Chapter 1

BarCor

1.1 Introduction

To detect changes of tens m s^{-1} or even smaller in the radial–velocity curves one needs to have high–quality data and besides, needs to know a very accurate barycentric corrections of the radial velocity. Because at a moment of arrival of star light to the observer the Earth orbits the Sun and rotates around its axis, one has to extract these movements from the radial velocity of the measured star. Thus the observed radial velocity is corrected for the motion of the observer in the direction of the observation. Besides, one must reduce also the time of the observation to the barycentre of the solar system.

To compute radial velocity and time corrections I use positions and velocities of the Earth obtained from the JPL Planetary and Lunar Ephemerides DE405 (see Standish [7], [8]). It is currently the JPL's latest ephemeris. For the given Julian Ephemeris Date one obtains positions and velocities by differentiation and interpolation of a set of Chebyshev coefficients. Both positions and velocities are rectangular and are related to the Earth Mean–equator and dynamical equinox of epoch J2000.0 of inertial reference frame. "Mean" indicates that the effects of nutation are ignored in the definition of the reference frame. The orientation of the inner planet system of DE405 onto the ICRF (International Celestial Reference Frame) is accurately determined mainly by the VLBI observations. Comparing to the origin of ICRF it is believed that the orientation of the whole inner planet ephemeris system of DE405 is now accurate to about 0.001 arcseconds. On the other hand ephemerides of the outer planets rely almost entirely upon optical observations.

1.2 The program usage

The program BarCor computes barycentric corrections of the radial velocity and the time. It is written in Fortran 77. For a successful program run there should be three files in the same directory as the program file. Primarily, there should be a file named JPLEPH which is necessary to obtain the Earth's positions and velocities. Further there should be two input files (identical to those used by the program Hec2 written by P. Harmanec). First is the file with star equatorial coordinates, which contains right ascension (in hours, minutes and seconds) and declination (in degrees, minutes and seconds of arc) of the observed star, the epoch of the coordinates and a numerical code of the observatory. This file has to be written in a fixed format (first record: maximum 26 characters, second record: format 6F10.4, 2I5). The example of such file is:

```
---------------------------------------------------------------------------
         10        20        30        40        50        60        70
....|....|....|....|....|....|....|....|....|....|....|....|....|....|....

51 Peg

22.       57.       27.9805   20.       46.       07.796     2000   10
```

Numerical codes for individual observatories are in Table 1.1.

The second file is the file with observed data and contains: a running number of each spectrum, year, month, day, hours, minutes and seconds of the observation (given in the Universal Time UT for the beginning of each exposure) and the exposure time in seconds. The file uses a free format. The example is:

```
8338 1997 10 13 05 18 43.08 900
8339 1997 10 13 05 34 27.32 900
```

The output file contains the printout of the star name, the name of the observatory and the star coordinates. For each spectrum in a given date and time there is the Barycentric Julian Date BJD and the barycentric correction of the radial velocity RVcorr. The example of the output file with input files shown above is:

```
  BarCor                                          Release  18 April 2006
  List of observations of star 51 Peg
    CFHT                            RA(2000) 22 57 28.0   DA(2000)  20 46  7

========================================================================
 N.    Date & UT start    exp[s]           BJD                  RVcorr
========================================================================

 8338 1997 10 13  5 18 43  900            50734.7311983      -11.867165
 8339 1997 10 13  5 34 27  900            50734.7421265      -11.894967
```

Since JPL's ephemerides are computed for the given Julian Ephemeris Date and our observation is in the UT time, it is necessary to reduce the UT time to the Ephemeris Time. The more detailed description of the problem is in the chapter 1.7, but now it is noteworthy to say that all users should update the most recent values of $\triangle$T in the Eq. (1.32), what can be done either by updating the actual source code of the program or by downloading the new source code[1], which will be continuously upgraded.

Code	Observatory
1	Ondřejov
2	DAO Victoria
3	Lick
4	Okayama
5	David Dunlap
6	Observatoire Haute Provence
7	Crimea
8	Zelenchuk
9	Kitt Peak National Observatory
10	Canada–France–Hawaii Telescope
11	Tautenburg
12	William Herschel Telescope
13	Nordic Optical Telescope
14	McDonald Observatory, Mt. Locke
15	McDonald Observatory, Mt. Fowlkes
16	Mercator, La Palma

Table 1.1: The numerical codes for individual observatories in the program BarCor.

