac{\rho_{\text{sw}}(r_{\text{min}})}{\rho_{\text{sw}}(r)}. \quad (5.22)$$

Since the line opacity $\chi(r)$ is proportional to ρ_{sw} (see Eq. 3.2), the depth dependent clumping factor can be written as

$$D(r) = D(r_{\text{min}}) \frac{r^2 v(r)}{r_{\text{min}}^2 v(r_{\text{min}})}, \quad (5.23)$$

The opacity inside a clump is

$$\chi_{\text{cl}}(r) = D(r)\chi(r), \quad (5.24)$$

and according to Eq. 5.7,

$$\chi_{\text{cl}} = \frac{\chi_0}{r_{\text{min}}^2 \frac{v(r_{\text{min}})}{v_{\infty}}} \phi(x), \quad (5.25)$$

which does not depend on radius r . Practically, this means that the clumps in this model can be distributed as in the case of the depth independent clumping factor, and after their distribution the radii of all clumps are set to the value at the reference depth r_{min} ,

$$l(r_{\text{min}}) = l_0 \sqrt[3]{r_{\text{min}}^2 \frac{v(r_{\text{min}})}{v_{\infty}}}. \quad (5.26)$$

5.4.2 Non-void interclump medium

As with the density inside clumps, which is defined by the clumping factor D , the inter-clump medium density ρ_{ic} can be defined by the inter-clump factor d . ρ_{ic} is assumed to be lower than the smooth wind density ρ_{sw} (the density of the wind without any clumping) by d (the third free parameter of the model), which can be assumed to be either constant or radius dependent.

Depth independent clumping factor In the case of a depth independent clumping factor,

$$\rho_{\text{ic}}(r) = d \rho_{\text{sw}}(r); \quad 0 \leq d < 1. \quad (5.27)$$

For a non-void inter-clump, the total mass of the wind is distributed between clumps and inter-clump. In this case, the total mass of the wind $m_{\text{w}} = \langle \rho_{\text{sw}} \rangle V_{\text{w}}$ can be expressed as

$$m_{\text{w}} = m_{\text{ic}} + \sum_{i=1}^{N_{\text{cl}}} m_{\text{cli}} = d \langle \rho_{\text{sw}} \rangle (V_{\text{w}} - V_{\text{cl}}) + D \langle \rho_{\text{sw}} \rangle \sum_{i=1}^{N_{\text{cl}}} V_{\text{cli}}. \quad (5.28)$$

From Eqs. (5.15) and (5.28), it then follows that

$$f_V = \frac{1-d}{D-d}. \quad (5.29)$$

Depth dependent clumping factor For constant clump properties and non-zero inter-clump density, it is natural to assume that the inter-clump density changes proportionally to the smooth wind density, i.e. that the inter-clump factor is constant in the wind and $\rho_{\text{ic}}(r)$ obeys a relation similar to Eq. 5.9,

$$\rho_{\text{ic}}(r) = \tilde{d} \rho_{\text{sw}}(r) \quad (5.30)$$

where $\tilde{d}$ is the depth-independent inter-clump factor taking into account a different total volume of clumps if all clumps have the radius given by Eq. 5.26. Let V_{ic} represent the “original” total inter-clump volume for the case of constant clumping factor D . Then the total mass of the inter-clump medium is thus

$$m_{\text{ic}} = \tilde{d} \rho_{\text{sw}} V_{\text{ic}}, \quad (5.31)$$

where ρ_{sw} is the mean density of the smooth wind. Similarly, the total mass of the inter-clump medium is

$$m_{\text{ic}} = \tilde{d} \rho_{\text{sw}} (V_{\text{ic}} + \Delta V), \quad (5.32)$$

where ΔV is the volume freed by lowering the clump radius. Combining Eq. 5.31 and Eq. 5.32 we obtain

$$\tilde{d} = d \frac{V_{\text{ic}}}{V_{\text{ic}} + \Delta V}. \quad (5.33)$$

The volume ΔV can be easily calculated, and d is then used as an inter-clump medium factor (see eq. 5.27).

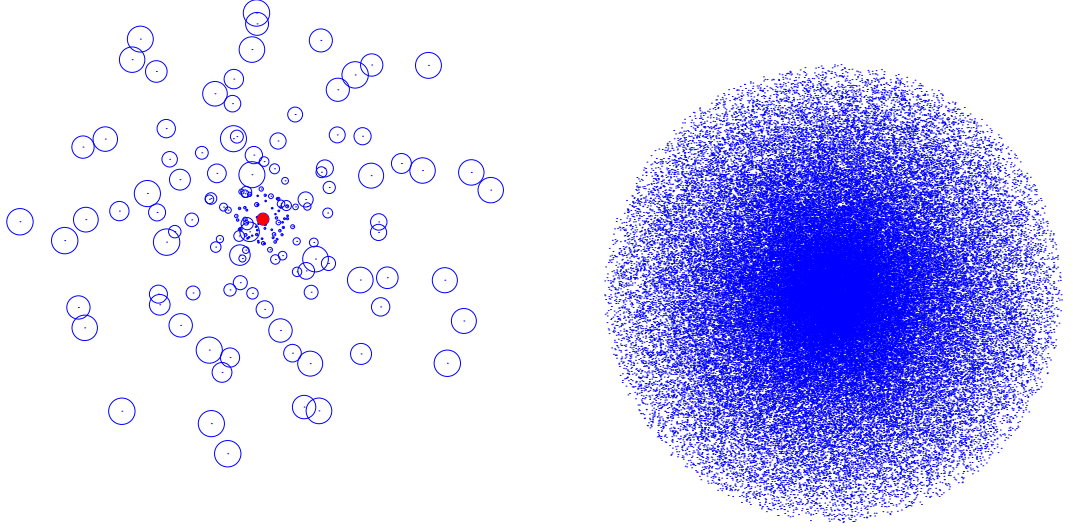


FIGURE 5.2: Two-dimensional projection of an example of a realization of our stochastic 3-D inhomogeneous wind model. **Left:** Clumps (blue circles) with the different size distributed around the star (red filled circle). **Right:** Distribution of the clumps; the filled red sphere at the center represents the star, while the blue dots represent the positions of the clump centers.

5.4.3 Clump distribution

The distance r_i of the i -th clump from the stellar center is chosen randomly by

$$r_i = (r_{\max} - r_{\text{cl}})\xi_i + r_{\text{cl}} \quad i = 1, N_{\text{cl}}, \quad (5.34)$$

where ξ_i is a random number uniformly distributed between 0 and 1. The clump positions are determined by employing the von Neumann rejection method (Press et al., 1992), described in Chapter 4.4.1, setting the probability density distribution function to the inverse of the β -velocity law (see Eq. 5.4), $1/v_r$. This reflects the equation of continuity for the number density of clumps. Consequently, more clumps are concentrated close to the star (see Fig. 5.2). Clumps are distributed uniformly and the position vector $\mathbf{r}_i$ of i -th clump is defined as

$$\mathbf{r}_i = r_i \sin \theta_i \cos \varphi_i \mathbf{e}_x + r_i \sin \theta_i \sin \varphi_i \mathbf{e}_y + r_i \cos \theta_i \mathbf{e}_z, \quad (5.35)$$

where θ_i and φ_i are randomly chosen with $\mu_i = \cos \theta_i = (2\xi_{i1} - 1)$ and $\varphi_i = 2\pi\xi_{i2}$ where ξ_{i1} and ξ_{i2} are two different random numbers (see Eq. 4.41), and $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are

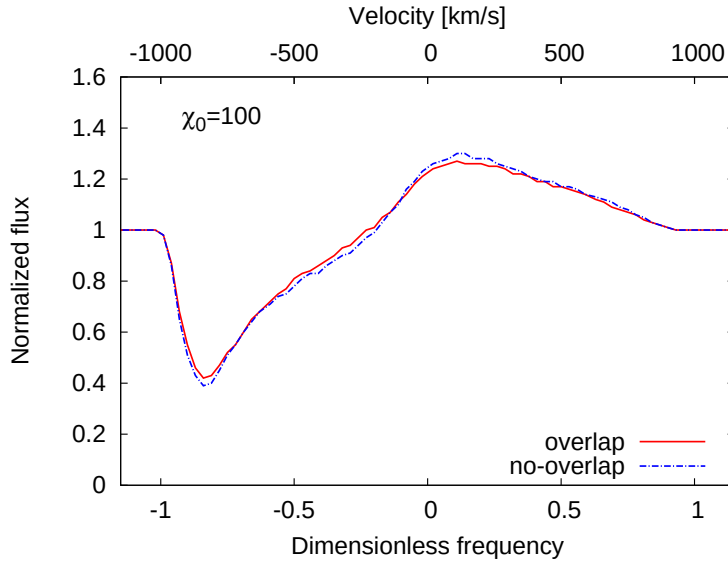


FIGURE 5.3: Line profiles calculated for the case of clump overlap (the dashed dotted red line) and no clump overlap (the full blue line). Model parameters (see Section 6) are $r_{\max} = 80$, $v_D = 50$, $v_{\infty} = 1000$, $v_{\min} = 10$, $\beta = 1$, and $\kappa_0 = 100$.

the unit vectors in Cartesian coordinates. For a given r_i , the radius of each clump l_i is determined according to Eq. 5.13. Clumps are either allowed or forbidden to overlap.

To avoid overlapping clumps, the clump-center coordinates are compared with all other clumps existing so far (using Pythagorean theorem) at the beginning when the clumps are set up and after a new clump is created. If any existing clump is closer to any other clump than the sum of both clump radii, the last clump is rejected and a new attempt is made. If clump overlapping is allowed, this rejection procedure is not applied. Differences in the line profiles calculated with and without clump overlapping are small (see Fig. 5.3).

To determine the total number of clumps and to fulfil the condition of mass conservation inside the wind, we create clumps one at a time, and when the total volume of created clumps, reduced by the volume filling factor, is higher than the total volume of the wind, i.e. if $V_{\text{cl}}/f_V > V_w$, then the number of created clumps represents the total number of clumps in our snapshot clump distribution.

If the process of random generation of clump positions using Eq. 5.34 and radii using Eq. 5.13 allows clump overlapping, $D = 1$ does not reproduce the smooth wind, as the density of overlapping clump parts is equal to the sum of the densities of overlapping clumps. Consequently, holes in the density structure are created, so that the medium is clumped even for $D = 1$. This contradicts the intuitive view that $D = 1$ means a smooth wind, and we have to understand the definition of f_V in a

generalized manner, which follows from the method of creating clump distribution.

5.4.4 Inhomogeneous velocity in clumps

In the wind model, the velocity of the smooth and the inter-clump medium is assumed to be monotonic as indicated by Eq. 5.4. However, the velocity inside the clumps is allowed to deviate from the monotonic wind, and a negative velocity gradient can consequently appear (see more in the Sect. 5.5). This assumption is based on the predictions of hydrodynamic wind simulations (e.g. Owocki et al., 1988; Feldmeier et al., 1997; Runacres and Owocki, 2002), which show that over-dense regions inside the wind are slower and the interclump medium has approximately the same velocity as the smooth wind.

The velocity inside the i -th clump can be approximated as

$$v(r) = v_\beta(r_i^c) - v_{\text{dis}}(r) \frac{r - r_i^c}{l_i}, \quad (5.36)$$

where r_i^c is the absolute position of the center of the i -th clump, $v_\beta(r_i^c)$ is the velocity determined according to Eq. 5.4 at the position r_i^c , and the velocity dispersion is $v_{\text{dis}}(r) = m v_\beta(r)$, where m ($0 < m \leq 1$) is the velocity deviation parameter (fourth free parameter of our model). Eq. 5.36 introduces a negative velocity gradient inside clumps, while the center of the clump has the same velocity as the smooth wind. The velocity structure of the model along one particular photon path is shown in Fig. 5.4.

5.5 Three-dimensional Monte Carlo radiative transfer code

To solve radiative transfer problem in inhomogeneous medium, a 3-D MCRT code has been developed, which can calculate resonance line profiles. The model does not use Sobolev approximation, and adopts the comoving frame method.

5.5.1 Creation of a photon

A photon is released from the lower boundary of the wind (surface of the star) and is followed through the wind until it reaches the outer boundary of the wind or is scattered back into the photosphere. The photons are uniformly distributed in $\cos \theta$ and φ over the whole surface area of the star. The position vector of the photon at the surface of the star ($r_{\text{min}} = 1$) is randomly chosen by $\mu_m = \cos \theta_m = (2\xi_{m1} - 1)$ and $\varphi_m = 2\pi\xi_{m2}$ (see Eq. 4.41; ξ_{m1} and ξ_{m2} are again two different random numbers), and it is defined by

$$\mathbf{r}(r_{\text{min}} = 1, \theta_m, \varphi_m) = \sin \theta_m \cos \varphi_m \mathbf{e}_x + \sin \theta_m \sin \varphi_m \mathbf{e}_y + \cos \theta_m \mathbf{e}_z, \quad (5.37)$$

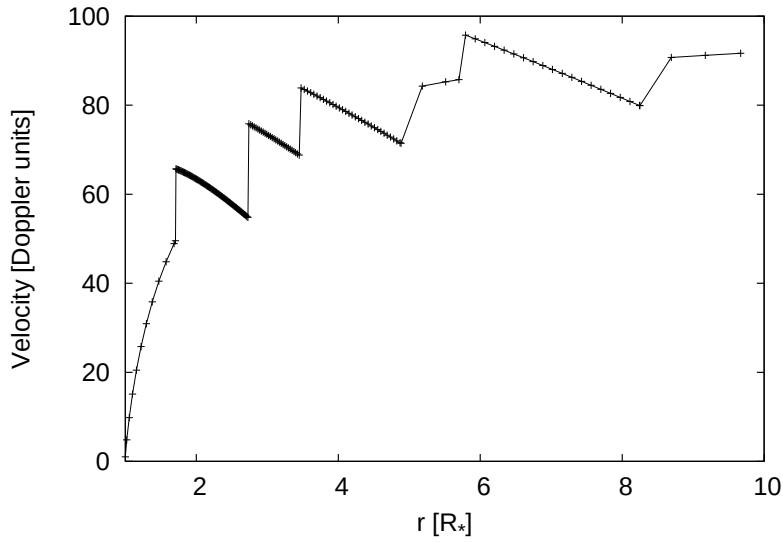


FIGURE 5.4: Velocity structure of the wind model along the considered photon path for the case of a non-monotonic velocity distribution inside clumps.

where θ_m and φ_m are the angles defining the position of the photon at the surface of the star with respect to the center of the star. From the lower boundary photons are released only upwards uniformly in φ and with a distribution function $\propto \mu d\mu$ in μ (see Chapter 4.4.2). The angular distribution function for photons emitted by the photosphere follows from the definition of flux (see, e.g., Lucy, 1983). Then the initial vector of photon propagation from the surface of the star is randomly chosen with $\mu_k = \cos \theta_k = \sqrt{\xi_{k1}}$ and $\varphi_k = 2\pi\xi_{k2}$ (see Eq. 4.35; (ξ_{k1} and ξ_{k2} are two different random numbers), and it is given by

$$\mathbf{k}(r_{\min} = 1, \theta_k, \varphi_k) = \sin \theta_k \cos \varphi_k \mathbf{e}_x + \sin \theta_k \sin \varphi_k \mathbf{e}_y + \cos \theta_k \mathbf{e}_z, \quad (5.38)$$

where θ_k and φ_k are the angles that define the direction of the photon propagation with respect to the center of the star.

In the MC calculations, frequencies of newly created photons are determined from the interval defined by the ratio v_∞/v_D (as in Hamann, 1981a). The initial frequencies of photons have values in the interval $\langle x_{\min}, x_{\max} \rangle$ where

$$x_{\min} = -\frac{v_\infty}{v_D} - x_{\text{band}}, \quad x_{\max} = \frac{v_\infty}{v_D} + x_{\text{band}}, \quad (5.39)$$

and x_{band} is the line width, assumed to be about 4.5 Doppler units (as in Mihalas et al., 1975).

To determine the initial photon frequency, a photon frequency is randomly selected from the frequency interval $\langle x_{\min}, x_{\max} \rangle$. However, to smooth the frequency

distribution of photons and to save computing time, we divide the whole frequency interval into N_{bin} subintervals of equal length, and release the same number of photons in each subinterval. We choose the photon frequency $x_n^{\text{obs}}(\mathbf{r}_{\text{min}})$ from the frequency interval of the n -th ($n = 1, \dots, N_{\text{bin}}$) bin, to be equal to the central frequency of the n -th bin (more photons with the same frequencies), and in the observer's frame it is given as

$$x_n^{\text{obs}}(\mathbf{r}_{\text{min}}) = (n - 0.5)x_{\text{bin}} + x_{\text{min}}, \quad (5.40)$$

where x_{bin} is the width of frequency bin (the same for all bins). Using this procedure we obtain smoother distribution of emergent radiation. To check to which extent this pre-defined set of frequencies influences finale results (and also to fulfil the wish of the referee of the paper Šurlan et al., 2012b), the frequencies of the photons were changed to a continuous random variable, i.e., the photon frequencies are randomly distributed within the bins rather than choosing a single frequency for each bin. In this case, each photon frequency $x_{n\xi}^{\text{obs}}(\mathbf{r}_{\text{min}})$, is randomly chosen from the frequency interval (producing more photons with different frequencies), and in the observer's frame is given as

$$x_{n\xi}^{\text{obs}}(\mathbf{r}_{\text{min}}) = (n - \xi)x_{\text{bin}} + x_{\text{min}}, \quad (5.41)$$

where $x_{\text{bin}} = (x_{\text{max}} - x_{\text{min}}) / N_{\text{bin}}$ is the width of the frequency bin (the same for all bins) and ξ is a random number.

Test calculations (see Fig. 5.5) show negligible difference in the line profiles calculated by releasing all photons from the midpoint (the blue dashed-dotted line in Fig. 5.5) of the frequency bins, or if the photon frequencies are set randomly “inside” those bins (the red full line in Fig. 5.5). Differences are due to the MC noise.

After a photon obtains its initial frequency, then the frequency is transformed to the comoving-frame frequency x_n^{cmf} according to Eq. 5.2 at the lower boundary by

$$x_n^{\text{cmf}}(\mathbf{r}_{\text{min}}) = x_n^{\text{obs}}(\mathbf{r}_{\text{min}}) - \mathbf{k}(\mathbf{r}_{\text{min}}) \cdot \mathbf{v}(\mathbf{r}_{\text{min}}) \quad (5.42)$$

where $\mathbf{k}(\mathbf{r}_{\text{min}})$ is the initial vector of photon propagation (given by Eq. 5.38), at some particular position on the surface of the star defined by the vector $\mathbf{r}_{\text{min}}$ (given by Eq. 5.37). $\mathbf{v}(\mathbf{r}_{\text{min}})$ is the initial velocity vector at $\mathbf{r}_{\text{min}}$. Note that $|\mathbf{r}_{\text{min}}| = r_{\text{min}}$.

5.5.2 Optical depth calculation

After its creation, each photon obtains information on how far it is allowed to travel before it undergoes interaction with some particle. Via a randomly chosen optical depth $\tau_\xi = -\ln \xi$ (see Eq. 4.37). The actual optical depth, which a photon passes on its travel, is calculated by summing each opacity contribution along its path.

To account for all inhomogeneities inside the wind in both velocity and density, adaptive integration steps are used. The photon path is defined by the vector of photon propagation $\mathbf{k}$, and the photon is followed along its path by stepwise adding a variable integration step Δr . If the current photon position is $\mathbf{r}_1$, then the vector

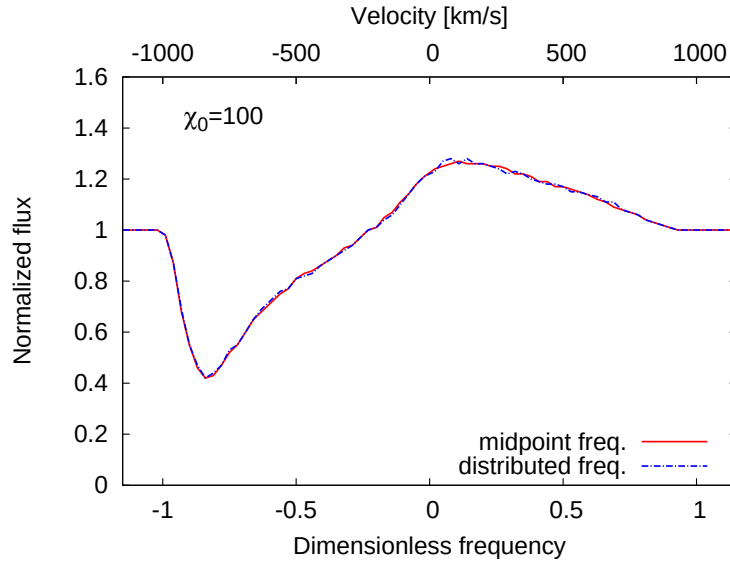


FIGURE 5.5: Line profiles calculated by releasing all photons with the frequency randomly distributed within the bins (the blue dashed-dotted line) or with the frequencies chosen at midpoints of the frequency bins (the red full line). Model parameters (see Section 6) are $L_0 = 0.5$, $D = 10$, $\kappa_0 = 100$, $r_{\max} = 80$, $v_D = 50$, $v_\infty = 1000$, $v_{\min} = 10$, and $\beta = 1$.

of the next photon position $\mathbf{r}_2$ is (see Fig. 5.1)

$$\mathbf{r}_2 = \mathbf{r}_1 + \Delta r \mathbf{k}. \quad (5.43)$$

After each integration step the coordinates of the current photon position are recalculated. When the photon is followed through the smooth part of the wind, Δr is set to be greater than 0.1 of the difference between the current photon position and surface of the star, and smaller than the velocity scale length $v(r)/v'$, i.e.,

$$0.1(r_1 - r_{\min}) \leq \Delta r \leq v(r)/v', \quad (5.44)$$

where v' is the velocity derivative (see Eq. 5.6). When the photon is in the smooth region very close to the star, the difference $(r_1 - r_{\min})$ is very small. So as not to have very small integration steps, we choose $\Delta r = \max[10^{-3}, 0.1(r_1 - r_{\min})]$. When the photon is close to the star, Δr is chosen to be equal to 10^{-3} . When the photon is far from the star, Δr is larger. To ensure that the photon does not miss any large changes in the velocity field, $\Delta r = \min[0.1(r_1 - r_{\min}), v(r)/v']$. Finally, condition for choosing integration step for the smooth part of the wind can be written as

$$\Delta r = \min\{\max[10^{-3}, 0.1(r_1 - r_{\min})], v(r)/v'\}. \quad (5.45)$$

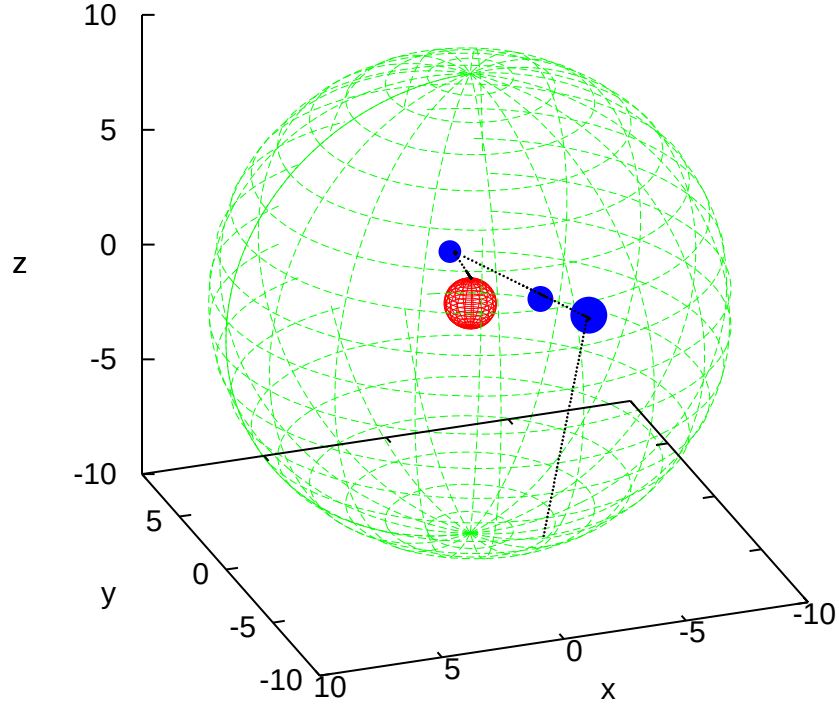


FIGURE 5.6: The path of one particular photon inside a realization of our clumped wind with the adaptive integration step. The bigger green sphere represents the outer boundary, while the red sphere in the center represents the lower boundary of the wind. Smaller blue balls represent the clumps and black dots denote integration steps.

When the photon is inside the interclump medium,

$$\Delta r = 0.3 l(r_1), \quad (5.46)$$

where $l(r_1)$ is the radius of the clump at r_1 (from Eq. 5.13).

For the photon path followed inside the clump, Δr is chosen larger than $0.1 l(r_1)$ and smaller than $0.3 / v'$, i.e.,

$$0.1 l(r_1) \leq \Delta r \leq 0.3 / v'. \quad (5.47)$$

Then, Δr is chosen as a minimum of $0.1 l(r_1)$ and $0.3 / v'$ in certain position of the photon. This adaptive integration step technique ensures that opacity calculation along the photon path is done properly without any possibility of skipping any inhomogeneity inside the wind and that it is fast enough. The illustration of the adaptive integration step is shown in the Fig. 5.6.

For the optical depth calculation, it is necessary to calculate the opacity. Ac-

According to Eq. 5.7, for the opacity calculation at the current position requires $v(r)$ and the Doppler shifted frequency of the photon at every position of the integration process. When the photon is traced inside the smooth or inter-clump regions, the velocity is calculated according to Eq. 5.4. When the photon is traced inside the i -th clump, the velocity is modified assuming that the velocity decreases there and that there is some small deviation from the monotonic velocity described with a help of a free parameter v_{dis} (see Sect. 5.4.4). The wind velocity at the point $\mathbf{r}_1$ according to Eq. 5.36 is then

$$v_r(r_1) = v_r(r_i^c) - v_{\text{dis}} \frac{r_1 - r_i^c}{l_i} \quad (5.48)$$

In the current version of the model, the photon can only experience resonance line scattering. Along the photon path, the Doppler shifted comoving-frame frequency of the photon is calculated as

$$x^{\text{cmf}}(\mathbf{r}_2) = x^{\text{cmf}}(\mathbf{r}_1) - \mathbf{k} \cdot (\mathbf{v}(\mathbf{r}_2) - \mathbf{v}(\mathbf{r}_1)), \quad (5.49)$$

where $x^{\text{cmf}}(\mathbf{r}_1)$ and $x^{\text{cmf}}(\mathbf{r}_2)$ are the comoving-frame frequency of the photon at the starting and at the final position of the integration step, respectively, and $\mathbf{v}(\mathbf{r}_1)$ and $\mathbf{v}(\mathbf{r}_2)$ are velocity vectors of the wind at those positions.

The optical depth between $\mathbf{r}_1$ and $\mathbf{r}_2$ for the actual integration step j is

$$\tau_j = \int_{r_1}^{r_2} \chi(r) \, ds \quad (5.50)$$

where s is taken along the path from $\mathbf{r}_1$ to $\mathbf{r}_2$ and is calculated using the trapezoidal rule. Along the entire path, the optical depth is accumulated ($\tau = \sum_j^J \tau_j$, where J is the total number of integration steps before a scattering event happens), and when $\tau \geq \tau_\xi$, then the condition for the photon interaction is fulfilled.