1.3 The Earth flattening

Because the Earth is rotating, it is slightly flattened. The exact shape is rather complicated, but for most purposes it can be approximated by an oblate spheroid. The reference spheroid is used to define the oblate spheroid which best defines the shape of the Earth. For the program computation I used the reference spheroid defined in the World Geodetic System WGS–84. In WGS–84 the equatorial radius, a in Fig. 1.1, is 6378137.0 meters. The Earth's polar radius, b, is related to the

[1]http://sirrah.troja.mff.cuni.cz/~mary/BarCor/

equatorial radius by the term called Earth flattening f, which equals:

$$f = \frac{a - b}{a}. \tag{1.1}$$

In WGS–84 the flattening is 1/298.257223563, which is a very small deviation from a perfect sphere. Using this value, the Earth's polar radius would be 6356752.3142 meters.

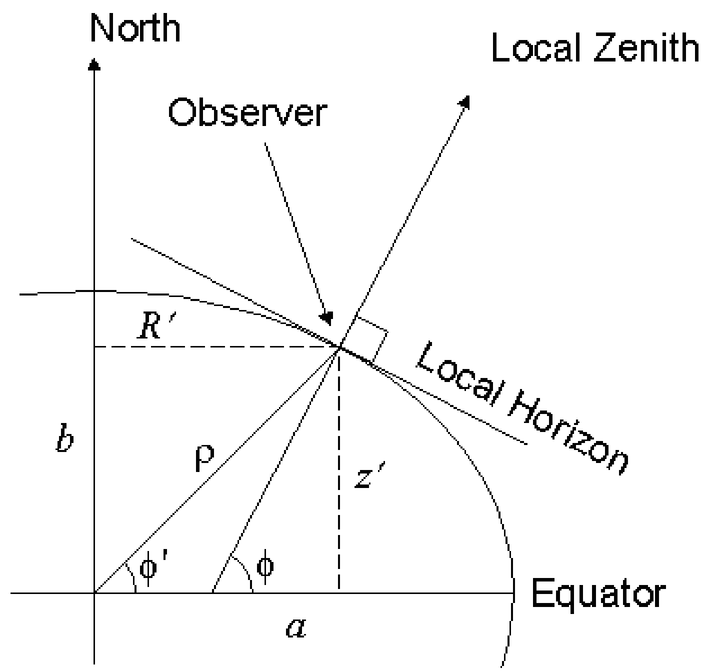

Figure 1.1: The cross–section of the oblate Earth. ϕ is the geodetic and ϕ' is the geocentric latitude, a is the equatorial and b is the polar radius, ρ is the radius at the place of the observer. Adopted from T. S. Kelso in Satellite Times.

In Fig. 1.1 there is the exaggerated view of the cross–section of the Earth. The local horizon is a plane which is tangent to the Earth's surface at the observer's position. In the direction from the Earth's centre perpendicular to the local horizon at the observer's position is the local zenith. On the sphere, this direction is always directly away from the Earth's centre, however on the oblate spheroid the flowline of the Earth's centre and the observer's position would not point, except of the equator and the poles, to the local zenith. In Fig. 1.1 ϕ is the geodetic latitude, what is the angle between the local zenith direction and the Earth's equatorial plane. This angle is the latitude used on maps and sometimes is also called the geographic latitude. $\phi^{'}$ is the angle between the line connecting the observer's position and the Earth's centre and the equatorial plane. The angle is called the geocentric latitude. ρ is the geocentric radius.

Since we use the geodetic latitude ϕ, it is necessary to convert it to the geocentric latitude $\phi^{'}$, which the program uses. The conversion can be carried out with the help of the following expressions. First the basic definition of an ellipse:

$$\frac{(R^{'})^2}{a^2} + \frac{(z^{'})^2}{b^2} = 1, \tag{1.2}$$

where

$$R^{'} = \rho \cos \phi^{'} \tag{1.3}$$

and

$$z^{'} = \rho \sin \phi^{'}. \tag{1.4}$$

It is clear that

$$\tan \phi^{'} = \frac{z^{'}}{R^{'}}. \tag{1.5}$$

The direction of the normal to the ellipse is given by

$$\tan \phi = -\frac{dR^{'}}{dz^{'}}. \tag{1.6}$$

Differentiating the equation of the ellipse, we have:

$$\frac{2R^{'} dR^{'}}{a^2} + \frac{2z^{'} dz^{'}}{b^2} = 0 \tag{1.7}$$

and after rearranging the terms:

$$\frac{z^{'}}{R^{'}} = -\frac{b^2}{a^2} \frac{dR^{'}}{dz^{'}}, \tag{1.8}$$

which can be written as:

$$\tan \phi^{'} = \frac{b^2}{a^2} \tan \phi = (1 - f)^2 \tan \phi = (1 - e^2) \tan \phi, \tag{1.9}$$

where $e^2 = 0.006694 = 2f - f^2$ is the eccentricity of the ellipse.