In the case of clump overlap, the opacities of overlapped clumps have to be properly accounted for. If the photon, once it leaves a clump, is inside another clump, the next integration step is set to start at the border of the new clump (i.e. the photon is moved back to the border of the entering clump) and the opacity of the new clump is also accounted for. In such way, it is ensured that the opacities of overlapped clumps are properly taken into account.

To check whether the photon is inside any clump and in which one, the following procedure is implemented. A cube with its centre at the clump centre and with sides equal to two clump radii is drawn. By a separate comparison of individual coordinates, it is checked if the photon is inside this cube. Such test defines the cube in the space, so additional test over distance to the clump centre is made. In the case that the photon is already in the clump, then first it is checked if the photon is still there. If not then the same procedure described above is again repeated. This procedure precisely defines in which clump the photon is and consequently to accurately accounts for the opacity of a particular clump.

5.5.3 Scattering events

In the winds of hot stars, different atomic processes occur, such as bound-free (bf), free-free (ff), bound-bound (bb) transitions and scattering of photons by free (Thomson limit valid for UV/optical radiation) or bound (Rayleigh scattering) electrons. All of these processes may contribute to the total opacity and emissivity. Thus, the total opacity and emissivity for a given frequency ν is a sum over the individual opacities of all contributing processes (see Section 4.5 for the case of electron and resonance line scattering).

Each time the condition for photon interaction is fulfilled, a random number is generated. Which kind of process occurs is determined according to the cross-section, which defines the probability that a particular process can happen. In this work, only resonance-line scattering is taken into account, so this step may be skipped.

After scattering, the photon obtains a new direction and a new optical depth τ_ξ , chosen randomly for the case of isotropic scattering (see Eq. 4.41) and a new optical depth τ_ξ is randomly chosen again. For determination of θ , ϕ , and τ_ξ , three different random numbers are used. Assuming complete redistribution (which is adequate, as discussed by Mihalas et al., 1976 and Lamers and Castor, 1979), the photon frequency after scattering is calculated randomly from Gaussian distribution with a mean of zero and the standard deviation equal to unity (Box and Muller, 1958).

5.5.4 Binning

When exiting the atmosphere each photon may have a different frequency from the other photons. To reproduce the emergent flux from the wind, all photons that reach the edge of the wind are collected and put into a proper frequency bin. More precisely, if $|\mathbf{r}_2| > r_{\max}$, then the photon frequency is recalculated to the observer's frame frequency (see Eq. 5.3) by

$$x^{\text{obs}} = x^{\text{cmf}} + \mathbf{k} \cdot \mathbf{v}. \quad (5.51)$$

The frequency x^{obs} is checked to determine which frequency bin of the N_{bin} bins it belongs (here N_{bin} is not necessarily the same as in Section 5.5.1), and is added into the proper frequency bin. The number of bins must be high enough to obtain satisfactory resolution of emergent profiles. If the photon is scattered back to the stellar surface (i.e. if $|\mathbf{r}_2| \leq r_{\min}$), then the photon is not counted and a new photon is released.

If the photon is out of the frequency interval $\langle x_{\min}, x_{\max} \rangle$, it is also not counted. However, if the frequency interval $\langle x_{\min}, x_{\max} \rangle$ is broader than the width of the line, this does not happen.

In this thesis, the emergent photons irrespective of their direction are counted, while the distant observer sees the 3-D clump configuration from one specific direction. By averaging over all directions, a mean emergent profile is effectively obtained,

similar to what would be get by averaging over many different random clump configurations seen from one specific direction.

However, this is only one special case. The photon binning can be done also in angle, i.e. we can count the emergent photons only from one specific direction. In this case, photons are placed into (μ, ϕ) bins of equal solid angle $d\Omega = \sin \theta d\theta d\phi = d\mu d\phi$. The number of angular bins is chosen depending on the symmetry of the radiation source and the medium where radiation interacts. For satisfactory resolution, the number of bins also has to be enough to resolve the smallest variation within solid angle. To obtain the emergent radiation from the top of the photosphere in the spherically symmetric case, the photons are binned in the range $0 < \mu < 1$ and $0 < \phi < 2\pi$.

5.6 Test calculations

To test the newly developed MCRT code, the results of the calculations were compared with results from the MCRT code of Sundqvist et al. (2010) (this comparison was done during Jon Sundqvist's visit in Ondřejov, 2010).

First, resonance line profiles obtained from the codes for the smooth wind case with monotonic velocity field and different values of parameter β were compared. Wind opacities were calculated using the parametrization (see Eq. 5.7) originally based on Hamann (1981a). As can be seen in Fig. 5.7 and Fig. 5.8, a very good agreement between calculated line profiles for $\beta = 0.8$ and $\beta = 2.0$, and for different values of κ_0 and v_D are achieved (the opacity parameter and the Doppler-broadening velocity, respectively – see Section 5.3.2).

To make additional tests, the MCRT code described in this thesis is developed to use given density and velocity structures of the wind. Using the density and velocity structures of a snapshot from a one-dimensional (1-D) time-dependent, radiation-hydrodynamic wind instability simulation computed by Feldmeier et al. (1997), resonance line profiles calculated by our codes are again compared. Results from a selected subset of models for the smooth and structured wind show almost perfect agreement between codes, see Fig. 5.9.

Comparing two independent codes producing nearly identical results ensures that both codes work correctly.

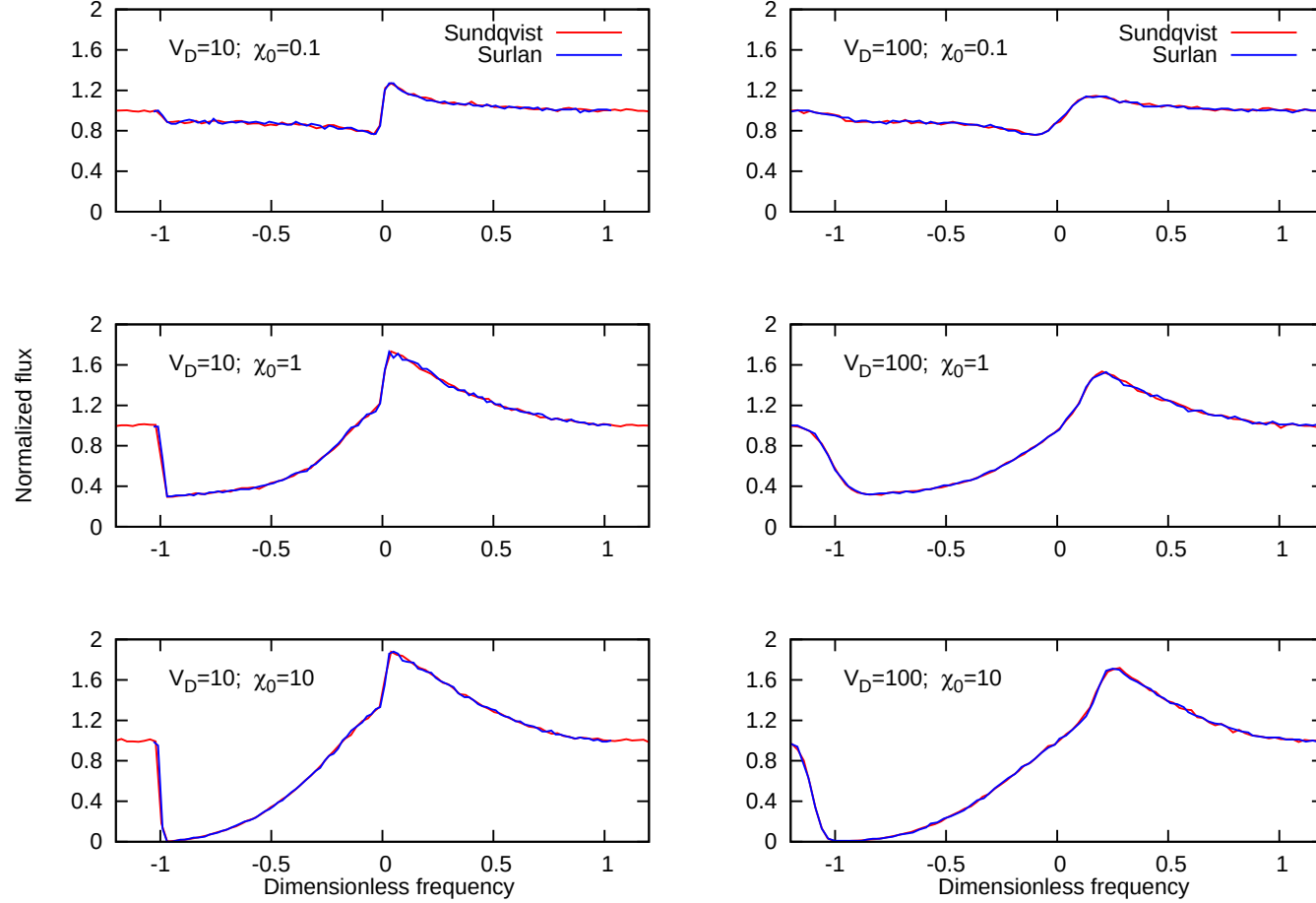


FIGURE 5.7: Comparison between weak (upper panels), intermediate (middle panels) and strong (lower panels) resonance line profiles calculation by Sundqvist (red lines) and Surlan (blue lines) MCRT codes for smooth wind case with $v_\infty = 1000 \text{ km s}^{-1}$, $v_{\min} = 10 \text{ km s}^{-1}$, and $\beta = 0.8$. The velocities v_D and opacity parameters κ_0 are indicated in the panels.

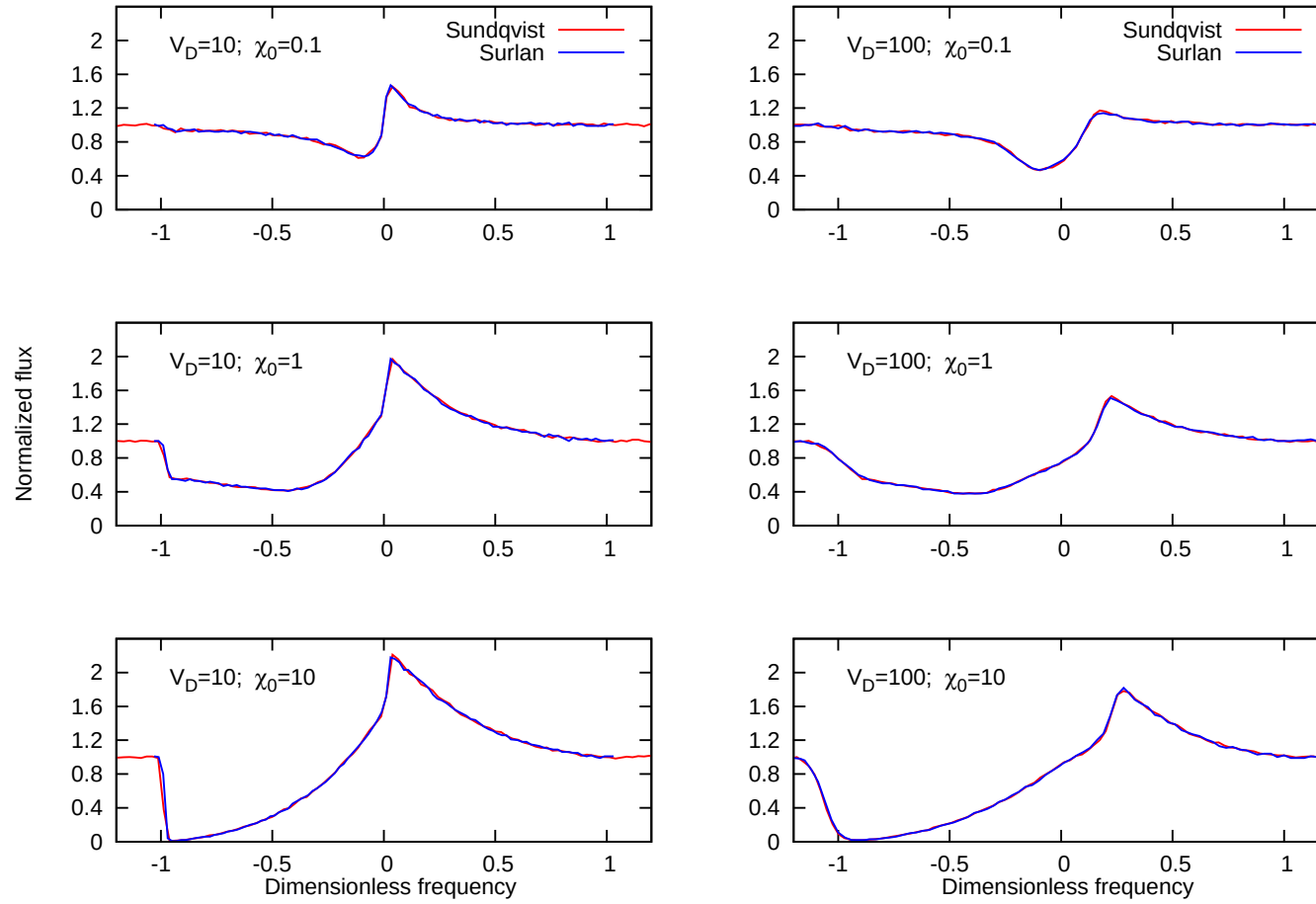


FIGURE 5.8: Comparison between weak (upper panels), intermediate (middle panels) and strong (lower panels) resonance line profiles calculated by Sundqvist (red lines) and Šurlan (blue lines) MCRT codes for smooth wind case with $v_\infty = 1000 \text{ km s}^{-1}$, $v_{\min} = 10 \text{ km s}^{-1}$, and $\beta = 2$. The velocities v_D and opacity parameters κ_0 are indicated in the panels.

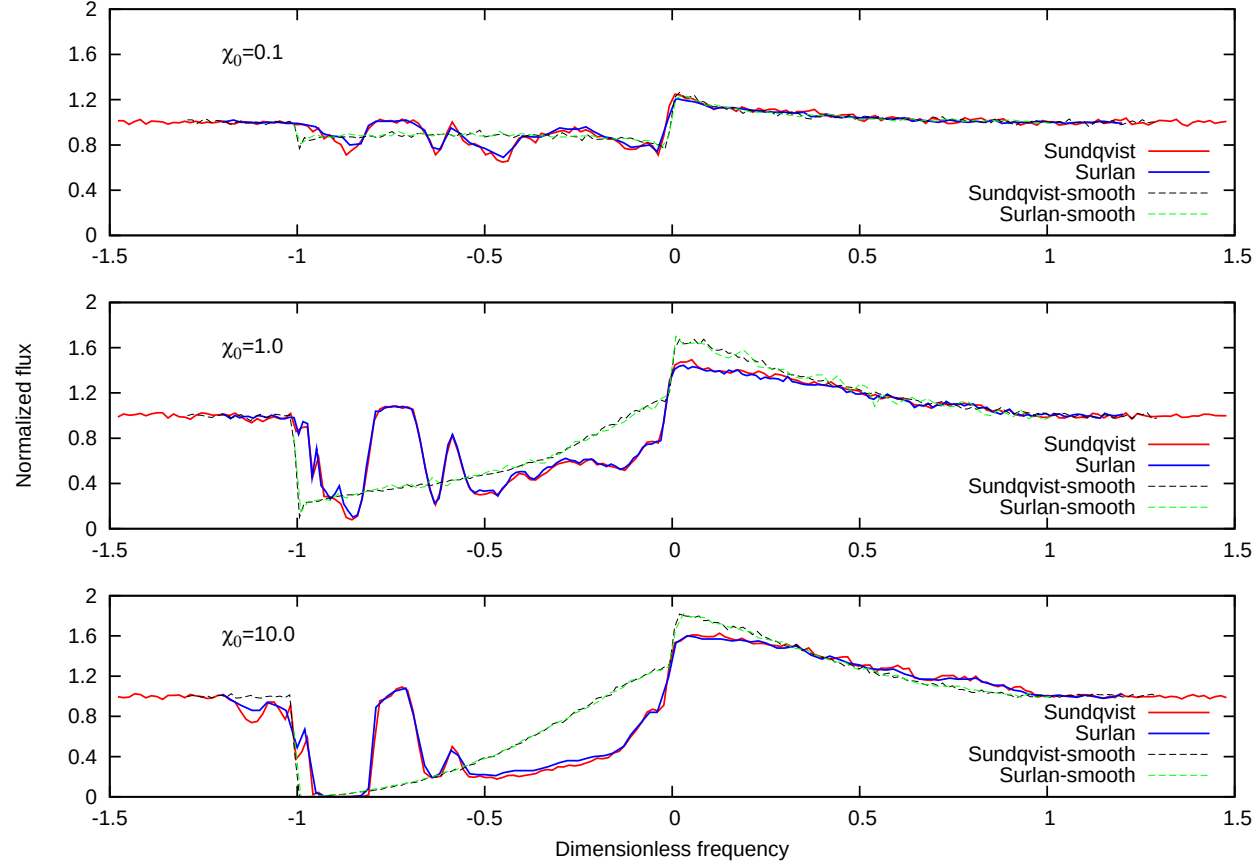


FIGURE 5.9: Comparison between weak (upper panel), intermediate (middle panel) and strong (lower panel) resonance line profiles calculated by Sundqvist (black lines for smooth wind case and red lines for clumped wind) and Šurlan (green lines for smooth wind case and blue lines for clumped wind) MCRT codes with using the density and velocity structures of a snapshot from a 1-D time-dependent, radiation-hydrodynamic wind instability simulation computed by Feldmeier et al. (1997). $v_\infty = 2200 \text{ km s}^{-1}$, $v_{\min} = 10 \text{ km s}^{-1}$, and $v_D = 11 \text{ km s}^{-1}$. The opacity parameters κ_0 are indicated in the panels.

Chapter 6

Resonance Line Calculations

In this Chapter, results of the resonance line calculation using the 3-D MCRT code for inhomogeneous hot star winds are presented.

6.1 Results of the model calculation

The basic effects on line profiles of both single resonance lines and resonance doublets are investigated by varying different model parameters. Depending on the effect that is wished to study, some parameters are kept fixed, while others are varied within considered ranges. The basic model parameters (see Table 6.1) are chosen to represent a typical O-type star. The varied model parameters (see Table 6.2) have their default values except for the one whose effect is aimed to be studied. The impact on the line profiles is shown for weak ($\chi_0 = 1$), intermediate ($\chi_0 = 10$), and strong ($\chi_0 = 100$) lines. If the variation in some model parameter has a similar effect on all three types of lines, the effect is shown for only the strong line case.

Each of the profiles shown in this thesis were calculated for one random configuration of clumps, corresponding to an observation with a short exposure time relative to the dynamical time-scale. Additionally, the line profiles represent mean emergent profiles (see Section 5.5.4). All calculations are performed using 10^5 photons distributed over 100 frequency bins, resulting in a signal-to-noise ratio of about 30 per bin due to Poisson statistics.

6.1.1 The effects of the macroclumping

Since microclumping is not generally justified for the line transfer in the wind, we consider only the macroclumping approximation. The total number of clumps in

Fixed model parameters	Value
Outer boundary of the wind	$r_{\max} = 80 R_*$
Beta parameter	$\beta = 1$
Velocity at the photosphere	$v_{\min} = 10 \text{ km s}^{-1}$
Terminal velocity	$v_{\infty} = 1000 \text{ km s}^{-1}$

Table 6.1: The fixed model parameters.

Varied model parameters	Considered range	Default value
Opacity parameter	$\chi_0 = 1, 10, 100$	100
Clump separation parameter	$L_0 = 1, 0.7, 0.5, 0.2 R_*$	0.5
Clumping factor	$D = 1, 5, 10$	10
Interclump medium density factor	$d = 0, 0.1, 0.5, 1$	0
Set-up of clumping	$r_{\text{cl}} = 1.01, 1, 1.3, 2 R_*$	1
Doppler velocity	$v_D = 10, 50, 100 \text{ km s}^{-1}$	50
Velocity deviation	$v_{\text{dis}}/v_{\beta} = 0, 0.1, 0.2, 1$	0

Table 6.2: The varied model parameters to study the influence of their variation on the resonance line profiles.

one snapshot of the clump distribution can be derived using the parameter L_0 (see Eq. 5.11 and Eq. 5.12) from

$$N_{\text{cl}} = \int_{r_{\text{cl}}}^{r_{\max}} n_{\text{cl}}(r) 4\pi r^2 dr. \quad (6.1)$$

For instance, if $0.2 \leq L_0 \leq 0.5$, then the total number of clumps within the clumped region is $10^5 \gtrsim N_{\text{cl}} \gtrsim 10^3$. A larger number of clumps (i.e smaller values of L_0) produces a less porous wind. If $L_0 \rightarrow 0$, the whole wind is filled with many little clumps and resembles a smooth wind.

In this subsection, all model parameters are set to their default values except for χ_0 and L_0 , which are varied as in Table 6.2. The main macroclumping effect on the line profile is the reduction in line strength relative to the smooth wind. In this case, where it is assumed that clumping starts from the surface of the star ($r_{\text{cl}} = 0$), the variation in the L_0 parameter has a strong influence on all three types of lines (see the left part of Fig. 6.1), as the clumps are optically thick for these lines.

According to Eqs. 5.13 and 5.14, the radius of the clumps depends on L_0 . For

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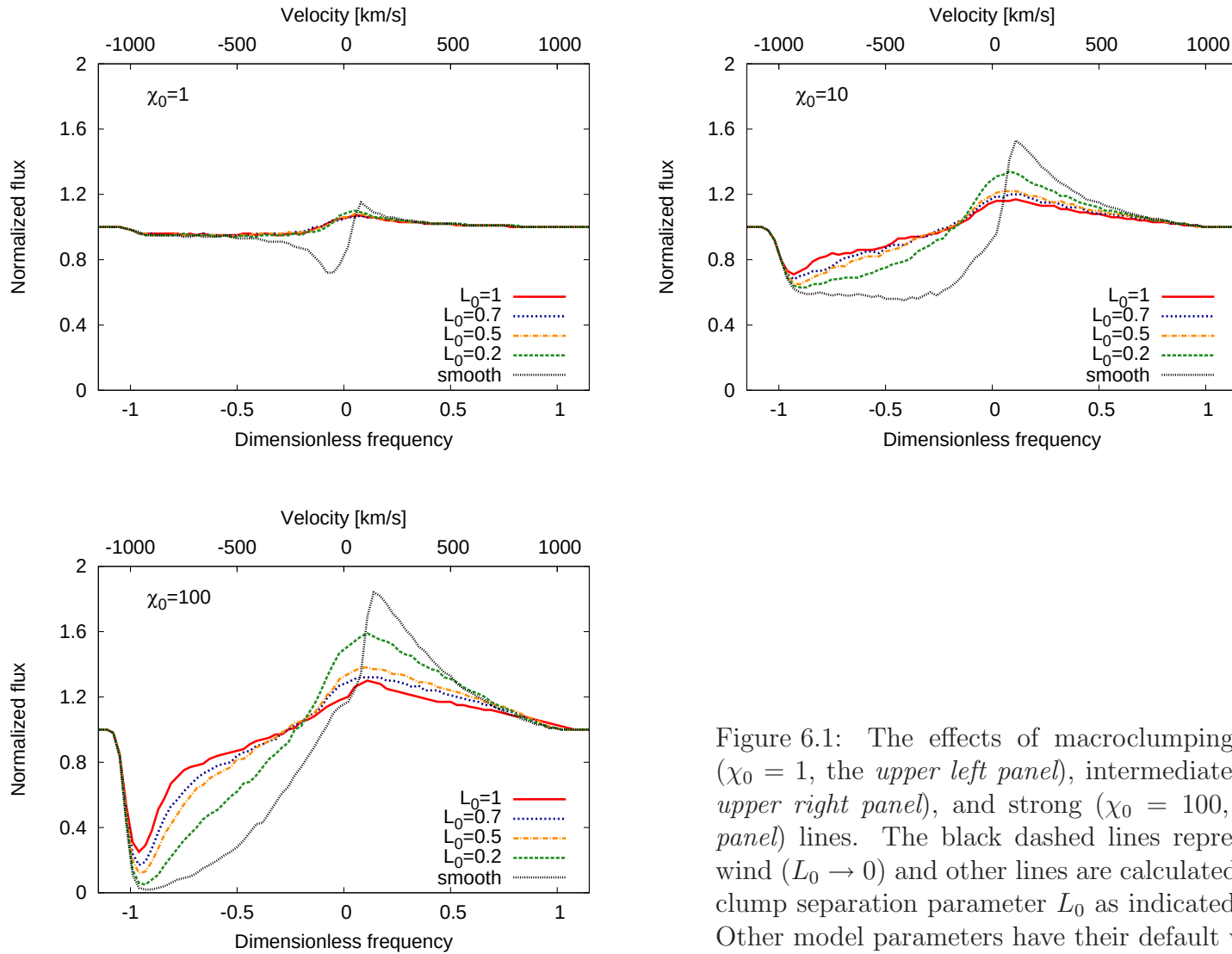


Figure 6.1: The effects of macroclumping on the weak ($\chi_0 = 1$, the *upper left panel*), intermediate ($\chi_0 = 10$, the *upper right panel*), and strong ($\chi_0 = 100$, the *lower left panel*) lines. The black dashed lines represent a smooth wind ($L_0 \rightarrow 0$) and other lines are calculated for a different clump separation parameter L_0 as indicated in the panels. Other model parameters have their default value.

very small L_0 , a huge number of clumps with small radii are created, and the clump contribution to the line opacity is almost the same as in the case of a smooth wind. There are not many “holes” between the clumps, and the photons cannot easily escape from the wind. However, for higher values of L_0 , fewer clumps with larger radii exist. Consequently, there are more “holes” in the wind, through which photons may freely propagate. This leads to lower absorption and weaker lines. Clumping lowers the effective opacity, to a lesser extent for the outer parts of the wind because the individual clumps become optically thin there, causing a bump in the blue part

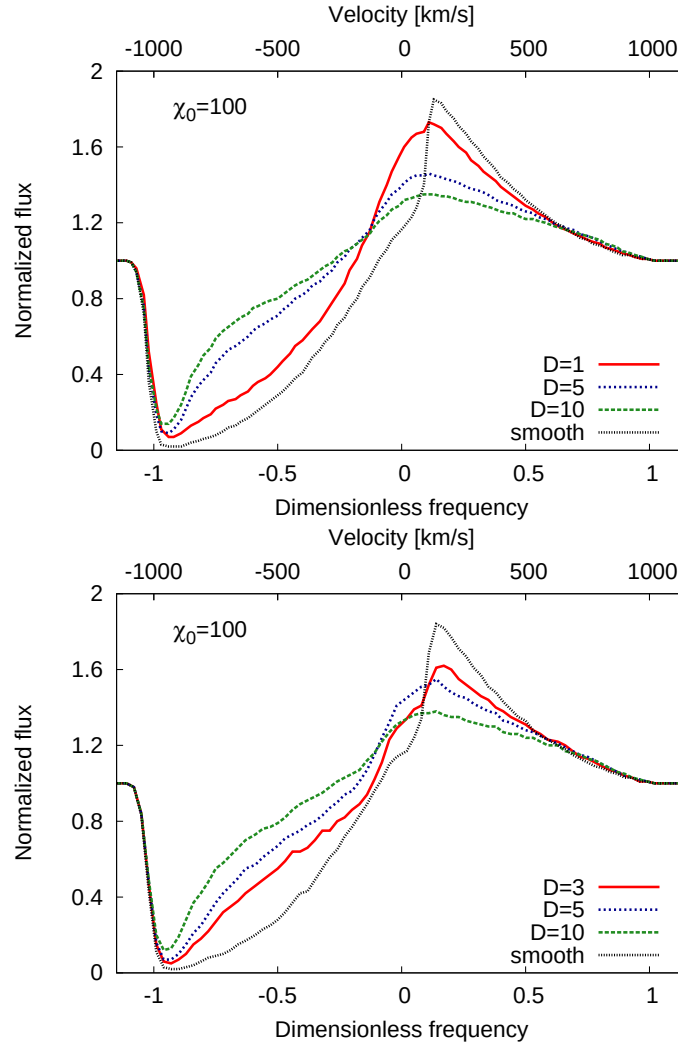


FIGURE 6.2: Effect of variation of the clumping factor D on the strong lines ($\chi_0 = 100$) in the case of overlapping clumps (upper panel) and non-overlapping clumps (lower panel). The model parameters have their default value (see Table 6.1 and Table 6.2).