Now we have the geocentric latitude, but in fact we need to compute R' in terms of the angle ϕ. It can be shown that

$$R' = \rho \cos \phi' = \frac{a \cos \phi}{\sqrt{1 - e^2 \sin^2 \phi}}, \tag{1.10}$$

where the term $1/\sqrt{1 - e^2 \sin^2 \phi}$ is just the effect of the Earth flattening.

Karttunen ([2]) indicates that the shape defined by the surface of the oceans, called the geoid, differs from the spheroid at most by about 100 m and that the difference $\phi - \phi'$ has a maximum $11\overset{\prime}{.}5$ at the latitude 45°. In case of $\phi \sim 49°$ the maximum error resulting from the omission of the Earth flattening in the radial–velocity correction is as large as 0,57 m s^{-1}.

1.4 Expressions for the Precession Quantities

To compare astronomical observations with calculated places of celestial objects one has to do reductions to refer either the observed or the calculated position to the same reference coordinate system. Such reductions are precession, nutation, aberration and parallax. As was mentioned above, these effects are, except the precession, ignored in the reference frame definition. Therefore one must use precession quantities to precess to and from an arbitrary epoch.

To define the basic equations for precession the readers are referred to Fig. 1.2 adopted from Lieske et. al. ([3]). There one can see the celestial sphere with mean ecliptics and equators shown for two epochs ε_F and ε_D. The first, ε_F, is an arbitrary fixed epoch (or a basic epoch) and the second is the mean epoch of date. $\overline{P}_0$ and P represent the mean pole of Earth's equator at the fixed epoch ε_F and the epoch of date ε_D, $\overline{C}_0$ and C represent the ecliptic pole at those two epochs. The vernal equinox at ε_F is denoted by $\overline{\Upsilon}_0$ while the mean equinox of date is denoted by Υ.

The equatorial precession quantities ζ_A, z_A and θ_A are also depicted in Fig. 1.2. These angles are most appropriate to precess from a fixed equinox and equator at epoch ε_F to the mean equinox and equator of date ε_D. The transformation in equatorial rectangular coordinates can be clearly seen in Fig. 1.2 and it reads as:

$$(x, y, z)_{\varepsilon_D} = (x, y, z)_{\varepsilon_F} R_z(\zeta_A) R_y(\theta_A) R_z(z_A) = (x, y, z)_{\varepsilon_F} A, \tag{1.11}$$

where

$$R_z(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.12}$$

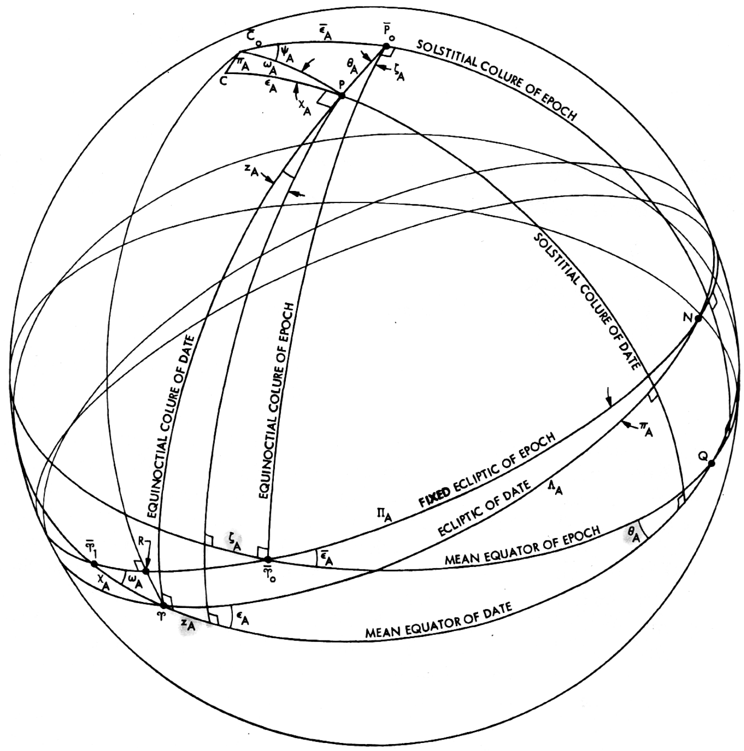

Figure 1.2: Celestial sphere with mean ecliptics and equators shown for two epochs, a fixed epoch ε_F and an epoch of date ε_D. $\overline{P}_0$ and P represent the mean pole of Earth's equator with $\overline{C}_0$ and C representing the ecliptic pole at those two epochs. The vernal equinox at ε_F is denoted by $\overline{\Upsilon}_0$ while the mean equinox of date is denoted by Υ. Adopted from Lieske et. al. ([3]).

is a rotation about the z axis by the angle α and by analogy

$$R_y(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{1.13}$$

is a rotation about the y axis. By the matrix multiplications one obtains the elements of the matrix A:

$$\begin{aligned}
a_{11} &= \cos\zeta_A \cos\theta_A \cos z_A - \sin\zeta_A \sin z_A \\
a_{12} &= \cos\zeta_A \cos\theta_A \sin z_A + \sin\zeta_A \cos z_A \\
a_{13} &= \cos\zeta_A \sin\theta_A \\
a_{21} &= -\sin\zeta_A \cos\theta_A \cos z_A - \cos\zeta_A \sin z_A \\
a_{22} &= -\sin\zeta_A \cos\theta_A \sin z_A + \cos\zeta_A \cos z_A \\
a_{23} &= -\sin\zeta_A \sin\theta_A \\
a_{31} &= -\sin\theta_A \cos z_A \\
a_{32} &= -\sin\theta_A \sin z_A \\
a_{33} &= \cos\theta_A .
\end{aligned} \tag{1.14}$$

The IAU precession–nutation model used before 2000 was composed of the IAU 1976 precession (see [3]) and IAU 1980 nutation (see [12], [6]). Lieske et. al. ([3]) developed expressions for the precession quantities at epoch J2000.0 as a function of the revised fundamental astronomical constants adopted by the International Astronomical Union at the XVI. General Assembly in Grenoble. The development of the usual precession quantities depends upon the dynamical motion of the ecliptic pole relative to a fixed ecliptic, due to planetary perturbations, and upon the dynamical motion of the celestial pole due to luni–solar torques on the oblate Earth. However the IAU 1976 precession model has a number of limitations (for more detail, see for example Capitaine et. al., [1]).

In 2000 IAU adopted new resolution and recommended previous models to be replaced, since 1 January 2003, by the IAU 2000 precession–nutation model, specifically by the MHB 2000 model provided in Mathews et al. ([4]). It includes a new nutation series for a non–rigid Earth and corrections to the precession rates in longitude and obliquity. This model is oriented with respect to the International Celestial Reference System (ICRS) through a fixed 3D rotation between the mean equatorial frame at J2000.0 and the Geocentric Celestial Reference System (GCRS). This rotation, called the *frame bias*, includes the numerical values for the pole offset at J2000.0 that MHB 2000 specifies and the equinox offset at J2000.0 that MHB 2000 does not specify. This equinox offset has only a second–order effect on the final transformation between celestial and terrestrial coordinates, however in fact it is dynamically inconsistent and the theory suffer, except the improvements in the precession rates, from the same limitations as the IAU 1976 precession. The only corrections that have been applied in the IAU 2000 model are the MHB corrections to precession rates in longitude and obliquity. But the expressions used for the motion of the ecliptic and other quantities of precession were the same as in Lieske et. al. ([3]). Woolard & Clemence ([13]) remarked that the motion of the equator and ecliptic are *kinematically* independent, but the motion of the equator depends *dynamically* upon the variations of the disturbing forces caused by changes in the positions of the Sun, the Moon and other planets in the Solar System with the motion of the ecliptic. Hence the improvement of the model for the precession of the equator requires also the use of an improved model for the ecliptic.