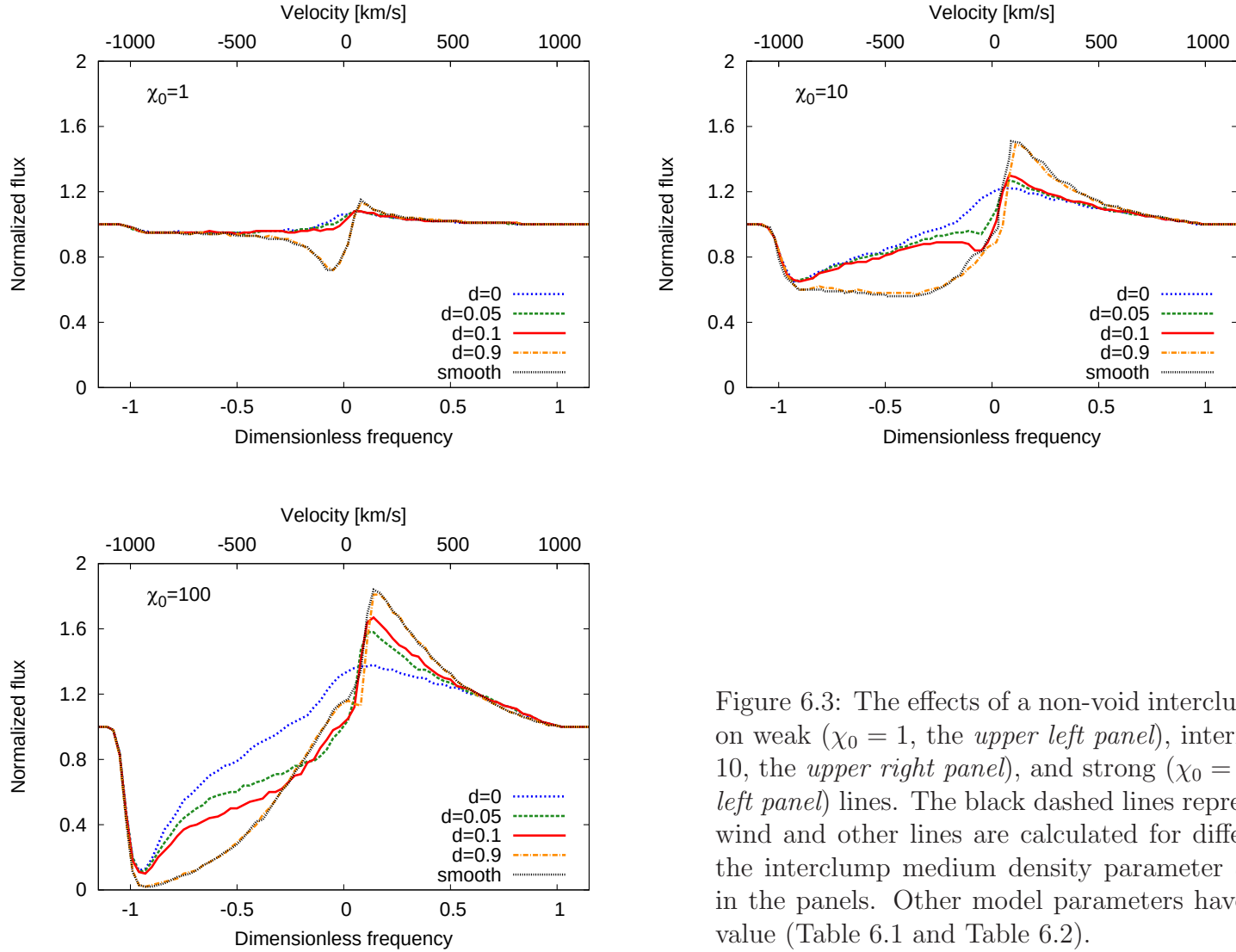


Figure 6.3: The effects of a non-void interclump medium on weak ($\chi_0 = 1$, the *upper left panel*), intermediate ($\chi_0 = 10$, the *upper right panel*), and strong ($\chi_0 = 100$, the *lower left panel*) lines. The black dashed lines represent a smooth wind and other lines are calculated for different values of the interclump medium density parameter d as indicated in the panels. Other model parameters have their default value (Table 6.1 and Table 6.2).

of the line. On the other hand, a significant part of the wind has approximately the same wind velocity as the wind terminal velocity. Consequently, despite the lower effective opacity there is still enough matter to absorb, the photon causing an absorption dip near v_∞ , especially for lower L_0 .

A similar effect of attenuation of the line strength can be obtained by varying the clumping factor D . In Fig. 6.2, the effect is shown only for the strong line ($\chi_0 = 100$), since the effects of varying the clumping factor D on line attenuation of weak and intermediate lines are very similar. The model parameters are again set to their default values except D . When enhancing the density within clumps relative to the smooth wind density (i.e. increasing the parameter D), the porosity effect is more pronounced and the line becomes weaker. When clump overlap is excluded, the largest filling factor that can be archived is the one for the densest set of spheres (on the order 0.7). The minimum value of D for which is possible to set distribution of clumps without overlapping in a reasonable amount of time is $D = 3$ (see lower panel of Fig. 6.2). Main reason for that is that clumps are allowed to be put only to free space, which, although in total quite large, may be formed of parts which are too small to allow additional clumps to fit there.

For the case of clump overlap, decreasing D to the minimum value ($D = 1$), produces a line profile close to the smooth one, but never the same. Even if the density inside the clumps is equal to the density of the smooth wind density, there are “holes” in density space and the opacity contribution to the line is smaller even if the density of the clumps is higher, see the upper panel of Fig. 6.2.

6.1.2 Effects of the non-void inter clump medium

In this section the changes caused by variations in the interclump medium density parameter d are investigated for different values of opacity parameter χ_0 , while other model parameters are set to their default values. The space between clumps is filled with matter of density defined by d (see Eq. 5.27). This matter fills the density “holes” between the clumps, and the photons are scattered there. This manifests itself as a strengthening of both the absorption and emission parts of the line profiles with respect to the model with void interclump medium (see Fig. 6.3). This effect is most pronounced for strong lines, where, even for a small d , the interclump medium contributes a lot to the absorption coefficient, making the lines more saturated and the emission stronger (see the lower panel in Fig. 6.3). For the same value of the parameter d , strong lines are saturated more than intermediate lines. A proper choice of d can saturate strong lines but still keep intermediate lines unsaturated (e.g. see lines for $d = 0.1$ in the upper right and lower panels of Fig. 6.3).

Assuming that the interclump medium is not void influences the strength of the line as a whole, particularly effecting the center of the line, where a small absorption dip appears. Since more clumps are concentrated close to the star, the inner part of the wind, together with the interclump medium, contribute to the line opacity more than the outer part of the wind.

6.1.3 Effects of velocity dispersion inside clumps

The absorption of an individual clump can be broadened by stochastic (thermal or micro-turbulent) motions inside the clump, but also by an additional velocity gradient in the clump as predicted by hydrodynamic simulations (Feldmeier et al., 1997).

These kinds of line broadening are described in our model by the Doppler-broadening velocity (v_D) and the velocity dispersion within the clumps (v_{dis}), respectively (see Sect. 5.4.4).

The effect of the Doppler-broadening velocity variation In Fig. 6.4, the effect of the v_D variation is shown. Normalization of the frequency-integrated absorption coefficient is compensated by changing χ_0 . For a higher value of v_D , the line profile is broader, and both the absorption and emission are stronger. When v_D decreases, the macroclumping effect becomes more pronounced because the clumps are more optically thick in the line center, leading to a smaller effective optical depth due to a larger macroclumping effect.

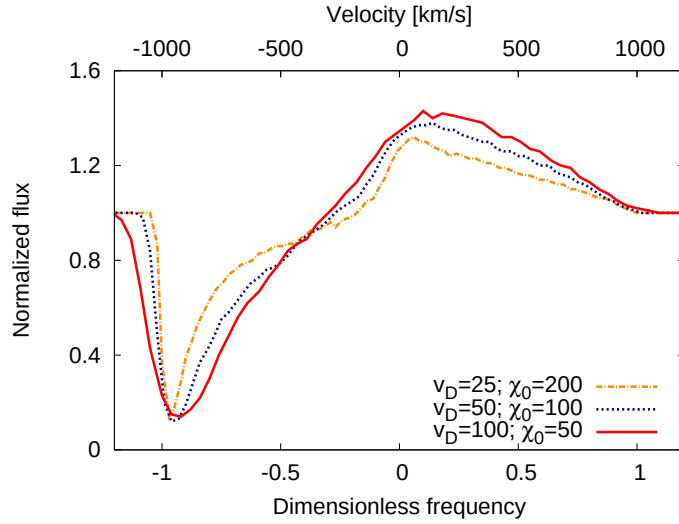


FIGURE 6.4: The effect of variable Doppler broadening on line profiles represented by three values of v_D and χ_0 as indicated in the figure. Other model parameters have their default value (Table 6.1 and Table 6.2).

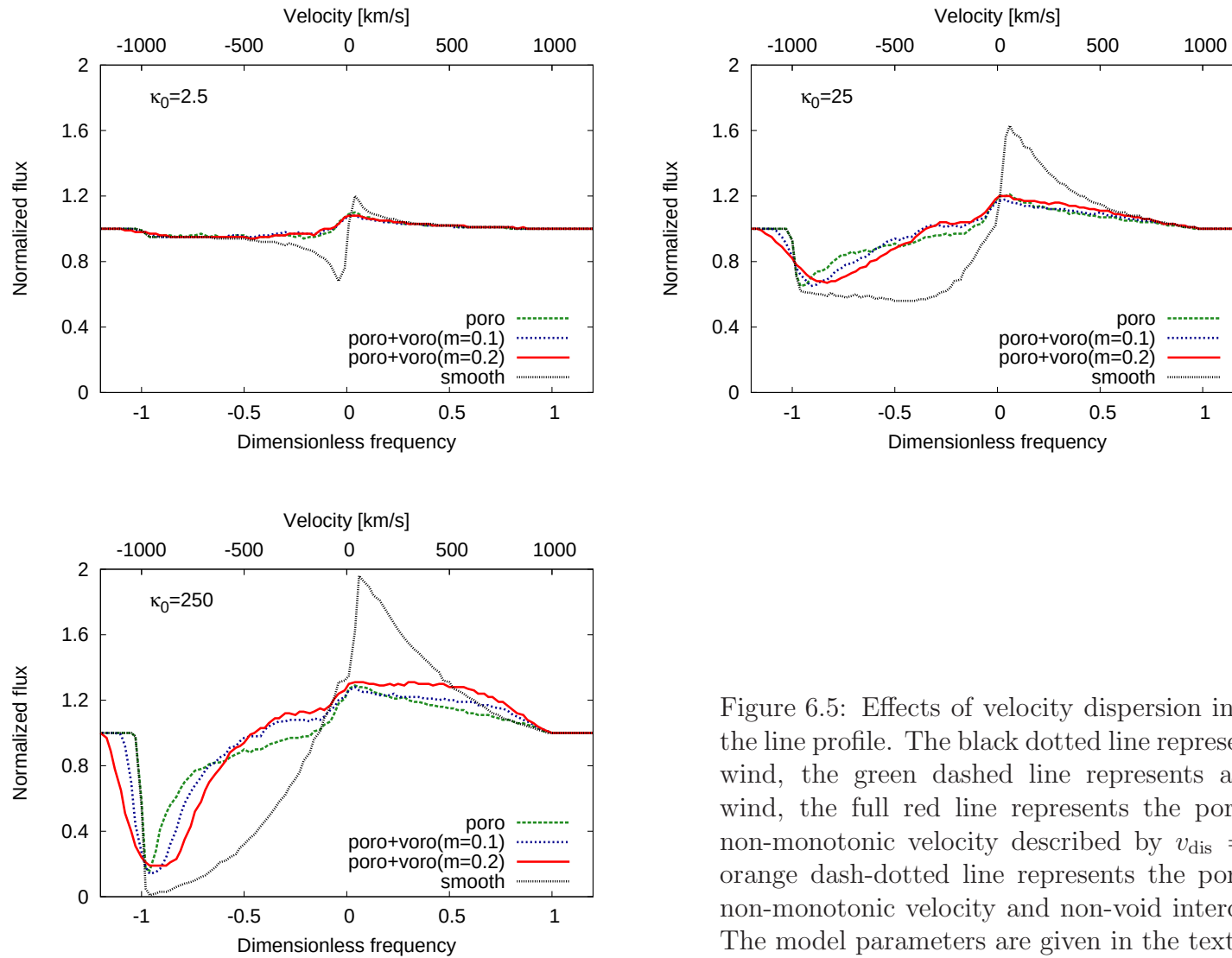


Figure 6.5: Effects of velocity dispersion inside clumps on the line profile. The black dotted line represents the smooth wind, the green dashed line represents a pure porosity wind, the full red line represents the porous wind with non-monotonic velocity described by $v_{\text{dis}} = 0.1$, and the orange dash-dotted line represents the porous wind with non-monotonic velocity and non-void interclump medium. The model parameters are given in the text.

The effect of the velocity dispersion variation To study the effect of the velocity dispersion within the clumps, $v_D = 20 \text{ km s}^{-1}$ is fixed and m (see Eq. 5.36) and χ_0 (Fig. 6.5) are varied. If the velocity dispersion inside the clumps is higher (i.e. when increasing the parameter m), the gaps in the velocity field are smaller (more velocities overlap) and the probability of photon escape is lower. This leads to some absorption at velocities higher than v_∞ . However, in the example shown the velocity gradient inside the clumps does not differ much from the velocity gradient of the smooth wind except for its sign, thus leaving the optical depth of the individual clumps roughly the same. Therefore, the “vorosity” has only a small effect on the total line strength.