Therefore there was need for an improved IAU 2000 precession model which was realized by Capitaine et. al. ([1]). They have clearly separated precession of the equator and precession of the ecliptic and obtained the developments of the quantities through two independent approaches. One of them uses the expressions for the primary precession angles to derive equatorial precession angles:

$$\begin{aligned}
\zeta_A &= 2''.650545 + 2306''.083227t + 0''.2988499t^2 \\
&\quad + 0''.01801828t^3 - 0''.000005971t^4 - 0''.0000003173t^5 \\
z_A &= -2''.650545 + 2306''.077181t + 1''.0927348t^2 \\
&\quad + 0''.01826837t^3 - 0''.000028596t^4 - 0''.0000002904t^5 \\
\theta_A &= 2004''.191903t - 0''.4294934t^2 - 0''.04182264t^3 \\
&\quad - 0''.000007089t^4 - 0''.0000001274t^5,
\end{aligned} \tag{1.15}$$

where the coefficients are in seconds of arc and the parameter t is the elapsed time in Julian centuries since J2000 of the Terrestrial time TT and is defined by:

$$t = (\ \mathrm{TT} - 2000 \text{ January 1d 12h TT})/36525, \tag{1.16}$$

with TT in days. It would be correct to use the Terrestrial dynamical barycentric time TDB instead of TT, but the difference TDB – TT is well below the required accuracy.

Another necessary quantity is an improved expression for the Greenwich Mean Sidereal Time GMST, because it directly depends on the precession in right ascension. Capitaine et. al. ([1]) derived the formula with a resolution of 0.1 microsecond:

$$\begin{aligned}
\mathrm{GMST} &= \mathrm{UT1} + 24110.5493771 + 8640184.79447825t_u \\
&\quad + 307.4771013(t - t_u) + 0.092772110t^2 \\
&\quad - 0.0000002926t^3 - 0.00000199708t^4 \\
&\quad - 0.000000002454t^5 \text{ s},
\end{aligned} \tag{1.17}$$

where t_u is the UT1 and t is the TT, both expressed in Julian centuries after J2000.

In 2006 the IAU recommended the precession theory of Capitaine et. al. ([1]) for the precession of the equator and the precession of the ecliptic. Not only on that account the program BarCor uses precession coefficients of Capitaine et. al. ([1]).

1.5 Radial–velocity correction

The equatorial coordinate system is defined such that the xy plane is consistent with the Earth Mean–equator related to the fixed epoch J2000.0 of inertial reference frame so that x axis points to the vernal equinox of epoch J2000.0. z axis perpendicular to the xy plane is the Earth rotational axis. Then the projected Earth velocity in the equatorial coordinate system is:

$$\begin{aligned} V_x &= V_x^{orb} + V_x^{rot} \\ V_y &= V_y^{orb} + V_y^{rot} \\ V_z &= V_z^{orb}, \end{aligned} \tag{1.18}$$

where V_x, V_y and V_z are sums of the velocity V_x^{orb}, V_y^{orb} and V_z^{orb}, respectively, caused by the Earth orbiting the Sun (computed from the JPL Ephemerides) and the velocity V_x^{rot}, V_y^{rot} and V_z^{rot}, respectively, caused by the Earth rotation. Since z axis is the Earth rotational axis, $V_z^{rot} = 0$.

The velocity of the observer due to Earth rotation V^{rot} is:

$$V^{rot} = \frac{2\pi K}{24 \cdot 3600 \cdot 1000} \left(h + \frac{6378137}{\sqrt{1 - e^2 \sin^2 \phi}} \right) \cos \phi \ (\mathrm{km\ s^{-1}}), \tag{1.19}$$

where h is the altitude of the observatory above sea level (in meters), ϕ is the geodetic latitude, $e^2 = 2f - f^2$, where f is the Earth flattening (for more details see section 1.3). K is the ratio of the mean solar to the mean sidereal day (in solar days) and is equal:

$$K = 1.002737909350795 + 5.9006 \cdot 10^{-11} t - 5.9 \cdot 10^{-15} t^2, \tag{1.20}$$

where t was already defined in Eq. (1.16). The Greenwich Mean Sidereal Time GMST is computed according to Eq. (1.17) and the Local Mean Sidereal Time LMST is:

$$\mathrm{LMST} = \left(\frac{\mathrm{GMST}}{86400} - \frac{l}{360} \right) 2\pi \ (\mathrm{rad}), \tag{1.21}$$

where l is the longitude of the observatory (in degrees, positive to east).