When the velocity dispersion is accounted for, the absorption extends to velocities higher than the terminal velocity v_∞ . This effect is important for the derivation of v_∞ from observations (i.e., see Prinja et al., 1990), and may provide an effective diagnostic for the possible existence of the velocity dispersion inside the clumps.

6.1.4 Effects of the onset of clumping

To illustrate the effect of variation of the clumping onset radius, now all parameters of the model are fixed to their default values except for this distance, r_{cl} . The main effect of the r_{cl} variation is the appearance of a strong absorption near the line center (see Fig. 6.6), which is due to the smooth part of the wind inside r_{cl} at low expansion velocity. If clumping starts higher in the wind, the absorption near the line center is broader, and partly attenuates the emission from the back hemisphere of the wind. This central absorption dip can therefore be used as a diagnostic for the onset of clumping above the photosphere.

It is noted that even a very small smooth part of the wind (between $r = 1$ and $r = 1.01$) causes significant central absorption, an effect that should be observable. As can be seen from the Fig. 6.6, only when $r_{cl} = 1$ does the absorption near the line center disappear. When the clumped part of the wind is larger (smaller value of r_{cl}), the reduction in the line strength is more pronounced. By setting $r_{cl} = r_{\max}$, the smooth wind is reproduced.

Clumping at the base of the wind Additionally, it is tested how the onset of clumping, which is not necessarily at the base of the wind, can influence the resonance line profiles.

For this purpose, it is examined how macroclumping (defined by L_0 and D), non monotonic velocity field (v_{dis}), non void interclump medium (d), and onset of clumping (r_{cl}) may influence weak ($\chi_0 = 0.5$), intermediate ($\chi_0 = 50$), and strong ($\chi_0 = 500$) resonance line profiles. Calculations are performed assuming $\sim 10^4$ clumps inside wind, corresponding to $L_0 = 0.5$ for a wind with $r = 10$. The clumping factor is set to $D = 10$ and the Doppler-broadening velocity is $v_D = 50 \text{ km s}^{-1}$. A snapshot of the distribution of clumps is shown in Fig. 6.7, upper right panel.

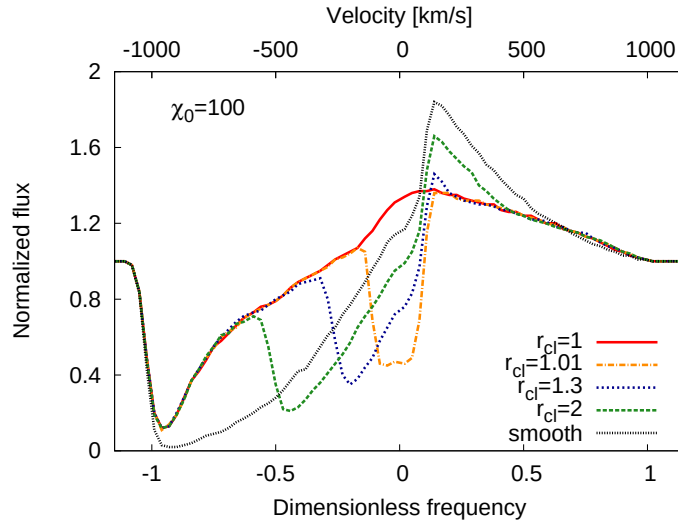


FIGURE 6.6: The effect of variation in the onset of the clumping r_{cl} for the case of strong lines ($\chi_0 = 100$). The dashed black line represents the smooth wind ($r_{cl} = r_{\max}$), and other lines are calculated for different r_{cl} as indicated in the figure. Other model parameters have their default value (Table 6.1 and Table 6.2).

The first set of line profiles (C1 in Fig. 6.7) is calculated assuming that clumping starts at the surface of the star ($r_{cl} = 1$) with a void interclump medium ($d = 0$), and a monotonic velocity field ($v_{\text{dis}} = v_{\beta}$). The weak (upper left panel), intermediate (lower left panel), and strong (lower right panel) lines show strong line strength reduction compared to the smooth wind (full lines). If the wind is clumped, more photons can escape through “holes” between clumps. This leads to lower effective opacities, and, consequently, less absorption. The line strength reduction depends on the number of clumps in the wind. The absorption dip close to v_{∞} is caused by a lower probability of photon escape through a lower number of velocity “holes”.

In order to investigate how the onset of clumping influences line profiles, a second set of line profiles (C2 in Fig. 6.7) are calculated assuming that clumping is only above $r_{cl} = 1.3$. Other model parameters are the same as for the first set of line profile calculations. A very strong and broad absorption near the line centers appears. Due to a higher absorption of the smooth part of the wind, in the region $1 < r < r_{cl}$. Such a deep absorption is not observed disappears if clumping starts closer to the wind base (perhaps even from the surface of the star) or if the interclump medium is sufficiently dense. The weak line nearly reproduces the smooth wind model in this case. For a small value of χ_0 , the clumps are optically thin, and have a very small influence on the line profile compared to the smooth wind.

Varying the interclump medium density parameter d creates a significant impact on line profiles, especially for strong lines. Keeping the same parameters as for

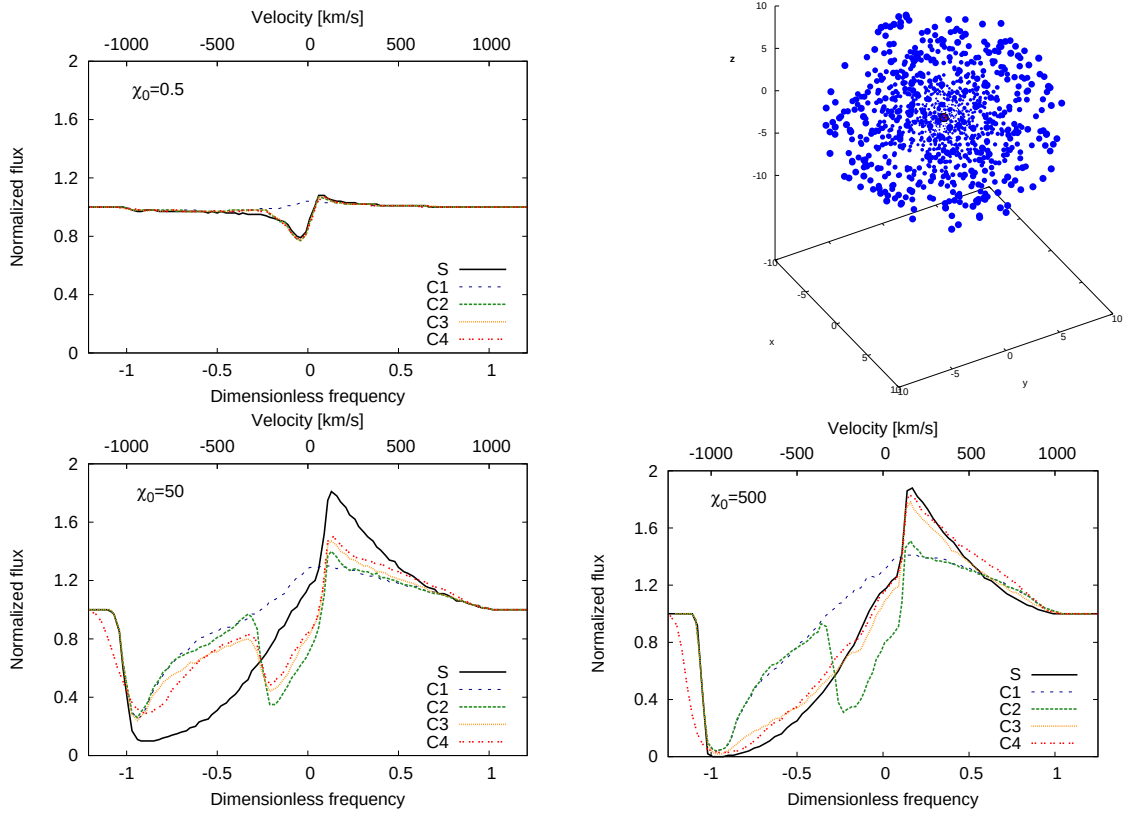


FIGURE 6.7: Line profiles for weak (upper left panel), medium (lower left panel), and strong (lower right panel) lines for the clump distribution in the upper right panel. S – the smooth model; C_n denotes the clumped models – C1 ($r_{\text{cl}} = 1$), C2 ($r_{\text{cl}} = 1.3$), C3 ($r_{\text{cl}} = 1.3$, $d = 0.05$), C4 ($r_{\text{cl}} = 1.3$, $d = 0.05$, $v_{\text{dis}} = 0.2 v_\beta$).

the second set of line profile calculation, but choosing $d = 0.05$, the third set of line profiles (C3 in Fig. 6.7) are calculated. While the weak line is unaffected, the intermediate and strong lines show an increase of absorption in the blue edge of the line and a decrease of absorption near the centre of the line. The degree of line saturation depends on the interclump medium density. For a particular value of d , saturation of strong lines and desaturation of the intermediate lines can be achieved.

Taking into account a non-monotonic velocity field, absorption at velocities higher than v_∞ appears. Setting $v_{\text{dis}} = 0.2 v_\beta$ and keeping other model parameters the same as for the third set of line profiles, we calculated a fourth set of line profiles (C4 in Fig. 6.7). The higher v_{dis} , the larger is the velocity span of clumps, and, consequently, there are more velocity overlaps (the “holes” in the velocity field are smaller), and the probability of photon escape is lower. These results were presented in Surlan et al. (2012a).

6.2 Doublet calculation

For the calculation of doublets, the profile ϕ_x in Eq. (5.7) is assumed to have the form

$$\phi_x = \frac{1}{\sqrt{\pi}} (e^{-(x+d_{\text{sep}}/2)^2} + p e^{-(x-d_{\text{sep}}/2)^2}), \quad (6.2)$$

where d_{sep} is the separation between doublet components. The ratio of line opacities has a fixed value $p = \chi_1/\chi_2$, which follows from the atomic line strengths. The zero point frequency is located in-between the components.

The frequency of the photon after scattering is chosen randomly with a Gaussian distribution in the same way as for singlets. However, if the co-moving frame frequency of the photon before scattering indicates that it belongs to the blue component ($x^{\text{cmf}} > 0$), the photon is redistributed only within the blue component of the doublet, and vice versa. In the case when lines overlap, the re-emitted photons may be redistributed only over the same doublet component in which they were absorbed, or more realistically, over both doublet components, in the ratio of the statistical weights of the levels, i.e. 2:1.

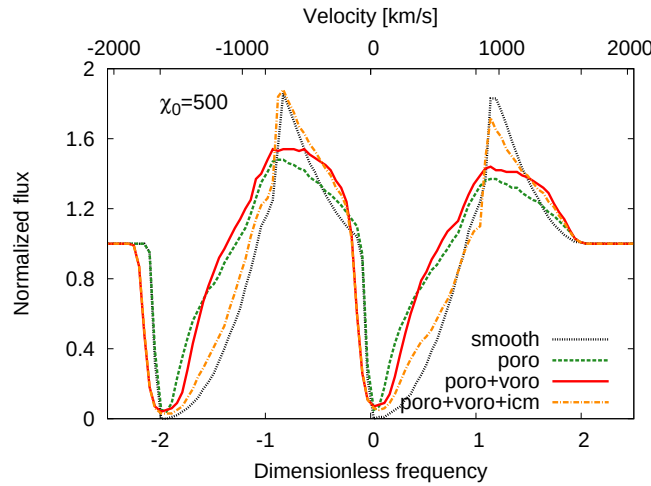


FIGURE 6.8: Effects of macroclumping (poro) on the line profile including velocity dispersion inside clumps (poro+voro) and non-void interclump medium (poro+voro+icm). The black dotted line represents the smooth wind, the green dashed line represents the macroclumping effect, the full red line represents the macroclumping effect including non-monotonic velocity described inside clumps with $v_{\text{dis}} = 0.1$, and the orange dash-dotted line represents the macroclumping effect including non-monotonic velocity and non-void interclump medium. The model parameters are given in the text.

In Fig. 6.8, one example of a strong doublet with $d_{\text{sep}} = 2000 \text{ km s}^{-1}$ and $\chi_0 = 500$

is presented. The different effects on the line profile by changing the properties of the clumps, which are analogous to the single-line case are demonstrated. The main effect of macroclumping is a reduction in the line strength. If the velocity dispersion inside clumps is taken into account, there is some absorption at velocities higher than v_∞ . Only in the case of non-void interclump medium it is possible to saturate the line.

Chapter 7

Comparison with observations

To compare the MCRT calculations with observations, first the P v resonance doublet for a few Galactic O4-O7 type stars is fitted to check the validity of mass-loss rates derived from H_α diagnostics.

Modeling H_α wind lines is not an easy task, as their strengths are influenced by photospheric-wind coupling. To verify the mass-loss rates derived from H_α lines, radio continuum and UV resonance lines diagnostics can be used. The P v doublet $\lambda\lambda 1118, 1128 \text{ \AA}$ is useful for this purpose. Theory predicts that P v is the dominant ion in the wind of O4-O7 type stars, and then it can be assumed that ionisation fraction of the P v ion, $q_{\text{P v}} \approx 1$ (also concluded from observations). Thus, the product $\dot{M}q_{\text{P v}}$ (see Section 2.2.3) is expected to agree with mass-loss rates obtained from radio ($\dot{M}_{\text{radio}}$) and H_α ($\dot{M}_{H_\alpha}$) diagnostics. However, as shown by e.g., Fullerton et al. (2006), $\dot{M}q(\text{P v})$ is smaller by a median factor of ~ 20 than $\dot{M}_{\text{radio}}$ and $\dot{M}_{H_\alpha}$. This means that mass-loss rates of O4-O7 type stars obtained from the H_α diagnostic, assuming a smooth wind, are overestimated and must be reduced by factor of ~ 20 . Lowering these mass-loss rates would have major consequences for massive, hot star evolution and the feedback that these stars provide to their interstellar environment.