The projected velocity of the observer due to Earth rotation V^{rot} in the equatorial coordinate system is:

$$\begin{aligned} V_x^{rot} &= -V^{rot} \sin(\mathrm{LMST}) \\ V_y^{rot} &= V^{rot} \cos(\mathrm{LMST}) \\ V_z^{rot} &= 0. \end{aligned} \tag{1.22}$$

To compute the barycentric correction RV_{corr} of the radial velocity one needs to project the Earth velocity V_x, V_y and V_z in Eqs. (1.18) to a line of sight of the observer, for what serve simple spherical coordinates:

$$\begin{aligned} x &= \cos\alpha \cos\delta \\ y &= \sin\alpha \cos\delta \\ z &= \sin\delta, \end{aligned} \tag{1.23}$$

where α and δ are stellar coordinates (right ascension and declination) at the epoch of the coordinates. Hereafter these star coordinates have to be corrected for the precession, that means one has to precess star coordinates from the epoch of the coordinates to the time of observation, as was described in detail in section 1.4. Therefore the projections p_1, p_2, resp. p_3, of the velocity V_x, V_y, resp. V_z, to a line of sight of the observer, using appropriate precession corrections are:

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \tag{1.24}$$

where A is the precession matrix from Eq. (1.11), but the parameter t in Eqs. (1.15) is:

$$t = (\text{ TT} - \text{EKV})/36525, \tag{1.25}$$

where EKV is the epoch of the coordinates in time units (Julian Date) that can be generally different from J2000.0 and ET is the Julian Ephemeris Date. For entireness I again remind that the program use precession quantities of Capitaine et. al. ([1]).

The Earth velocity components computed from the JPL's ephemerides are related to the epoch J2000.0, therefore they have to be precessed from the epoch J2000.0 to the time of the observation:

$$\begin{pmatrix} V_x^{orb} \\ V_y^{orb} \\ V_z^{orb} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} V_x^{JPL} \\ V_y^{JPL} \\ V_z^{JPL} \end{pmatrix}, \tag{1.26}$$

where V^{JPL} are velocities from JPL's ephemerides and the parameter t is the same as in Eq. (1.16). Finally the barycentric correction RV_{corr} of the radial velocity is:

$$RV_{corr} = V_x p_1 + V_y p_2 + V_z p_3 \tag{1.27}$$

with V_x, V_y and V_z defined in Eqs. (1.18).

1.6 Time correction

The barycentric coordinates of the Earth in the equatorial coordinate system computed from the JPL Planetary and Lunar Ephemerides are related to the epoch J2000.0 as well as the Earth velocity components. Hence one only has to beware of the epoch of the coordinates α and δ. In case of the epoch other than J2000.0 one has to precess the star coordinates from the arbitrary epoch to the epoch J2000.0:

$$\begin{pmatrix} x_{2000} \\ y_{2000} \\ z_{2000} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{1.28}$$

The formula is similar as in Eq. (1.24), but now the parameter t is:

$$t = (2000 - \mathrm{EKV})/36525 \tag{1.29}$$

with both J2000 and EKV expressed in Julian Dates.

Then the barycentric correction of the time is:

$$T_c = L(C_x x_{2000} + C_y y_{2000} + C_z z_{2000})\ (\mathrm{d}), \tag{1.30}$$

where C_x, C_y and C_z are individual barycentric coordinates of the Earth computed from the JPL's ephemerides and L is the time in which the light exceeds one Astronomical Unit in vacuum, expressed in days:

$$L = \frac{499.004782}{86400}\ (\mathrm{d}). \tag{1.31}$$

1.7 Others

Computation of the Geocentric Julian Date is done with use of the modified function GEO written by J. Vondrák in 2001.

Positions and velocities obtained from the JPL Planetary and Lunar Ephemerides DE405 are computed for the given Julian Ephemeris Date JED, which is based on the dynamic Terrestrial time and is defined:

$$\mathrm{JED} = \mathrm{TAI} + \frac{32,184}{86400} = \mathrm{UT1} + \triangle\mathrm{T} + \frac{32,184}{86400}\ (\mathrm{d}), \tag{1.32}$$

where TAI is the Atomic time and $\triangle$T is a correction resulting from a non–uniformity of the Earth rotation. It is the difference between the uniformly passing time TT and the time UT1, which is the proper rotational time of the Earth. $\triangle$T is determined from observations of the International Earth Rotation Service. UTC is the coordinated Universal time UT and the difference $\mathrm{TAI} - \mathrm{UTC} = n$ is an integer number that means a number of seconds of the time difference. The value n serves to save the difference $|\mathrm{UT1} - \mathrm{UTC}| < 0.8$ s by adding leap seconds.

In a determination of $\triangle$T in the Julian Ephemeris Time in Eq. (1.32) rises maximum error 0.15 s, what makes maximum error in the Barycentric Julian Date 0.000002 d, although the real error is even smaller. For more details see the end of the next section.

1.8 Program tests and error estimates

	α (h)								
δ (°)	0	4	8	12	16	20	21	22	23
90	35 6/1985	39 7/1992	34 7/1992	36 7/1992	36 7/1992	37 7/1992	35 7/1992	39 7/1992	37 7/1992
-90	44 12/1994	43 12/1994	47 12/1994	47 12/1994	46 12/1994	45 12/1994	47 12/1994	42 12/1994	46 12/1994
70	33 5/2003	44 7/1992	46 8/2002	44 8/2002	25 8/2002	23 3/1998	25 3/1998	28 4/2002	31 5/2003
-70	51 2/1996	31 2/1996	35 12/1994	53 12/1994	64 12/1994	58 12/1994	56 2/1996	56 2/1996	54 2/1996
50	49 5/2002	52 5/1991	57 8/2002	59 9/2002	40 10/1997	38 1/1991	39 4/2002	45 4/2002	48 4/2002
-50	63 3/1988	32 2/2006	36 8/1994	59 11/2000	73 12/2000	69 2/1988	70 2/1988	69 2/1996	66 3/1988
30	61 5/2002	60 5/1991	61 8/2000	70 9/2002	51 10/1997	54 1/1991	55 2/1988	60 3/1988	61 4/2002
-30	74 3/1988	47 7/2000	51 8/2000	62 11/2000	77 12/2000	79 2/1988	80 2/1988	78 3/1988	78 3/1988
10	70 4/1988	61 7/2000	66 8/2000	72 9/2002	66 12/2002	71 2/1988	72 2/1988	75 3/1988	75 3/1988
-10	77 3/1988	58 7/2000	63 8/2000	65 9/2002	74 12/2002	79 2/1988	80 2/1988	81 3/1988	82 3/1988

Table 1.2: The program testing. For right ascension α in the range from $0-24$ h and declination δ in the range from $-90-90°$ there are two numbers. The upwards is the maximum difference in radial–velocity corrections computed by programs BarCor and Brvel, in cm s^{-1}. The bottom number is the date when the difference between the two programs attained maximum.

First let us say something about the programs and theories for computations the barycentric corrections. In fact there are two approaches how to compute these corrections, either the analytical theory using developments of series of many terms or numerical integration of N–body system.

Among the analytical theories there is the widely used program Brvel written by S. Yang & J. Amor in 1984. They used procedures written by Stumpff (see [9], [10], [11]). Another independent procedure using the analytical theory is the procedure AABER1 written by C. Ron & J. Vondrák in 1986 for the computation of the Earth barycentric velocity components (see [5]). There might be other programs and procedures, which are used locally at some observatories and unpublished.

The most expanded and also the most accurate theory using the numerical integration of N–body system is already mentioned JPL Planetary and Lunar Ephemerides. The most accurate is their latest ephemeris DE405, but also JPL's former ephemerides (DE200, DE96 etc.) are very accurate.

Theory	Program	Accuracy (cm s^{-1})
Analytical theory	Brvel	42
	AABER1	17
N–body integration, JPL/DE405	BarCor	1–2

Table 1.3: The accuracy of the Earth velocity components for different programs and theories.

Let us compare the accuracy of the above–mentioned theories. In Table 1.3 there is a comparison of the accuracy of the Earth velocity components for individual programs. Stumpff ([11]) compared the Earth barycentric velocity components with the JPL's ephemeris DE96 and he found the maximum error 42 cm s^{-1}. Ron & Vondrák ([5]) compared their velocity components with the JPL's ephemeris DE200 and they declared that the maximum error does not exceed 17 cm s^{-1}. That sounds pretty good, but for the precise measurements requiring high accuracy this is not enough. Thus what can we do? It is quite simple: we can use the JPL's Planetary and Lunar Ephemerides with the accuracy of the velocity components about few cm s^{-1}. Such accuracy is especially achieved by the continuous measurements of the Earth–Moon distance by the laser telemeter, with the accuracy of 1–2 cm.