One of the possible ways to explain such a large discrepancy between different diagnostics is to relax the commonly used assumption that the wind is smooth, and instead consider that the hot star wind is clumped (see Chapter 3). In the limit of optically thin clumps (the microclumping approach, see Section 3.3.3), any density squared diagnostics (such as H_α), assuming a smooth wind model, are reduced by a factor of $\sqrt{D}$. The overestimation of derived mass-loss rates from H_α lines has been shown by many authors (see e.g., Hillier and Miller, 1998; Hillier et al., 2003; Crowther et al., 2002; Bouret et al., 2003, 2005).

Assuming the macroclumping approach (using the macroclumping factor C_{macro}

– see Eq. 3.9), Oskinova et al. (2007) found that the discrepancy between the H_α and P v diagnostic, reported by Fullerton et al. (2006) can be resolved by introducing macroclumping in this parametric way, i.e. assuming that wind is clumped and allowing clumps to have any optical depth. This authors showed that the optically thin H_α line is not affected by wind porosity, while the P v resonance doublets become significantly weaker when macroclumping is added. This leads to the conclusion that mass-loss rates derived from density square diagnostics are actually correct, and do not needed to be reduced. Assuming the wind is clumped, both diagnostics result in the same mass-loss rate, i.e. the recombination (H_α) and resonance (UV) lines can be simultaneously fit with observations with satisfactory agreement.

Following the conclusions of Oskinova et al. (2007), the detailed 3-D treatment of clumping is used to show how wind clumping with certain properties can affect line strength reduction of the P v UV resonance lines without a need to reduce mass-loss rates. For this purpose, several Galactic O4-O7 type stars with a theoretically predicted and empirically estimated value of $q_{Pv} \approx 1$ is selected. The list of stars from the sample together with stellar and wind parameters taken from Fullerton et al. (2006) are listed in Table 7.2. To calculate the P v resonance doublets of each star, the opacity parameter is needed. This parameter can be calculated if the mass-loss rate of the star and abundance of the element is known.

7.1 Parametric line opacity for a given mass-loss rate

Mass-loss rates (see Eq. 2.5) follows from the equation of mass continuity for a homogeneous, stationary, spherically symmetric wind (see Eq. 2.1). Let us consider a resonance line of the ion j of the element with an atomic number i . Let the mass of the particle of the element i be m_i , and its abundance by weight (mass fraction) be X_i . Then, if $n_{j,i}(r)$ and $n_i(r)$ represent the number density of the ion and its parent element, respectively, they are connected via

$$n_{j,i}(r) = q_{j,i}(r)n_i(r), \quad (7.1)$$

where $q_{j,i}$ is the ionization fraction of the ion j . The number density of the element i can be expressed as

$$n_i(r) = \frac{X_i}{A_i m_H} \rho(r), \quad (7.2)$$

where A_i is atomic weight of the element i and m_H is atomic mass of the hydrogen. Combining equations (2.5), (7.1), and (7.2) we obtain

$$n_{j,i}(r) = q_{j,i}(r) \frac{X_i}{A_i m_H} \frac{\dot{M}}{4\pi r^2 v(r)}. \quad (7.3)$$

The line opacity for the case of pure scattering is

$$\chi_L(r, \nu) = \frac{\pi e^2}{m_e c} f_{l,u} n_{l,i}(r) \phi(\nu), \quad (7.4)$$

where $f_{l,u}$ is the oscillator strength of the transition between levels $l \leftrightarrow u$, $n_{l,i}(r)$ is the occupation number of the lower level l of this transition, e is the elementary charge, m_e is the mass of electron, and c is the speed of light. The Doppler profile is assumed to be as

$$\phi(\nu) = \frac{1}{\sqrt{\pi}} \frac{1}{\Delta \nu_D} e^{-\frac{(\nu - \nu_0)^2}{\Delta \nu_D^2}}, \quad (7.5)$$

where $\Delta \nu_D = \nu_0 v_D / c = v_D / \lambda_0$ is the Doppler width, ν_0 is line-centre frequency, $\lambda_0 = c / \nu_0$ is the line-centre wavelength of considered transition, and v_D is the thermal velocity. The profile function (7.5) is normalized to unity,

$$\int_0^\infty \phi(\nu) d\nu = 1. \quad (7.6)$$

Using the dimensionless frequency

$$x = \frac{\nu - \nu_0}{\Delta \nu_D}, \quad (7.7)$$

the opacity (7.4) can be expressed as

$$\chi(r, x) = \frac{\pi e^2}{m_e c} f_{l,u} n_{l,i}(r) \frac{\lambda_0}{v_D} \phi(x), \quad (7.8)$$

where

$$\phi(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}, \quad (7.9)$$

which is again normalized to unity,

$$\int_{-\infty}^\infty \phi(x) dx = 1.$$

The frequency integrated line opacity can be expressed as

$$\chi_L(r) = \frac{\pi e^2}{m_e c} f_{l,u} n_{l,i}(r) \frac{\lambda_0}{v_D}. \quad (7.10)$$

Assuming that all ions j are in the state l , i.e. $n_{j,i} \approx n_{l,i}$, we can combine (7.3) and (7.10) to yield

$$\chi_L(r) = \frac{\pi e^2}{m_e c} f_{l,u} q_{j,i}(r) \frac{X_i}{A_i m_H} \frac{\dot{M}}{4\pi r^2 v(r)} \frac{\lambda_0}{v_D}. \quad (7.11)$$

The opacity parametrisation of Hamann (1980) is recalled

$$\chi_L(r) = \frac{\chi_0}{\frac{r^2}{R_*^2} \frac{v(r)}{v_\infty}} q_{j,i}(r), \quad (7.12)$$

where χ_0 is a “line strength parameter”. Assuming this parameter is expressed in units of R_*^{-1} , Eq. 7.11 and Eq. 7.12 give

$$\chi_0 = \frac{\pi e^2}{m_e c} f_{l,u} \frac{\lambda_0}{v_D v_\infty} \frac{\dot{M}}{4\pi R_*} \frac{X_i}{A_i m_i}. \quad (7.13)$$

This equation can calculate the parametric opacity for a given mass-loss rate and abundance of the element.

7.2 Comparison with observations

Using Eq. 7.13 the opacity parameter χ_0 for each star from the sample (see the last column of Table 7.2) is calculated. Constants and atomic data used for this purpose are listed in Table 7.1. Stellar (R_* and $v \sin i$) and wind ($\log \dot{M}_{H_\alpha}$, v_∞ and β) parameters are given in Table 7.2. The line profiles are calculated assuming the smooth wind model with $v_D = 20 \text{ km s}^{-1}$ (see the blue full lines with crosses in Fig. 7.3, Fig. 7.1, and Fig. 7.2). There are large discordances between synthetic and observed line profiles. To achieve a satisfactory fit with observation, namely the strengths of the doublets components, the opacity parameter (i.e. mass-loss rates, $\chi_0 \propto \dot{M}$) we reduced and line profiles are again calculated assuming the smooth wind. All profiles are corrected by rotational velocities using `WRplot` and moved in the frequency space to obtained the best fit. The best agreement with observations is obtained if we reduce χ_0 by 20 times. This leads to the conclusion that derived mass-loss rates from H_α diagnostics must also be reduced by the factor of 20 (similar results also were obtained by Fullerton et al., 2006).

Constant	Value
Cross-section of the classical oscillator	$\pi e^2 (m_e c)^{-1} = 0.0265 [\text{cm}^2 \text{s}^{-1}]$
Oscillator strength for blue component of P v	$f_{l,u,\text{blue}} = 2 f_{l,u,\text{red}} = 0.473$
Wavelength	$\lambda_{Pv} = 1117.979 \cdot 10^{-8} [\text{cm}]$
Mass fraction	$X_{Pv} = 5.87 \cdot 10^{-6}$
Atomic weight	$A_{Pv} = 31$
Atomic mass of hydrogen	$m_H = 1.67 \cdot 10^{-24} [\text{g}]$

Table 7.1: The data and constant used for the opacity parameter χ_0 calculations.

Star	Spectral Type	R_* [$M_\odot$]	$v \sin i$ [km s^{-1}]	v_∞ [km s^{-1}]	$\log \dot{M}_{H_\alpha}$ [$M_\odot/\text{yr}$]	β	χ_0
HD 66811	O4 I(n)f	19.4	220	2250	-5.06	0.9	1130.27
HD 190429A	O4 If+	20.8	105	2300	-4.85	0.7	1672.54
HD 14947	O5 If+	16.8	133	2350	-5.07	1.0	1221.67
HD 210839	O6 I(n)fp	21.1	219	2250	-5.16	0.7	821.58
HD 34656	O7 II(f)	24.1	91	2150	-5.62	1.2	251.44

Table 7.2: Stellar and wind parameters of Galactic O-star sample from Fullerton et al. (2006).

Model parameters	Value
Outer boundary of the wind	$r_{\min} = 1$
Outer boundary of the wind	$r_{\max} = 10$
Clump separation parameter	$L_0 = 0.5$
Clumping factor	$D = 10, 400$
Interclump medium density factor	$d = 0.05$
Set-up of clumping	$r_{\text{cl}} = 1$
Velocity deviation	$v_{\text{dis}}/v_\beta = 0.1$
Velocity at the photosphere	$v_{\min} = 10 \text{ [km s}^{-1}\text{]}$
Doppler velocity	$v_D = 20 \text{ [km s}^{-1}\text{]}$

Table 7.3: The model parameters used to fit the observed P v lines of all Galactic O-stars from our sample.

The question is, whether it is necessary to reduce mass-loss rates, or if the mass-loss rates obtained from H_α diagnostics can be kept and make the wind clumping to see what the profile looks like. Oskinova et al. (2007) used a similar procedure. Assuming the same mass-loss rates, the authors simultaneously fit H_α and P v line profiles using a “standard” clumping factor of $D = 10$, corresponding to a filling factor $f_V = 0.1$. Following this idea, D is setted to $D = 10$, and a void interclump medium and a monotonic velocity field are assumed and a drastic reduction of line strengths is obtained. However, the fit is not good enough (see the full purple lines in Fig. 7.3, Fig. 7.1, and Fig. 7.2). There are more emission in both components of P v which suggest that some more amount of matter should be presented in the wind to achieve better fit. Thus the interclump medium is setted to $d = 0.05$ and $v_{\text{dis}} = 0.1 v$. Filling the interclump medium with some matter, the line profiles near the rest wavelength fit better, but the deep absorption at the edges of the blue part of the profiles remain (see the full green lines in Fig. 7.3, Fig. 7.1, and Fig. 7.2). To

reduce the opacity in the outer wind and therefore reduce the blue “troughs”, the parameter D is increased until good fit also at the blue edges of the line profiles is succeed to obtain. Model parameters used in the calculations are given in Table 7.3.

The best agreement is found for all the stars from the sample with $D = 400$ (see the full red lines in Fig. 7.3, Fig. 7.1, and Fig. 7.2). Square root of D corresponds to the factor of 20, i.e. $\sqrt{400} = 20$, the value obtained by Fullerton et al. (2006) in their analysis of O4-O7 type stars, to make $\dot{M}_{H_\alpha}$ consistent with $\dot{M}_{P\,V}$.

With such a high D , a large number of small and dense clumps are created and significantly reduce the probability for photon interaction, the total opacity is also reduced even in the outer part of the wind that becomes transparent for photons, and deep blue “troughs” disappear in all line profiles. The solution of the problem of high parameter D is to assume that D (and d) is depth dependent.

This first comparison with observations show that the winds of the stars from the sample are clumped, and that with appropriate adjustment of clumping parameters it is possible to fit the strength of the P V resonance lines assuming the same mass-loss rates obtained from H_α diagnostics.

From the comparison performed here, it can be conclude that macroclumping affects UV resonance lines (ρ -diagnostics), contrary to the results of Fullerton et al. (2006). Further tests are needed to check H_α line profiles with clumping as well as radio continuum with clumping to resolve the discrepancy between H_α and radio mass-loss rates mentioned briefly in Fullerton et al. (2006).