To find out more details about the accuracy of the radial–velocity corrections I did detailed study of the comparison of the two programs: BarCor and Brvel (mentioned above). The results are displayed in Table 1.2 and my technique was following: for each right ascension α in the range from $0 - 24$ h, declination δ in the range from $-90 - 90°$, furthermore for the first day of each month (from January 1d 0h to December 1d 0h) and for each year between 1980–2006, I computed the radial–velocity correction using of both programs BarCor and Brvel. The two numbers in Table 1.2 for each α and δ are the maximum difference in radial–velocity corrections between the two programs (the upwards number) and the date of the maximum difference (the bottom number). All radial–velocity corrections were computed for the Observatory Ondřejov in the Czech Republic (longitude

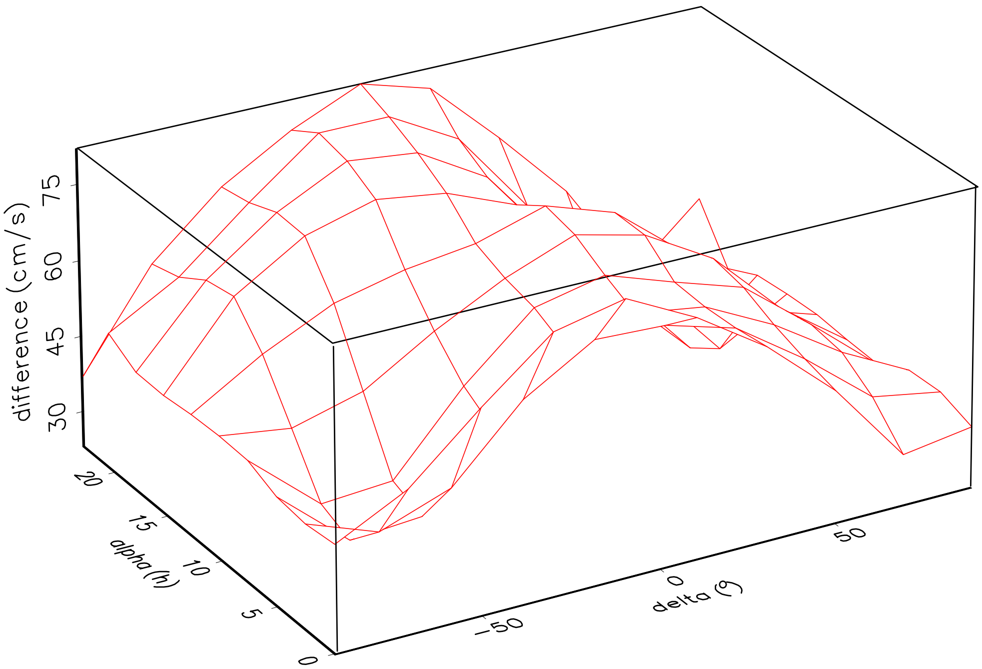

Figure 1.3: The graphical representation of the differences in radial–velocity corrections between the programs BarCor and Brvel.

0h 59m 8.1s, latitude $+49°54'38''$, altitude 528 m) and for the epoch of the coordinates J2000.0. In the epoch B1950.0 the differences between the two programs appears even smaller.

In Fig. 1.3 there is the graphical representation of the differences in radial–velocity corrections between the programs BarCor and Brvel. One can see that the maximum difference occurs near the celestial equator (declination close to $-10°$) and right ascension near 22–23 h. These differences correspond to the error estimates by Stumpff ([11]). He found the maximum error of the velocity components from his procedure compared with the JPL's ephemeris DE96 42 cm s^{-1}, what amounts errors in the radial–velocity corrections similar to those in Table 1.2. The differences in right ascension $\alpha = 0$ h are of course the same as in 24h. But the intent reader could notice that there is something strange with the differences at the declination δ in $-90°$ and $90°$. The differences should be obviously the same for all values of right ascension, because it is always the same point – the Earth north and south pole! It is encouraging that the problem is not in BarCor, but in Brvel. Namely BarCor gives for the Earth pole every time the same value of the radial–velocity correction, but Brvel not.

Hereafter, I made following findings: when I changed either of the Earth velocity components from the JPL by the number 42 cm s^{-1} and then computed the radial–velocity correction with use of the program BarCor, the differences in corrections of the programs BarCor and Brvel were significantly higher or lower (depending on whether 42 cm s^{-1} was added or subtracted) than those in Table 1.2. This could be also the check of the program accuracy.

Further I compared the Barycentric Julian Dates computed by the programs BarCor and Brvel. The maximum difference was 0.00000031 d, but in most cases the difference did not exceed 0.0000001 d.

One would also like to see the differences in the radial–velocity corrections between the program BarCor and the procedure AABER1, but the procedure computes only the Earth velocity components. That is why there is no detail comparison. But to be perfect the maximum difference between the velocity components of the two was about 19 cm s^{-1}. This corresponds to the assumptive errors in AABER1 17 cm s^{-1}. The reader should then realize that errors in the radial–velocity corrections are anymore higher.

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