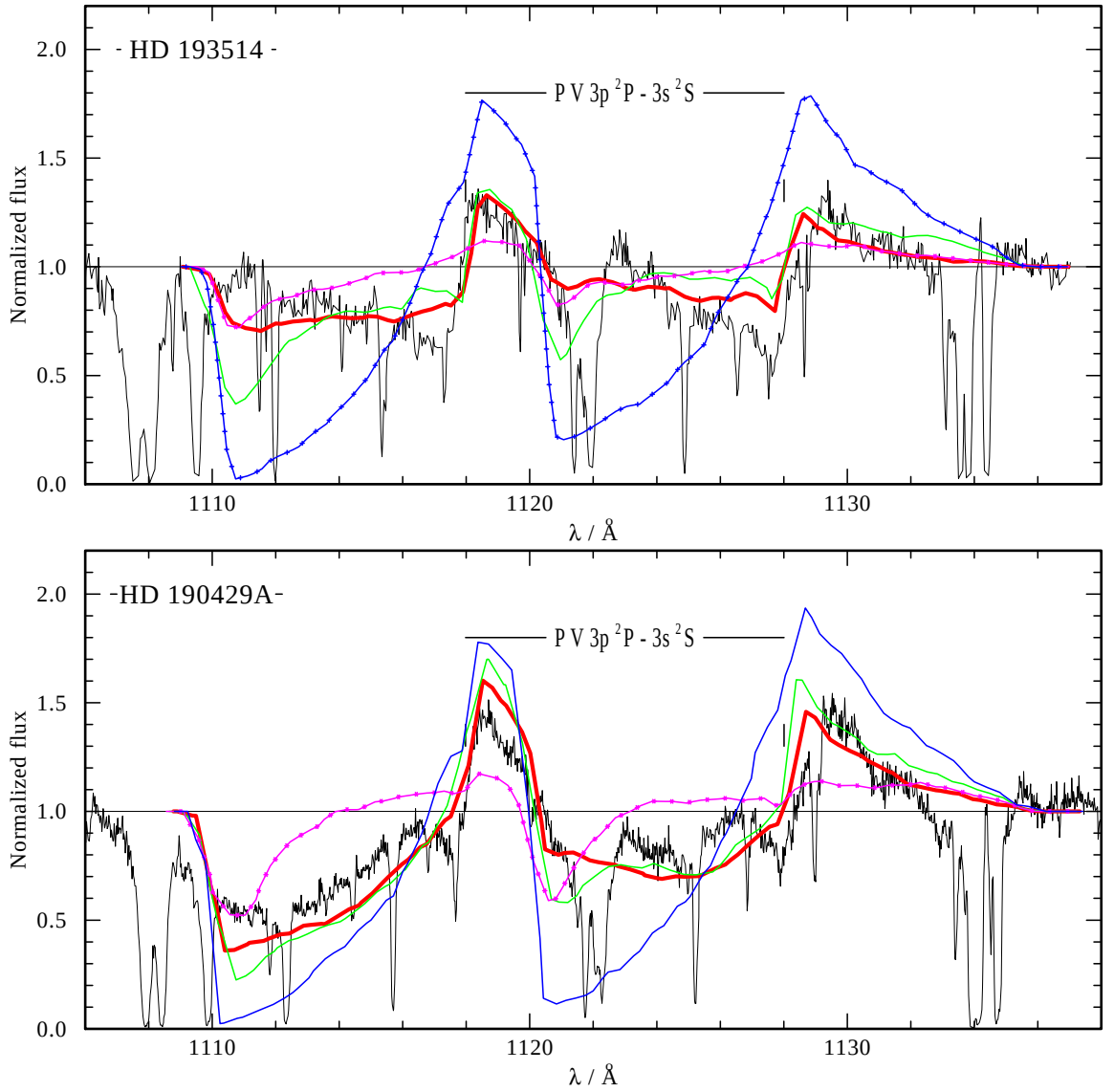


FIGURE 7.1: Comparison of FUSE observations with theoretical line profiles for stars HD 190429A and HD 14947. Black – observations; blue – smooth wind models; Purple – clumped winds with $D = 10$ and void interlump medium and monotonic velocity; green – clumped winds with $D = 10$ and with $d = 0.05$ and $v_{\text{dis}} = 0.1 v_{\beta}$; red – clumped winds with $D = 400$ and with $d = 0.05$ and $v_{\text{dis}} = 0.1 v_{\beta}$. Other model parameters are given in Table 7.3.

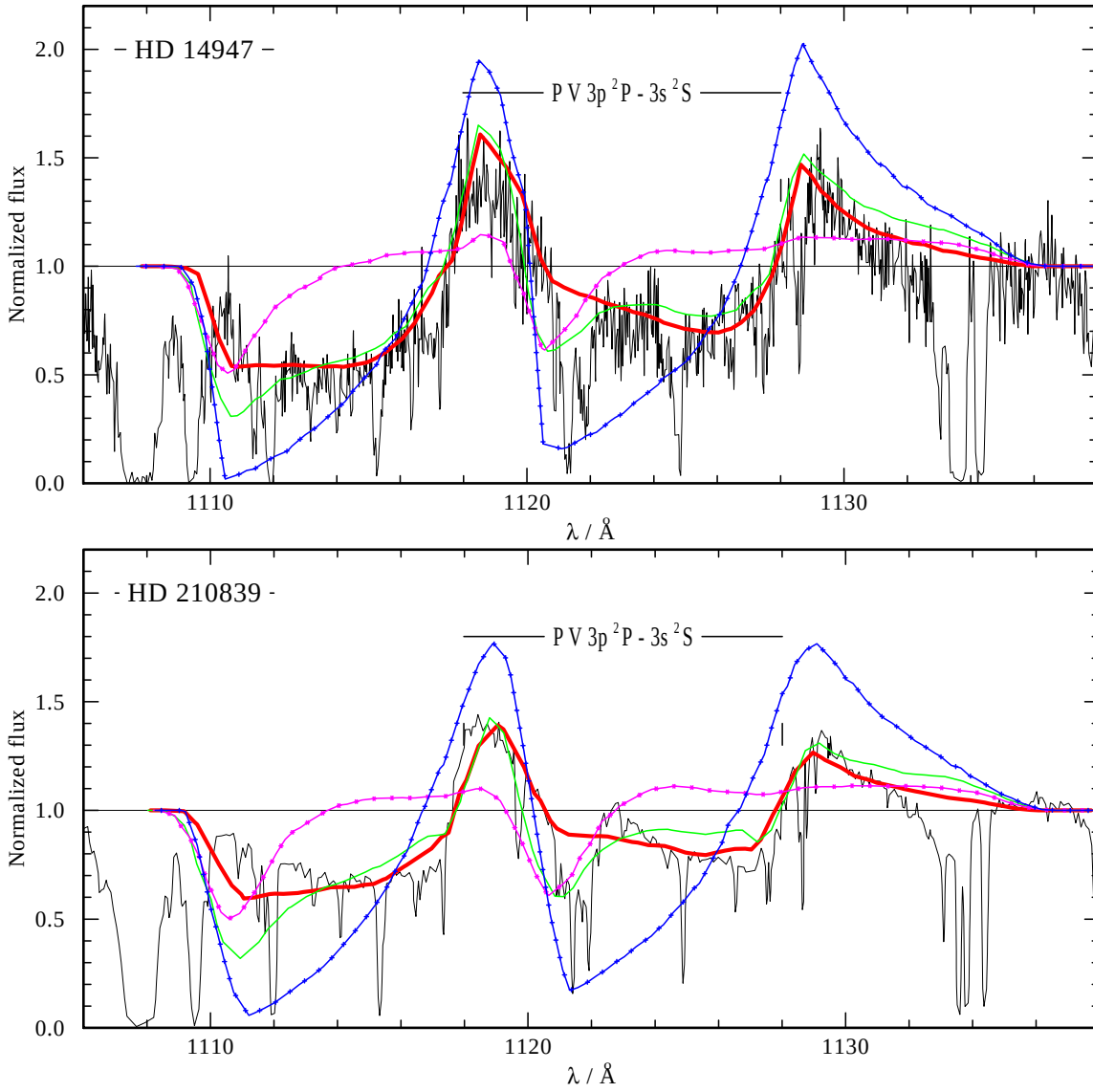


FIGURE 7.2: Comparison of FUSE observations with theoretical line profiles for stars HD 210839 and HD 34656. Black – observations; blue – smooth wind models; Purple – clumped winds with $D = 10$ and void interlump medium and monotonic velocity; green – clumped winds with $D = 10$ and with $d = 0.05$ and $v_{\text{dis}} = 0.1 v_{\beta}$; red – clumped winds with $D = 400$ and with $d = 0.05$ and $v_{\text{dis}} = 0.1 v_{\beta}$. Other model parameters are given in Table 7.3.

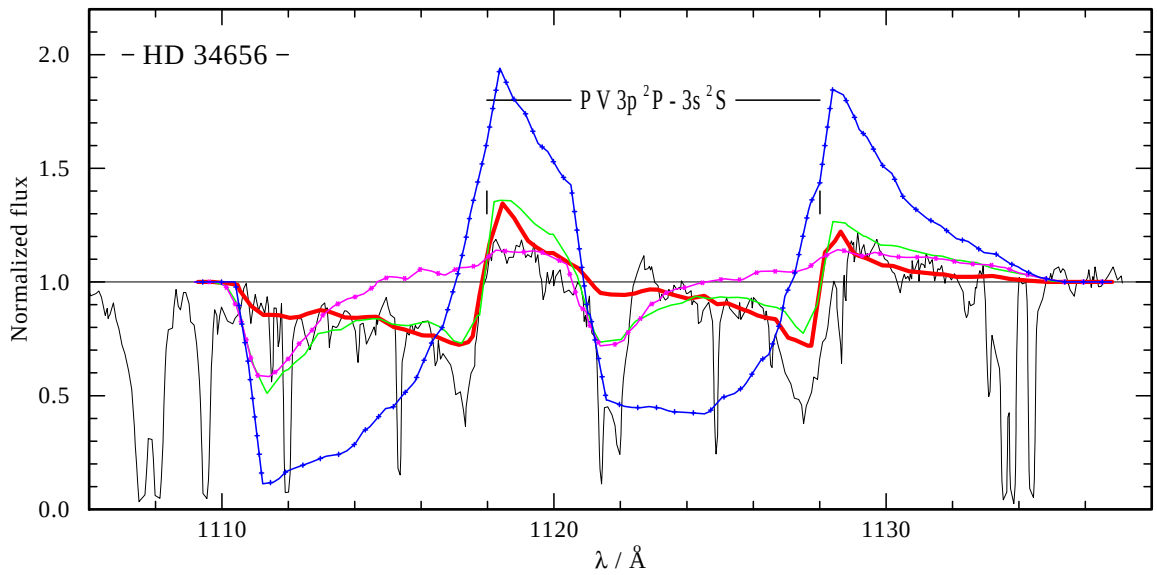


FIGURE 7.3: Comparison of FUSE observations with theoretical line profiles for HD 193514. Black – observations; blue – smooth wind model; Purple – clumped wind with $D = 10$ and void interlump medium and monotonic velocity; green – clumped wind with $D = 10$ and with $d = 0.05$ and $v_{\text{dis}} = 0.1 v_{\beta}$; red – clumped wind with $D = 400$ and with $d = 0.05$ and $v_{\text{dis}} = 0.1 v_{\beta}$. Other model parameters are given in Table 7.3.

Chapter 8

Summary and conclusions

In this thesis, a full 3-D inhomogeneous stellar wind model is presented and the radiative transfer equation using MC method is solved to model resonance lines (Šurlan et al., 2012b). To our knowledge, this is the first work where the problem of resonance line formation in stellar winds is solved in full 3-D, while previous work was restricted to 2-D or pseudo-3-D geometries only.

The radiative transfer is also calculated for the formation of doublets. This is important, because UV resonance doublets are a key diagnostic tool for stellar winds. This work demonstrates the role that stellar wind clumping plays in the formation of spectral lines, and how it affects the empirical wind diagnostics.

The method developed is very flexible, as it is capable to account for different 3-D shapes of clumps, for arbitrary 3-D velocity fields, and for interclump medium at the same time. Below we briefly summarize the main results of the models.

- Allowing for clumps of any optical thickness (macroclumping) causes a significant reduction of the effective opacity in the lines. Consequently, there is less absorption in the wind. Since this reduction is weaker for the outer parts of the wind, the line profiles show an absorption dip near v_∞ , which is probably reduced if the clumping factor is higher further outwards. This will be tested in further work.
- For a given clumping factor D , the key model parameter affecting the effective opacity is L_0 , the clump separation parameter. The opacity reduction is largest for the largest value of L_0 (Fig. 6.1). Therefore, the mass-loss rate empirically obtained by fitting the UV resonance lines becomes larger when the macroclumping effect is taken into account.

- The onset of clumping, r_{cl} , affects the line shape: the closer to the stellar surface clumping starts, the weaker is the absorption dip at the line center (Fig. 6.4). This absorption dip may provide an effective diagnostic for the onset of clumping.
- Line saturation is strongly affected by the interclump medium. A non-void interclump medium is required to reproduce the saturated lines simultaneously with non-saturated lines (Fig. 6.3).
- Adding a velocity dispersion within the clumps to the mean velocity law extends absorption to a larger blue-shift than corresponding to v_{∞} . This effect has to be taken into account when deriving v_{∞} from observations.
- In any clumped wind, non-monotonic velocities will always appear together with density inhomogeneities. Therefore, their combined effect must be taken into account for the line formation modeling (see e.g., Šurlan et al., 2012a).
- In case of resonance doublets, the basic clumping effects are analogous to the case of single lines.
- The problem of P v mass-loss rate determination has not been satisfactorily solved yet. While X-rays do not seem to alter the ionization balance of P v, the case of XUV radiation has not yet been tested by detailed consistent non-LTE calculations. The dependence of mass-loss rates on clumping has still not been analysed in a big detail yet. Available the results show that clumping, below the critical point, may have strong influence on predicted mass-loss rates. Although results of hydrodynamical simulations do not support clumping below the sonic point, there is a possibility of creating clumps by other mechanisms. Detailed hydrodynamical calculations are needed (Kubát and Šurlan, 2012).

The main conclusion of this work is that in a realistic 3-D wind with density inhomogeneities and non-stationary velocity, the P-Cygni profiles from resonance lines are significantly different from those from smooth and stationary 3-D winds (see also Šurlan et al., 2012b,a). Any mass-loss diagnostics which do not account for wind clumping must underestimate the actual mass-loss rates. This can explain the reported discrepancies between the mass-loss rates obtained from ρ - and ρ^2 -based diagnostics, respectively. Using the general description of clumping presented in this work and with comparison with observations, it will be possible to gain deep insight to the structure of stellar winds, and obtain reliable mass-loss rate measurements.

The most important principal problem in clumping is the clump creation mechanism. All current models rely on results of 1-D stability analysis and 1-D hydrodynamical wind modeling. However, these models do not describe how clumps are formed in 3-D. 3-D models of clumping must be based on free parameters, which help to fit observations, but do not answer the essential questions of clump formation. It

is possible that an additional mechanism other than radiation line driving may be responsible for clump creation, however, this work is still in progress. Another issue is the evolution of clumps after they are formed. They may expand to follow the local density via the clumping factor, or they may save their mass, volume, and shape, and leave the star as they were formed, or something in between may be true. Detailed non-LTE modeling including clumping will provide a more sophisticated tool which is necessary to understand the formation of spectral lines in an expanding inhomogeneous massive, hot star wind. To further understand how clumping may influence predicted mass-loss rates, a detailed description of clumping has to be incorporated in hydrodynamical calculations. Future research should include greater detail analysis of the physics of clumps.

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